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QUANTIFYING IRREVERSIBILITY AT THE QUANTUM LEVEL

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Quantum thermodynamics

quantum transport

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September 06th, 2018

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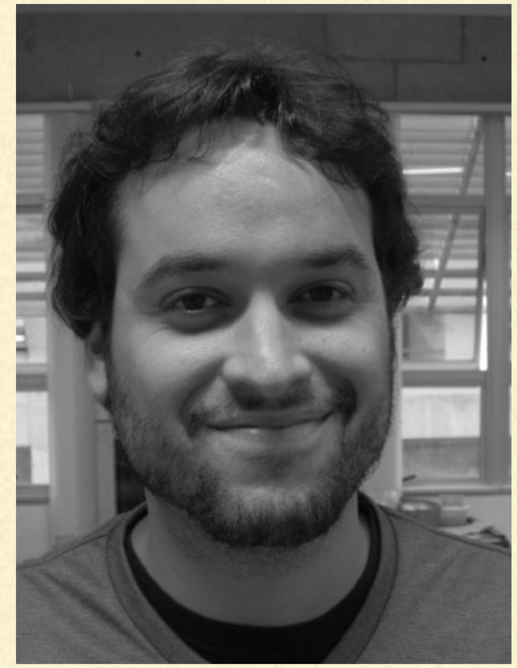

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IRREVERSIBILITY

- Consider a system *S* connected to an environment *E*, undergoing some process.
- Information about S is diluted in the environment and some part (or all of it) may never return.
- Irreversibility ≔ the irretrievable loss of *any* resource.

ENTROPY PRODUCTION

- In thermodynamics the resources are heat and work, and irreversibility is quantified using the *entropy production*.

(Clausius inequality) $$\Delta S \geq \frac{\delta Q}{T} \qquad \longrightarrow \qquad \Sigma := \Delta S - \frac{\delta Q}{T} \geq 0$$

(entropy flux)

(entropy production)

- We also express this in terms of rates:

$$\Pi = \frac{d\Sigma}{dt} \qquad \boxed{\frac{dS}{dt} = \Pi - \Phi} \qquad \Phi = -\frac{1}{T}\frac{dQ}{dt}$$

(entropy production *rate*)

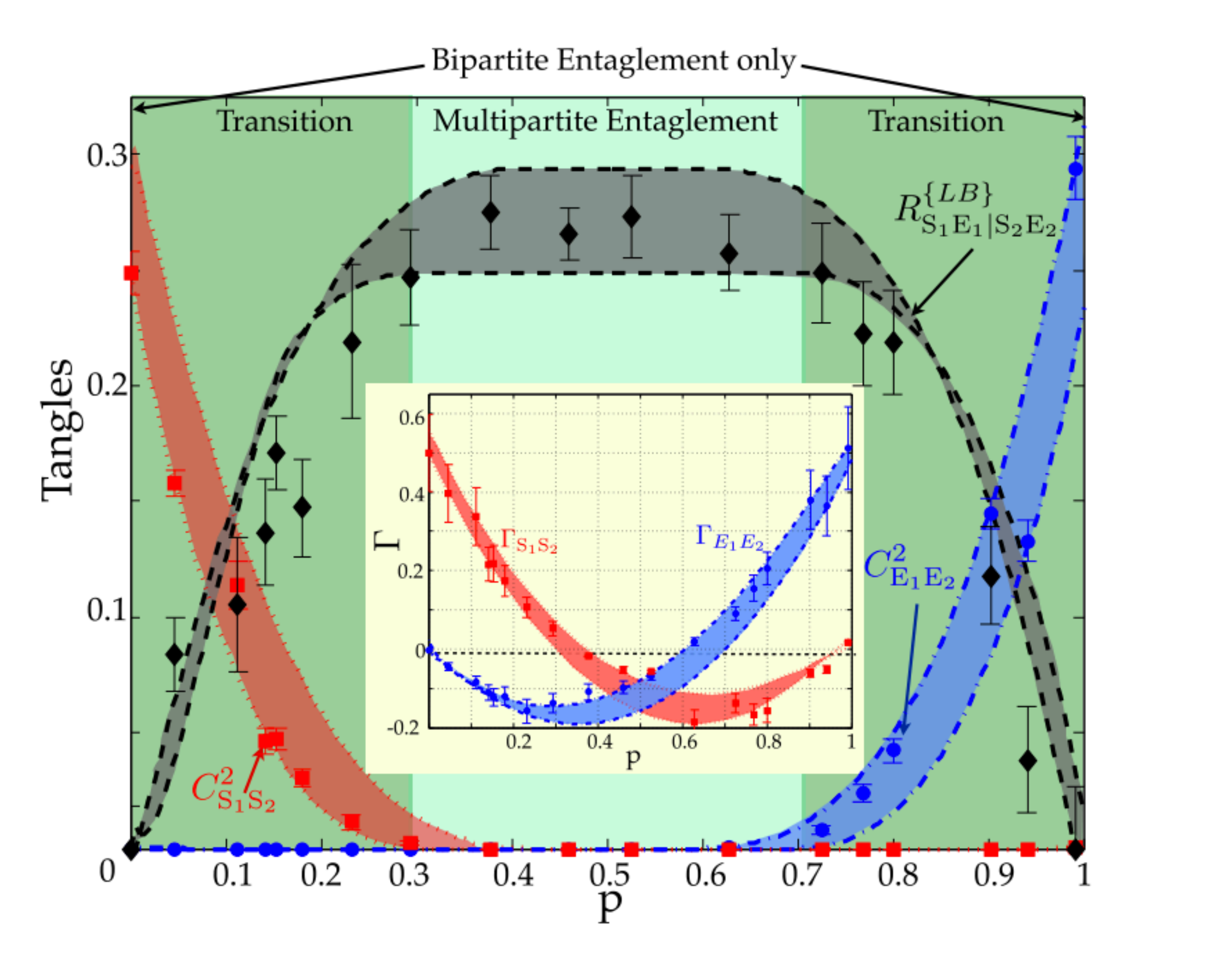
(entropy flux *rate*)

WHAT IS DIFFERENT IN QUANTUM SYSTEMS?

WHAT IS DIFFERENT IN QUANTUM SYSTEMS?

1. In quantum systems there are also other resources, such as entanglement and coherence.

   - They are also irretrievably lost due to the contact with the environment.



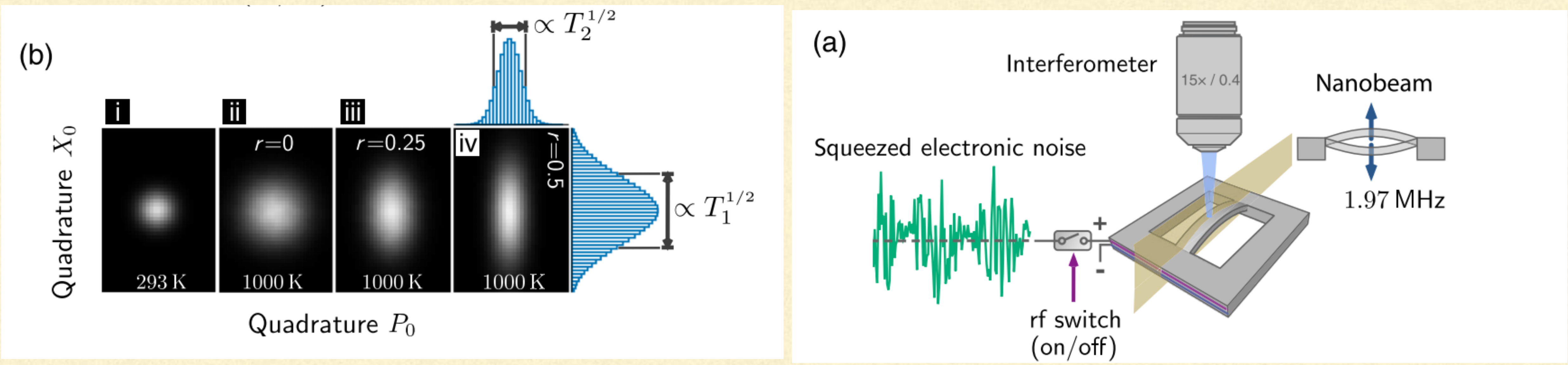
Aguilar, Valdés-Hernández, Davidovich, Walborn, Souto Ribeiro, *Phys. Rev. Lett*, **113**, 240501 (2014)

WHAT IS DIFFERENT IN QUANTUM SYSTEMS?

