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THERMODYNAMICS AND INFORMATION IN QUANTUM NON-EQUILIBRIUM STEADY-STATES

Gabriel T. Landi

Instituto de Física da Universidade de São Paulo

www.fmt.if.usp.br/~gtlandi

Quantum thermodynamics

quantum transport

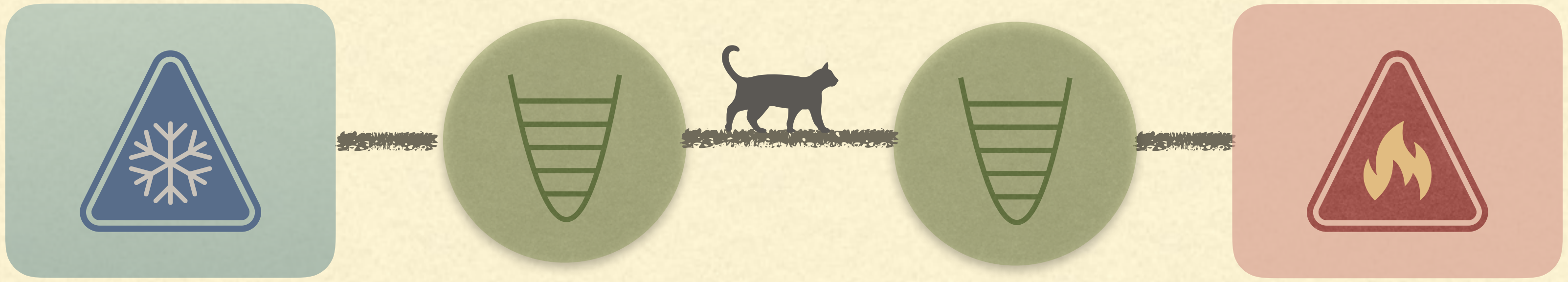
IFUSP

Queen's University in Belfast

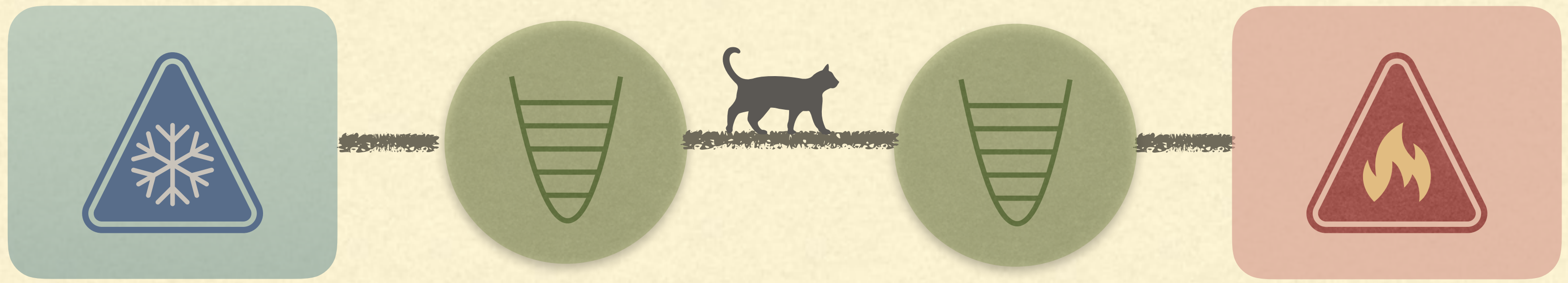
Trinity College, Dublin

December, 2018

NON-EQUILIBRIUM STEADY STATE (NESS)