2. We are no longer restricted to equilibrium baths.

   - It is possible to work with *engineered environments*.

- Example: *squeezed thermal bath:*



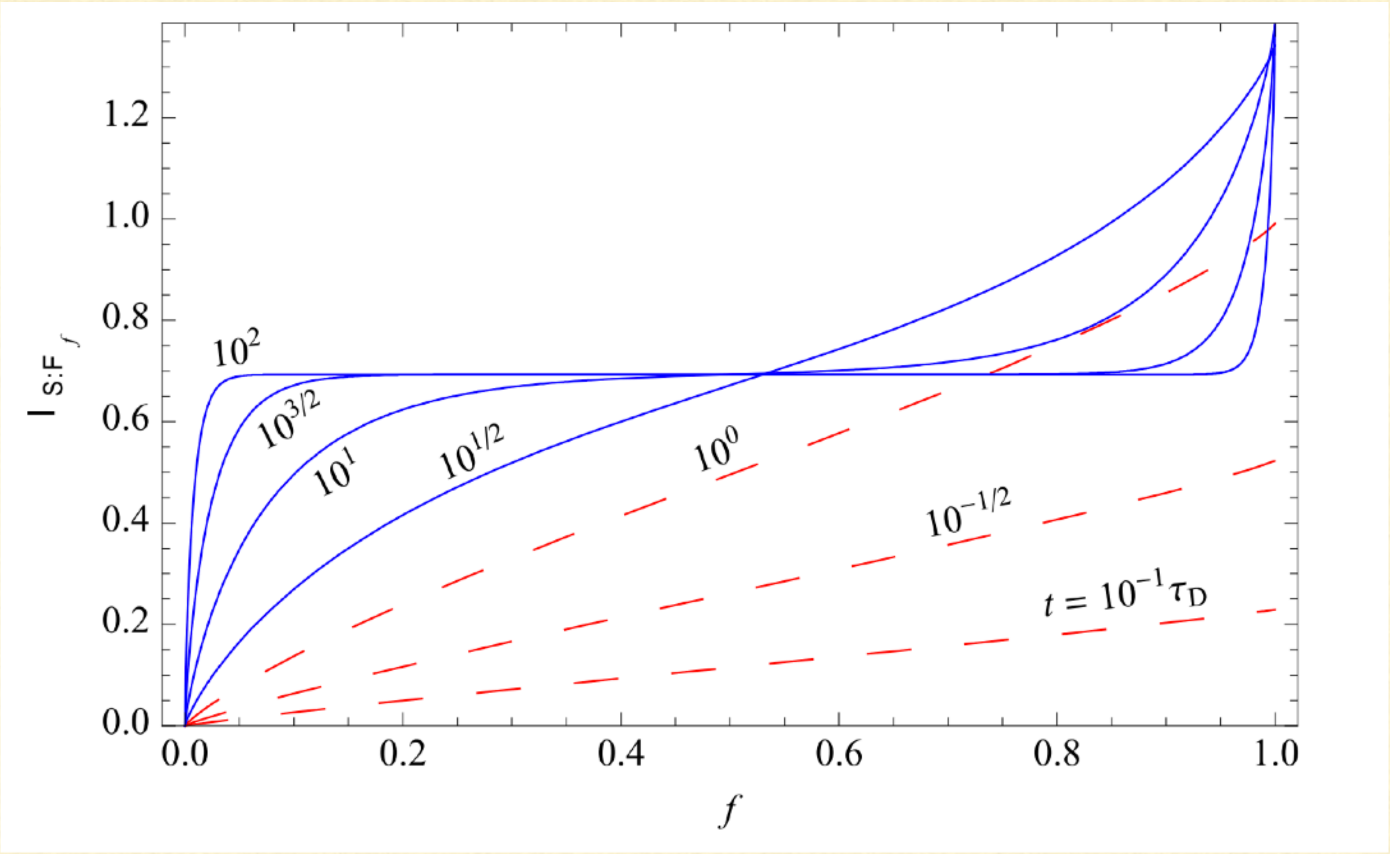
Klaers, Faelt, Imamoglu, Togan, *Phys. Rev. X*, **7**, 031044 (2017)

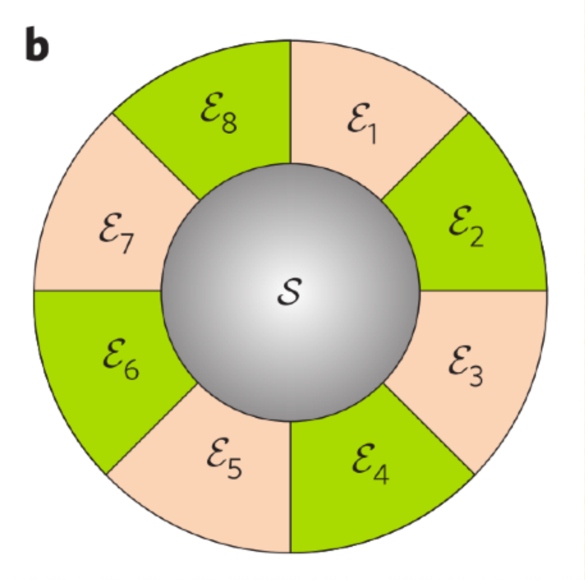
Move beyond the standard paradigms of thermodynamics.

WHAT IS DIFFERENT IN QUANTUM SYSTEMS?

3. Information becomes an essential concept:

   - *"The fragility of states makes quantum systems very difficult to isolate. Transfer of information (which has no effect on classical states) has marked consequences in the quantum realm. So, whereas fundamental problems of classical physics were always solved in isolation (it sufficed to prevent energy loss), this is not so in quantum physics (leaks of information are much harder to plug)."*

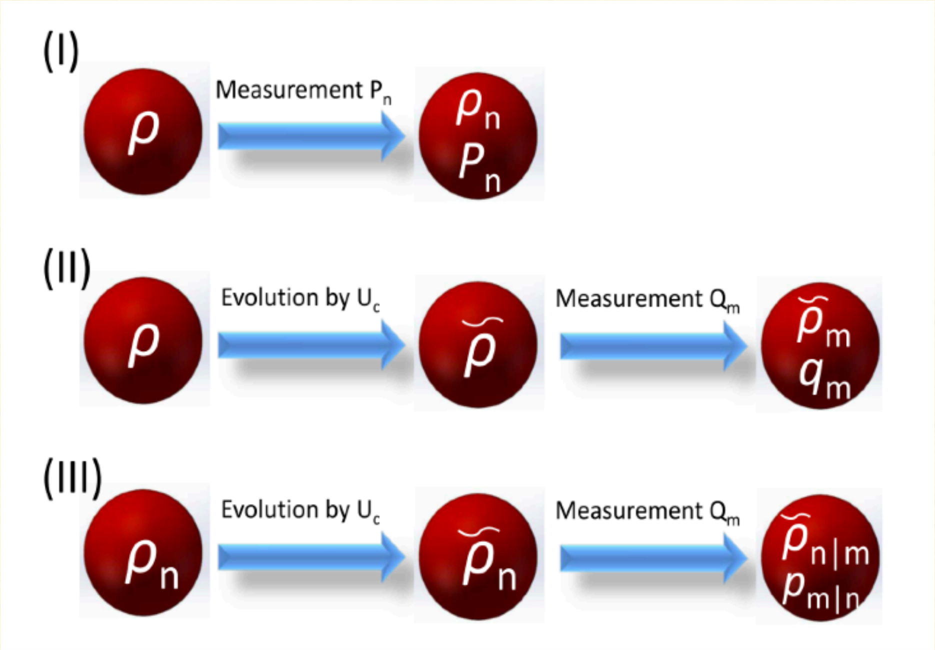




W. J. Zurek,
Nature Physics, **5**, 181 (2009)

WHAT IS DIFFERENT IN QUANTUM SYSTEMS?

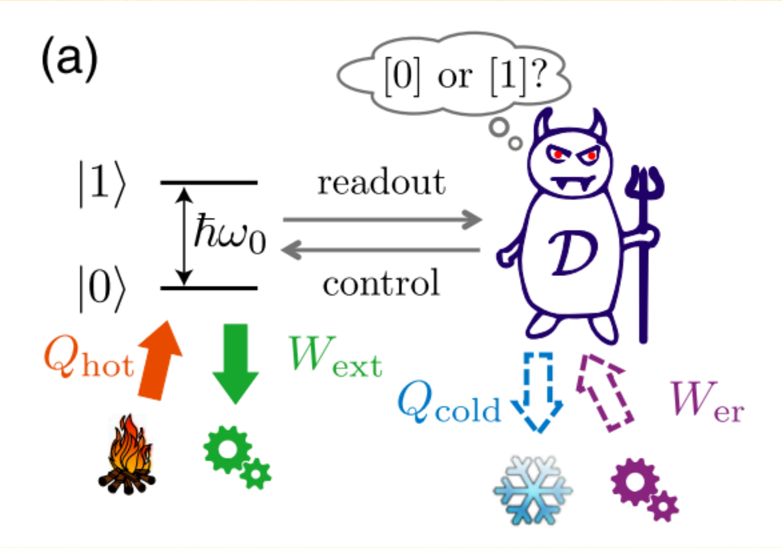
4. Measurement plays a central role:



Xiong, *et. al.*, *Phys. Rev. Lett.* **120**, 010601 (2018)

- Processes depend on deltas.
- Back-action (state collapse) affects how we extract thermodynamic information.

- Measurements can be directly implemented in thermodynamic engines.
- Maxwell's demons and information engines.



Elouard, Herrera-Martí, Huard, Auffèves, *Phys. Rev. Lett*, **118**, 260603 (2017)

CLASSICAL VS. QUANTUM MASTER EQUATIONS

Jader P. Santos, Lucas C. Céleri, Gabriel T. Landi and Mauro Paternostro
The role of quantum coherence in non-equilibrium entropy production
arXiv 1707.08946 (submitted to *Nature Quantum Information*)

- Consider a system with discrete energy levels and let p_n denote de probability of being found in state *n*.

- In a classical approach, the dynamics of the system in contact with a bath would be described by a Pauli master equation:

$$\frac{dp_n}{dt} = \sum_m \left\{ W(n|m)p_m - W(m|n)p_n \right\}$$

- Let us assume the steady-state is thermal equilibrium $\quad p_n^{\text{eq}} = \frac{e^{-\beta E_n}}{Z}$

- Using the Shannon entropy, Schnakenberg proposed the following expression for the entropy production [Rev. Mod. Phys., **48**, 571 (1976)].

$$\Pi = -\frac{dS(\mathbf{p}(t)||\mathbf{p}^{\text{eq}})}{dt} \qquad S(\mathbf{p}||\mathbf{p}^{\text{eq}}) = \sum_n p_n \ln p_n / p_n^{\text{eq}}$$

(relative entropy)

Π due to system adapting to new population imposed by the bath.

QUANTUM MASTER EQUATION

- Now consider a quantum master equation: $\dfrac{d\rho}{dt} = -i[H,\rho] + D(\rho)$
- This equation will describe the evolution of both populations *and* coherences.
- e.g.: $D(\rho) = \gamma(1-f)\left[\sigma_-\rho\sigma_+ - \frac{1}{2}\{\sigma_+\sigma_-,\rho\}\right] + \gamma f\left[\sigma_+\rho\sigma_- - \frac{1}{2}\{\sigma_-\sigma_+,\rho\}\right]$

$$f = \frac{1}{e^{\beta\Omega}+1}$$

$$\rho = \begin{pmatrix} p_0 & q \\ q^* & p_1 \end{pmatrix}$$

$$\frac{dp_0}{dt} = \gamma f p_1 - \gamma(1-f)p_0$$

$$\frac{dp_1}{dt} = \gamma(1-f)p_0 - \gamma f p_1$$

(Pauli master equation)

$$\frac{dq}{dt} = -\frac{\gamma}{2}q$$

ENTROPY PRODUCTION

- Here we consider **Thermal Operations** (or Davies maps), which have simple thermal properties.
  - Thermalize correctly.
  - Populations evolve according to classical M Eq.
- The entropy flux does not depend on the coherences: $$\Phi = -\frac{1}{T}\frac{dQ}{dt}$$
- But the entropy production, on the other hand, becomes

$$\boxed{\Pi = -\frac{dS(\rho||\rho_{\text{eq}})}{dt}}$$

$$S(\rho||\rho_{\text{eq}}) = \text{tr}\Big\{\rho(\ln\rho - \ln\rho_{\text{eq}})\Big\}$$

GLOBAL UNITARY DYNAMICS

- We can instead think about entropy production in terms of the *global* unitary dynamics of S+E. Then one may show that

$$\Pi = -\frac{d\mathcal{I}_{SE}}{dt} + \frac{dS(\rho_E(t)||\rho_E^{\text{th}})}{dt}$$

- Thus, entropy production stems from:

1. *Mutual information* built up between S and E that is lost.

2. The state of the environment being pushed away from equilibrium.

1707.08946 and 1804.02970
see also: M. Esposito, K. Lindenberg, and C. Van Den Broeck, NJP **12**, 013013 (2010).

CONTRIBUTION FROM QUANTUM COHERENCES

- But now we can separate: $S(\rho||\rho_{\rm eq}) = S(\mathbf{p}||\mathbf{p}^{\rm eq}) + \mathcal{C}(\rho)$

  $\mathcal{C}(\rho) = S(\Delta_H(\rho)) - S(\rho)$ (Entropy of coherence)

- As a result, we find that the entropy production can be divided in two parts:

$$\boxed{\Pi = -\frac{dS(\mathbf{p}(t)||\mathbf{p}^{\rm eq})}{dt} - \frac{\mathcal{C}(\rho)}{dt}}$$

- One part is the classical: entropy production due to population change.
- But the other is genuinely quantum mechanical:
  - Entropy production due to loss of coherence.

QUANTUM PHASE SPACE FORMULATION

Jader P. Santos, Gabriel T. Landi and Mauro Paternostro
The Wigner entropy production rate
Phys. Rev. Lett, **118**, 220601 (2017)

- We shall consider the relaxation of a bosonic mode in contact with a bath:

$$\frac{d\rho}{dt} = -i[H, \rho] + D(\rho)$$

$$H = \omega(a^\dagger a + 1/2)$$

$$\boxed{D(\rho) = \gamma(\bar{n}+1)\left[a\rho a^\dagger - \frac{1}{2}\{a^\dagger a, \rho\}\right] + \gamma\bar{n}\left[a^\dagger \rho a - \frac{1}{2}\{a a^\dagger, \rho\}\right]}$$

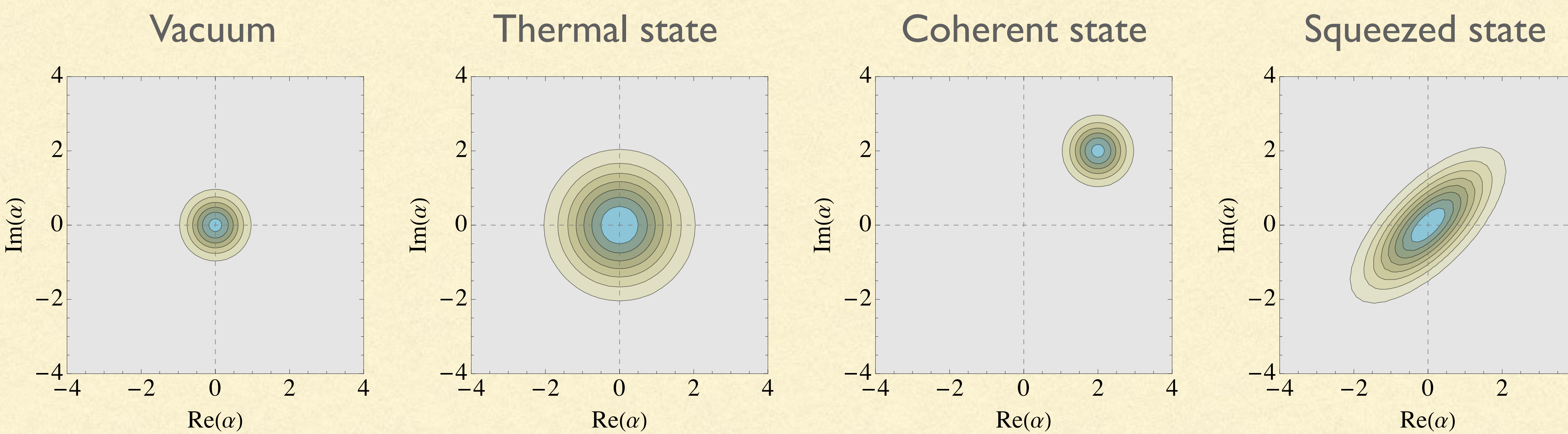
$$\bar{n} = \frac{1}{e^{\beta\omega} - 1}$$

- Instead of working with density matrices, we work with the **Wigner function:**

$$W(\alpha,\alpha^*) = \frac{1}{\pi^2}\int d^2\lambda e^{\lambda^*\alpha-\lambda\alpha^*}\mathrm{tr}\left\{\rho e^{\lambda a^\dagger-\lambda^* a}\right\}$$

$$\mathrm{Re}(\alpha) = q$$
$$\mathrm{Im}(\alpha) = p$$

Introduces the notion of quantum phase space



- The Wigner function will satisfy a *Quantum Fokker-Planck equation*

$$\partial_t W = \partial_\alpha J(W) + \partial_\alpha^* J^*(W)$$

$$J(W) = \frac{\gamma}{2}\Big[\alpha W + (\bar{n} + 1/2)\partial_{\alpha^*} W\Big]$$

- This is a *continuity equation* for the quasi-probability.

- J(W) is a probability current.

- The current is zero if and only if the system is in thermal equilibrium.

$$J(W_{\text{eq}}) = 0 \qquad\qquad W_{\text{eq}}(\alpha, \alpha^*) = \frac{1}{\pi(\bar{n} + 1/2)} e^{-\frac{|\alpha|^2}{\bar{n}+1/2}}$$

WIGNER ENTROPY

- Instead of using the von Neumann entropy, we adopt instead the entropy of the Wigner function:

$$S = -\int d^2\alpha \, W \ln W$$

- This will be real for Gaussian states (because then W > 0).

- Moreover, it coincides with the Rényi-2 entropy

$$S_2 = -\ln \mathrm{tr}\rho^2$$

Adesso, Girolami, Serafini, *PRL*, **109**, 190502 (2012)

- With the Wigner entropy we can now separate

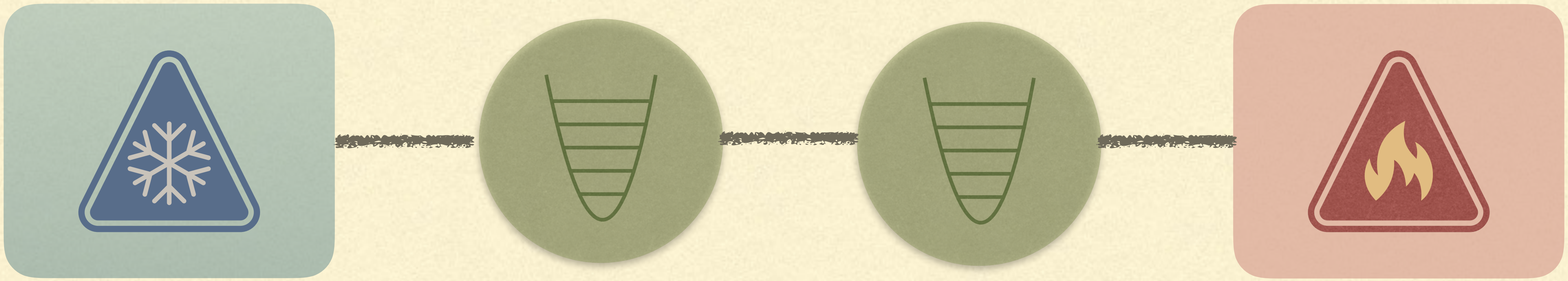
$$\frac{dS}{dt} = \Pi - \Phi$$

- As a result, we find

$$\Pi = \frac{4}{\gamma(n+1/2)} \int d^2\alpha \, \frac{|J(W)|^2}{W} = -\frac{dS(W||W_{\text{eq}})}{dt}$$

$$\Phi = \frac{\gamma}{n+1/2}\left[\langle a^\dagger a\rangle - n\right] = -\frac{1}{\omega(n+1/2)}\frac{dQ}{dt}$$

- At high temperatures $\omega(n+1/2) \simeq T$ which leads to $\Phi \simeq -\frac{1}{T}\frac{dQ}{dt}$



NON-EQUILIBRIUM STEADY-STATES (NESS)

William B. Malouf, Jader P. Santos, Mauro Paternostro and Gabriel T. Landi
Wigner entropy production in quantum non-equilibrium steady-states
In preparation *(2018)*.

- We now consider a Fokker-Planck equation of the form

$$\partial_t W = \mathcal{U}(W) + \sum_{k=1}^{N} \partial_{\alpha_k} J_k(W) + \partial_{\alpha_k^*} J_k^*(W)$$

- where *U*(W) is the unitary interaction between subsystems.
- The evolution of the entropy then becomes

$$\frac{dS}{dt} = \sum_k (\Pi_k - \Phi_k)$$

- Each term describes the production and flux to each dissipation channel.

$$\Pi_k = \frac{4}{\gamma_k(\bar{n}_k + 1/2)} \int d^2\alpha_1 \ldots d^2\alpha_N \, \frac{|J_k(W)|^2}{W}$$

$$\Phi_k = \frac{\gamma_k}{\bar{n}_k + 1/2} \left[\langle a_k^\dagger a_k \rangle - \bar{n}_k \right]$$

- In the case of the NESS, the unitary interactions play an essential role:

$$\Pi = -\frac{dS(W||W_{\text{eq}})}{dt} - \int U(W) \ln W_{\text{eq}}$$

- In the NESS the first term is zero and only the second survives.

- Unitary part is required to allow a current to flow from one bath to another.

- In the NESS we obtain an Onsager expression:

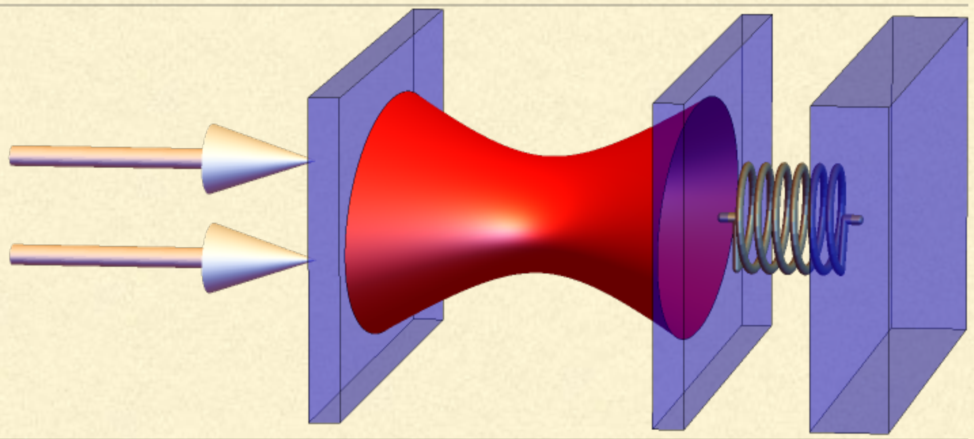
$$\Pi_{\text{NESS}} = \sum_{k,\ell} \mathcal{J}_{k,\ell} \left(\frac{1}{\bar{n}_k + 1/2)} - \frac{1}{\bar{n}_\ell + 1/2} \right)$$

- Where $\mathcal{J}_{k,\ell}$ is the current from *k* to ℓ

PHYSICAL IMPLEMENTATIONS

M. Brunelli, L. Fusco, R. Landig, W. Wieczorek, J. Hoelscher-Obermaier, G. T. Landi, F Semião, A. Ferraro, N. Kiesel, T. Donner, G. De Chiara, and M. Paternostro
Measurement of irreversible entropy production in mesoscopic quantum systems out of equilibrium.
arXiv 1602.06958.
To appear in Phys. Rev. Lett. 2018.

OPTOMECHANICS

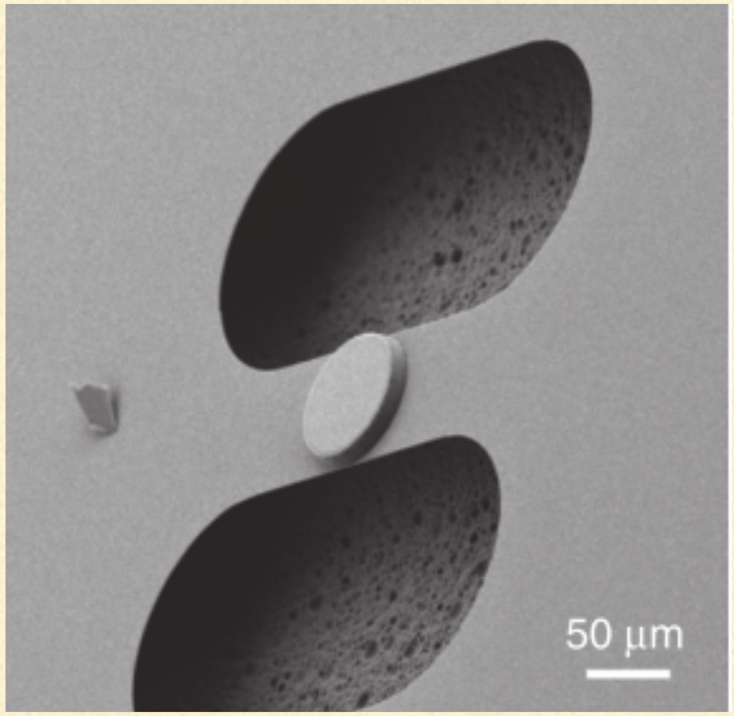


- A thin membrane is allowed to vibrate in contact with radiation trapped in a cavity.

$$H = \omega_c a^\dagger a + \left(\frac{p^2}{2m} + \frac{1}{2} m \omega_m^2 x^2 \right)$$

$$-g a^\dagger a x + \epsilon (a^\dagger e^{-i\omega_p t} + a e^{i\omega_p t})$$

$$\frac{d\rho}{dt} = -i[H, \rho] + D_c(\rho) + D_m(\rho)$$



Aspelmeyer group
Viena

Groeblacher, *et. al.*, Nature Communications, **6**, 7606 (2015)

$$\frac{d\rho}{dt} = -i[H,\rho] + D_c(\rho) + D_m(\rho)$$

- The system tends to a NESS because there are two dissipation channels.
- The mechanical oscillator has the usual damping:

$$D_m(\rho) = \gamma(n_m+1)\left[b\rho b^\dagger - \frac{1}{2}\{b^\dagger b,\rho\}\right] + \gamma n_m\left[b^\dagger\rho b - \frac{1}{2}\{bb^\dagger,\rho\}\right]$$

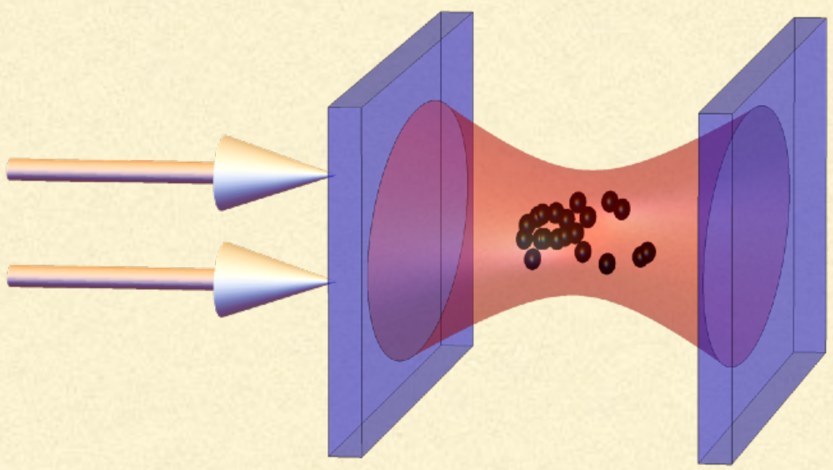
- where γ is the coupling rate to the environment and $n_m = \dfrac{1}{e^{\omega_m/T}-1}$
- On the other hand, the cavity can also loose photons (this is how they measure the cavity), which is described by

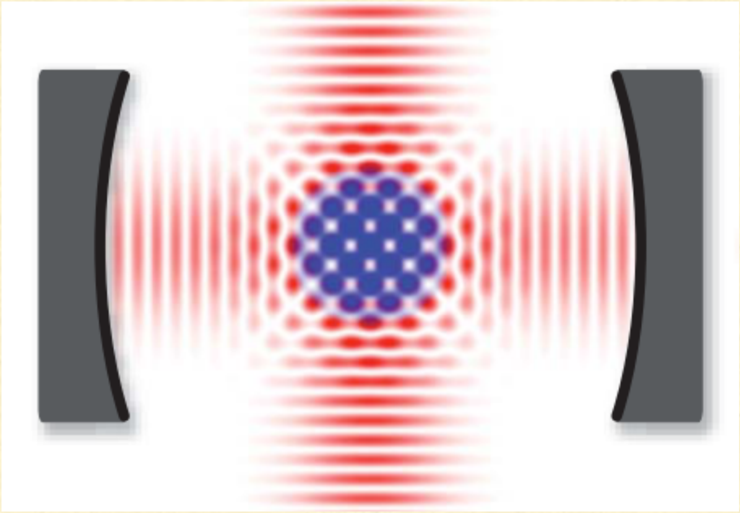
$$D_c(\rho) = 2\kappa\left[a\rho a^\dagger - \frac{1}{2}\{a^\dagger a,\rho\}\right]$$

DRIVEN-DISSIPATIVE BEC

Esslinger group
ETH

- Another interesting quantum NESS is that of a BEC interacting with a cavity field.





b_0 and b_1 are bosonic operators of the ground-state and first excited state of the BEC

$$H = \omega_c a^\dagger a + \frac{\omega_0}{2}(b_1^\dagger b_1 - b_0^\dagger b_0) + \frac{2\lambda}{\sqrt{N}}(a + a^\dagger)(b_0^\dagger b_1 + b_1^\dagger b_0)$$

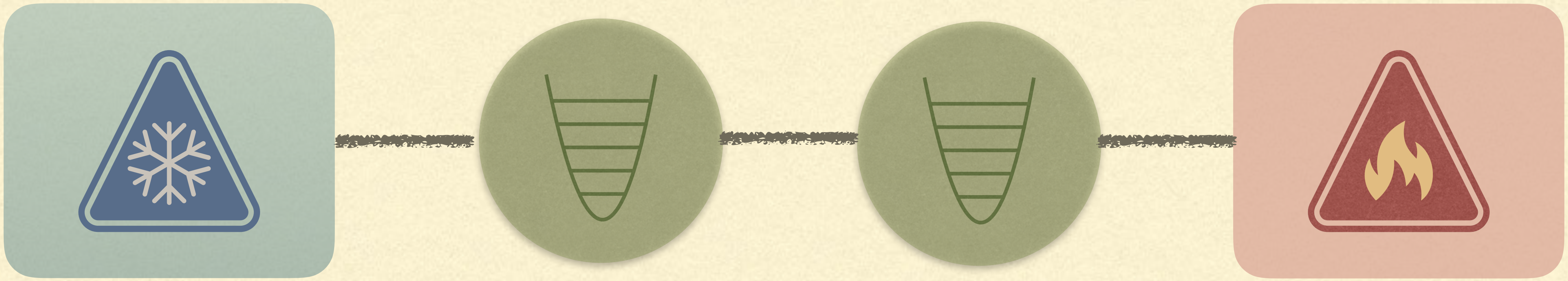
Baumann, *et. al.*,Nature, **464**,1301 (2010)

GAUSSIANIZATION

- Both models can be Gaussianized for large drive and converted into an effective system of two harmonic oscillators

$$H = \omega_a a^\dagger a + \omega_b b^\dagger b + g(a + a^\dagger)(b + b^\dagger)$$

- For the BEC, the mode *b* is a collective (Schwinger) mode of the atomic system.

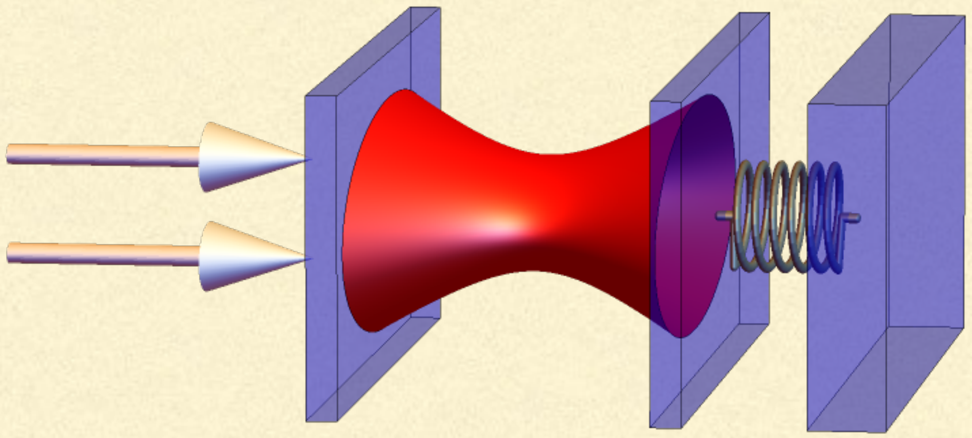


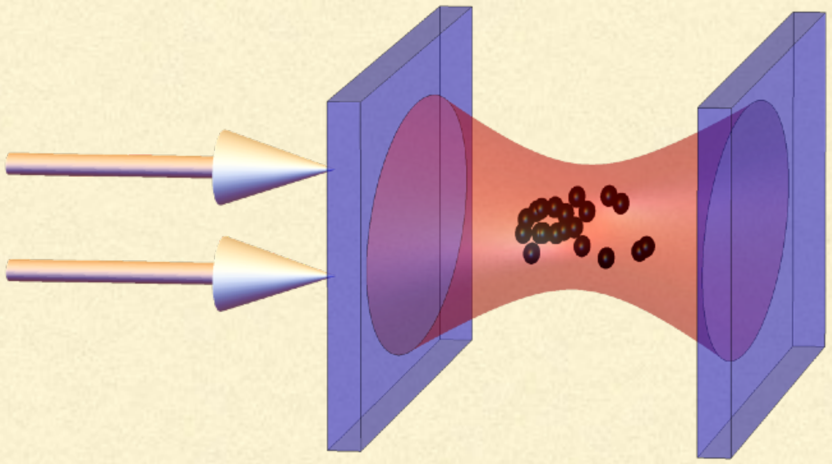
TROUBLE @T = 0

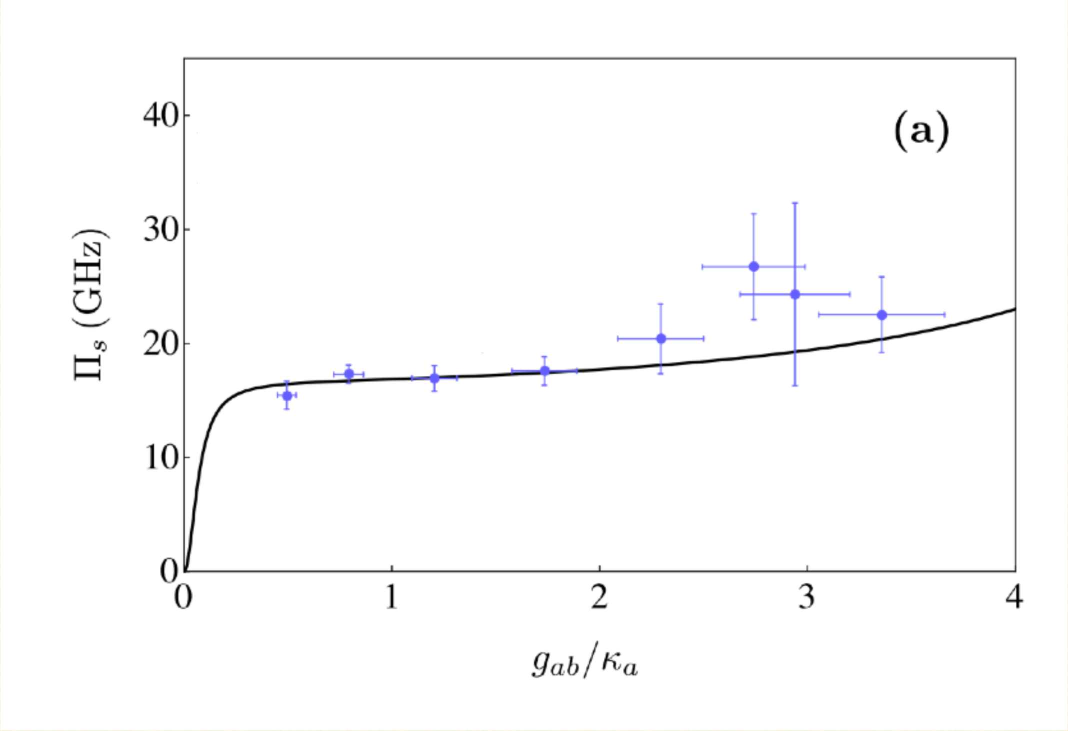
$$D_c(\rho) = 2\kappa\left[a\rho a^\dagger - \frac{1}{2}\{a^\dagger a, \rho\}\right]$$

- Both models clearly correspond tend to a quantum NESS.
- However, in both cases one of the reservoirs is the photon loss bath.
  - which behaves exactly as a thermal bath at zero temperature.
- The von Neumann description of breaks down at T = 0.
  - Both production and flux diverge.
- The Wigner framework is therefore *the* only framework capable of describing entropy production in these systems.

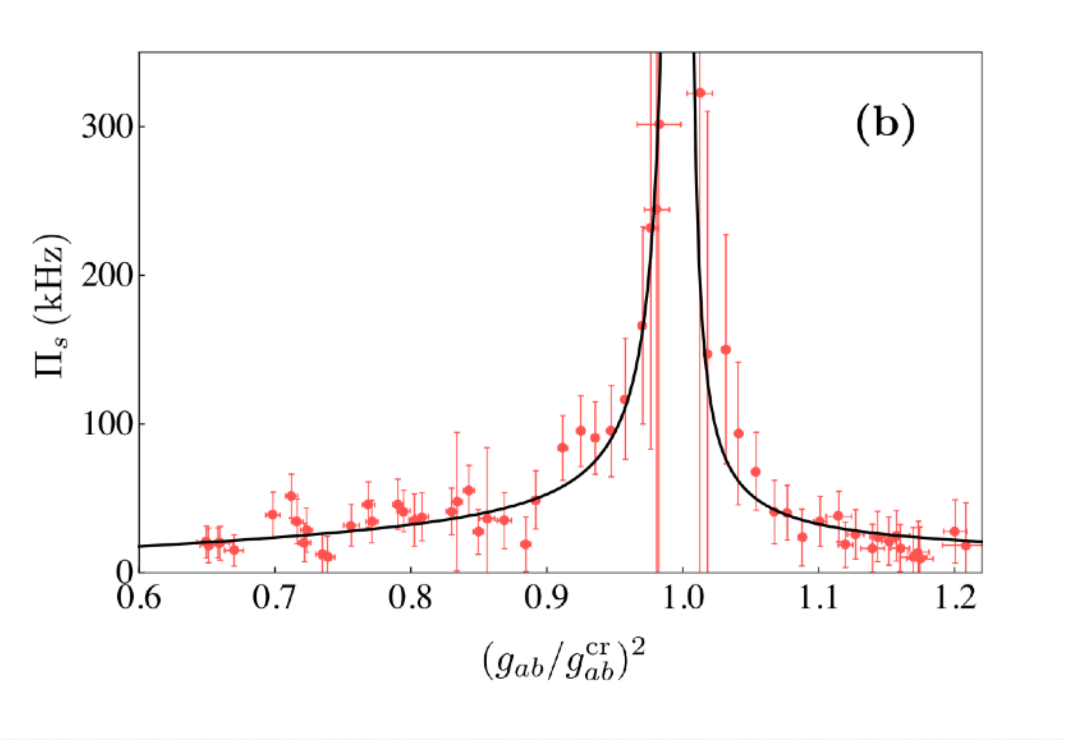
RESULTS







optomechanics



BEC

IRREVERSIBILITY FROM THE PERSPECTIVE OF THE ENVIRONMENT

Jader P. Santos, Alberto L. de Paula, Raphael Drumond, Gabriel T. Landi and Mauro Paternostro

Irreversibility at zero temperature from the perspective of the environment.

arXiv 1804.02970
Phys. Rev. A. Rapid Communications, ***97****, 050101 (2018).*

- We have also studied models of exactly soluble open system dynamics.

$$H = \omega a^\dagger a + \sum_k \Omega_k b_k^\dagger b_k + \sum_k \lambda_k (a^\dagger b_k + b_k^\dagger a)$$

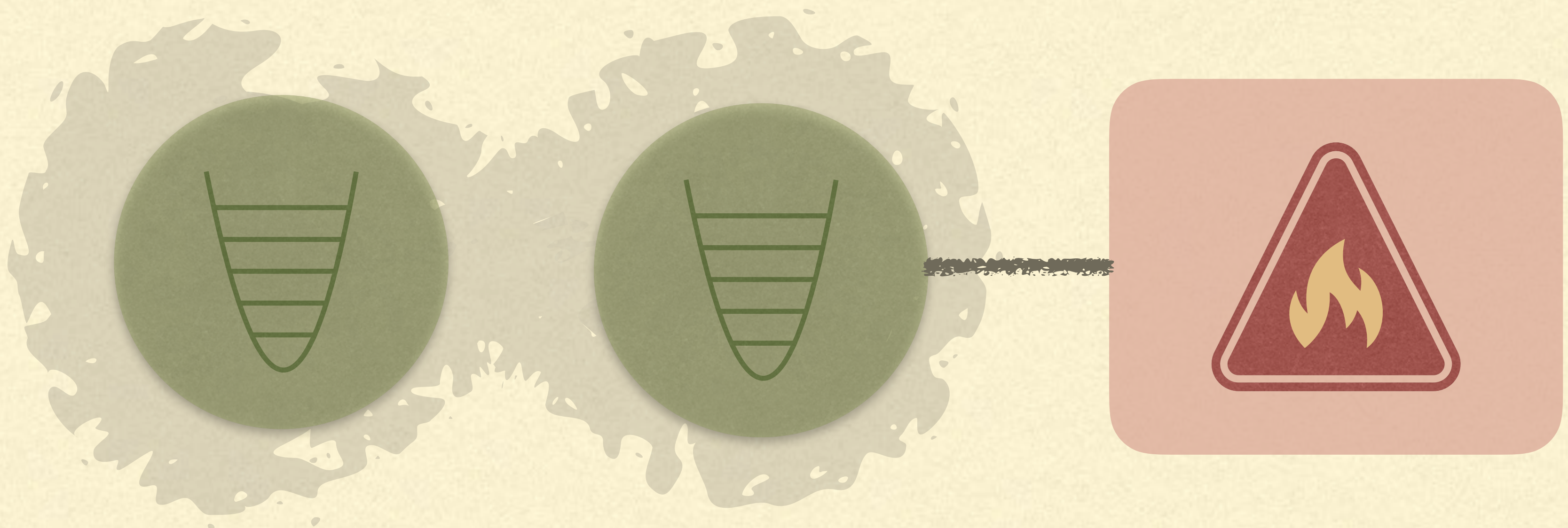
- It is possible to have *full* access to all properties of system and environment, for arbitrary environment sizes.
- Extremely versatile. Allows control over:
  - non-Markovianity.
  - Role of system-environment correlations.
  - Quantum Darwinism and objective reality.

- The entropy production is divided as:

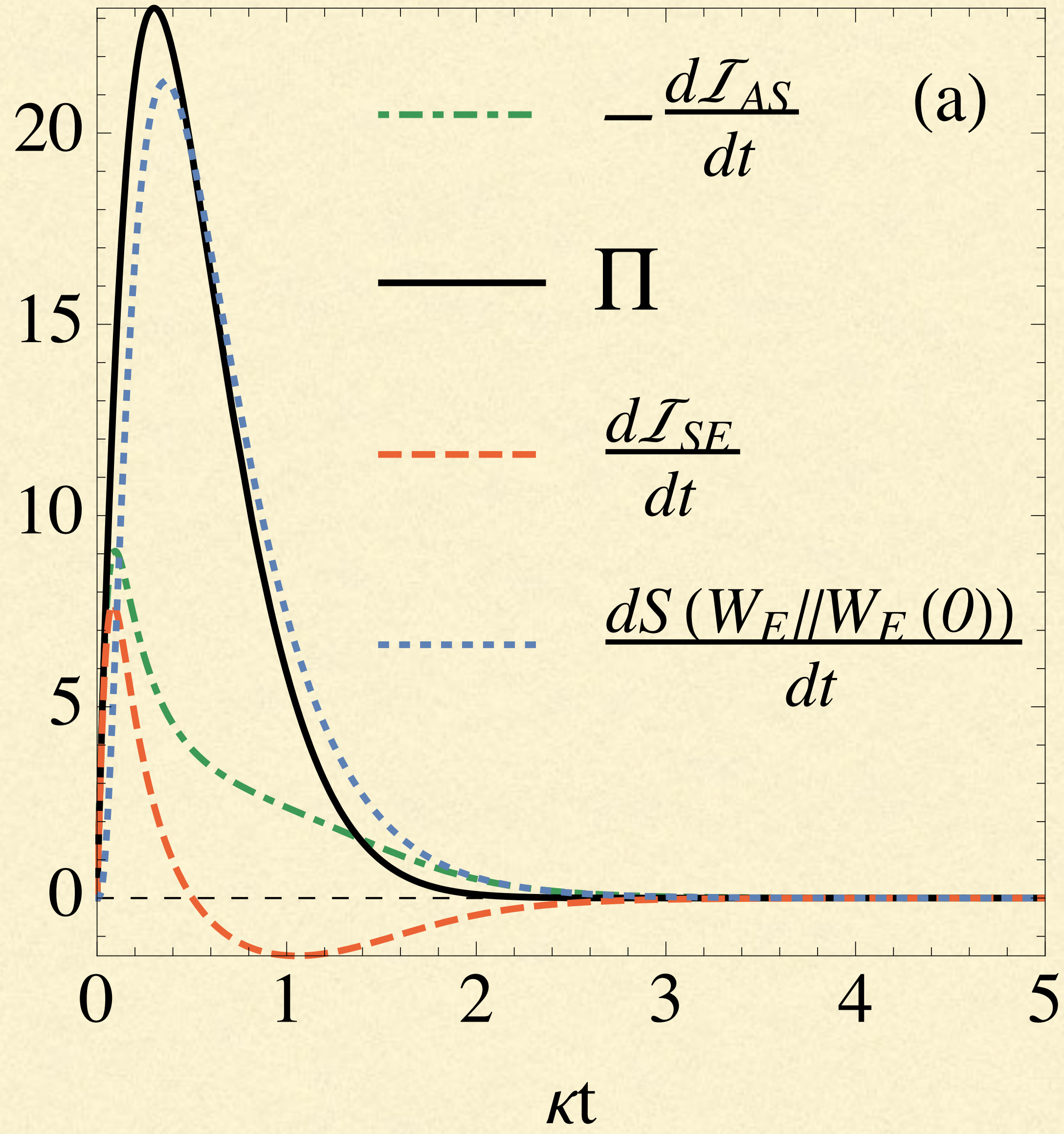
$$\Pi = \frac{d\mathcal{I}_{SE}}{dt} + \frac{dS(W_E||W_E(0))}{dt}$$

- Can be used to study the role of system-environment correlations in the emergence of irreversibility.

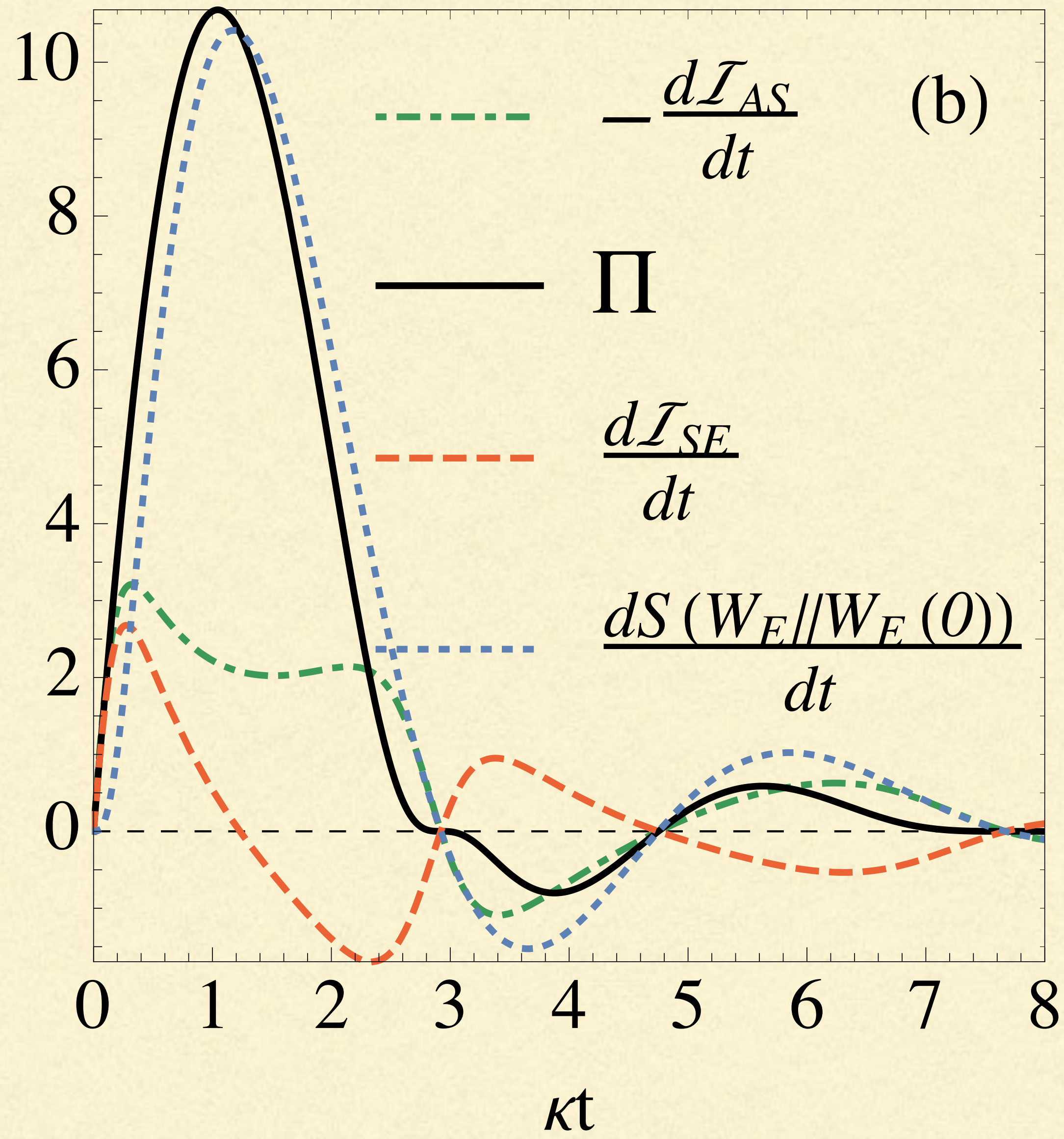
- Example: monitoring the entanglement between a system and an ancilla.



Rivas, Huelga, Plenio, PRL, **105**, 050403 (2010)



Markovian



non-Markovian

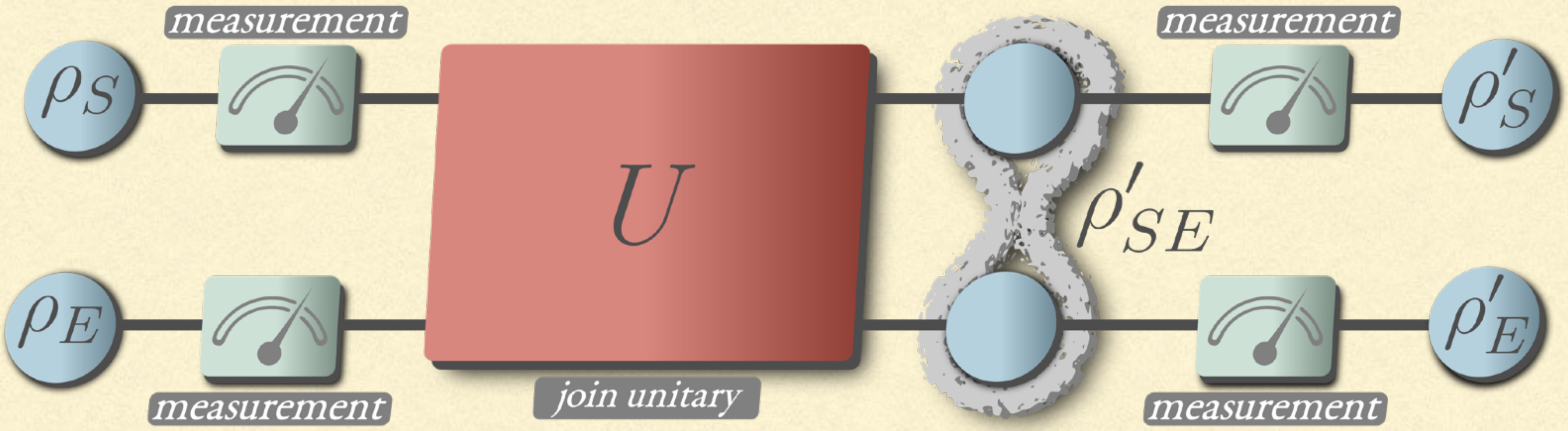
Before I finish…

2 PERMANENT POSITIONS @ USP

- *"Quantum Information" & "Materials at the nanoscale"* (i.e. condensed matter)
  - Average of 4h a week of teaching only.
  - Large number of students interested in masters and PhD.
  - Good funding from the São Paulo Funding Agency.
  - If you are already an experienced professor, you can jump to Associate professor in one year.
- For more information, see www.fmt.if.usp.br/~gtlandi

QUANTUM TRAJECTORIES

- Entropy production is not an observable.
  - But in certain cases it can be related to observables (e.g. currents in Onsager's theory).
- Otherwise, to access the entropy in the lab, we need to perform 2 quantum measurements.



- Initially the environment is thermal and the system is in an arbitrary state:

$$\rho_E(0) = \sum_\mu q_\mu^{\mathrm{th}} |\mu\rangle\langle\mu|, \qquad \rho_S(0) = \sum_\alpha p_\alpha |\psi_\alpha\rangle\langle\psi_\alpha|$$

- In general the system is *not* diagonal in the energy eigenbasis:

$$H_S|n\rangle = E_n|n\rangle$$

- **Step 1**: At $t = 0$ we then measure both S and E in the basis $|\psi_\alpha\rangle \otimes |\mu\rangle$
- Obtain outcomes with probability $p_\alpha q_\mu^{\mathrm{th}}$
- **Step 2**: evolve with a unitary *U* to obtain a final state ρ'_{SE}
- Now define $\rho'_S = \mathrm{tr}_E \rho'_{SE} := \sum_\beta p'_\beta |\psi'_\beta\rangle\langle\psi'_\beta|$
- **Step 3**: measure again S and E in the basis $|\psi'_\beta\rangle \otimes |\nu\rangle$

- Quantum trajectory: $\mathcal{X} = \{\alpha, \mu, \beta, \nu\}$

$$\mathcal{P}[\mathcal{X}] = p(\beta, \nu|\alpha, \mu) p_\alpha q_\mu^{\mathrm{th}} = |\langle \psi'_\beta, \nu | U | \psi_\alpha, \mu\rangle|^2 p_\alpha q_\mu^{\mathrm{th}}$$

- Now we define the stochastic entropy production

$$\boxed{\sigma[\mathcal{X}] = -\ln\left(\frac{p'_\beta q_\nu^{\mathrm{th}}}{p_\alpha q_\mu^{\mathrm{th}}}\right)}$$

- Its average gives the entropy production we had before: $\langle \sigma[\mathcal{X}]\rangle = \Sigma$

- And it satisfies a fluctuation theorem: $\langle e^{-\sigma[\mathcal{X}]}\rangle = 1$

CONTRIBUTION FROM QUANTUM COHERENCES

- But now we can ask, on this *stochastic level*, what is the meaning of separating the entropy production in two parts?

$$\Pi = -\frac{dS(\mathbf{p}(t)||p^{\text{eq}})}{dt} - \frac{d\mathcal{C}(\rho)}{dt}$$

- Define an *augmented* quantum trajectory: $\tilde{\mathcal{X}} = \{\alpha, n, \mu, \beta, m, \nu\}$

$$\mathcal{P}[\tilde{\mathcal{X}}] = \mathcal{P}[\mathcal{X}] p_{n|\alpha} p'_{m|\beta}$$

- where we defined the conditional probabilities $p_{n|\alpha} = |\langle n|\psi_\alpha\rangle|^2$

$$p'_{m|\beta} = |\langle m|\psi'_\beta\rangle|^2$$

- We then find that $\boxed{\sigma[\tilde{\mathcal{X}}] = \sigma_{\text{classical}}[\tilde{\mathcal{X}}] + \xi[\tilde{\mathcal{X}}]}$

- where

$$\sigma_{\text{classical}}[\tilde{\mathcal{X}}] = -\ln\left(\frac{p'_m q^{\text{th}}_\nu}{p_n q^{\text{th}}_\mu}\right)$$

$$\boxed{\xi[\tilde{\mathcal{X}}] = -\ln\left(\frac{p_n}{p_\alpha}\right) - \ln\left(\frac{p'_m}{p'_\beta}\right)}$$

- The coherence contribution is precisely the *information gain:*
  - That is, the amount of information that the bases $|n\rangle$ and $|\psi_\alpha\rangle$ share with each other.

- This is therefore related to the fundamental incompatibility of different basis sets.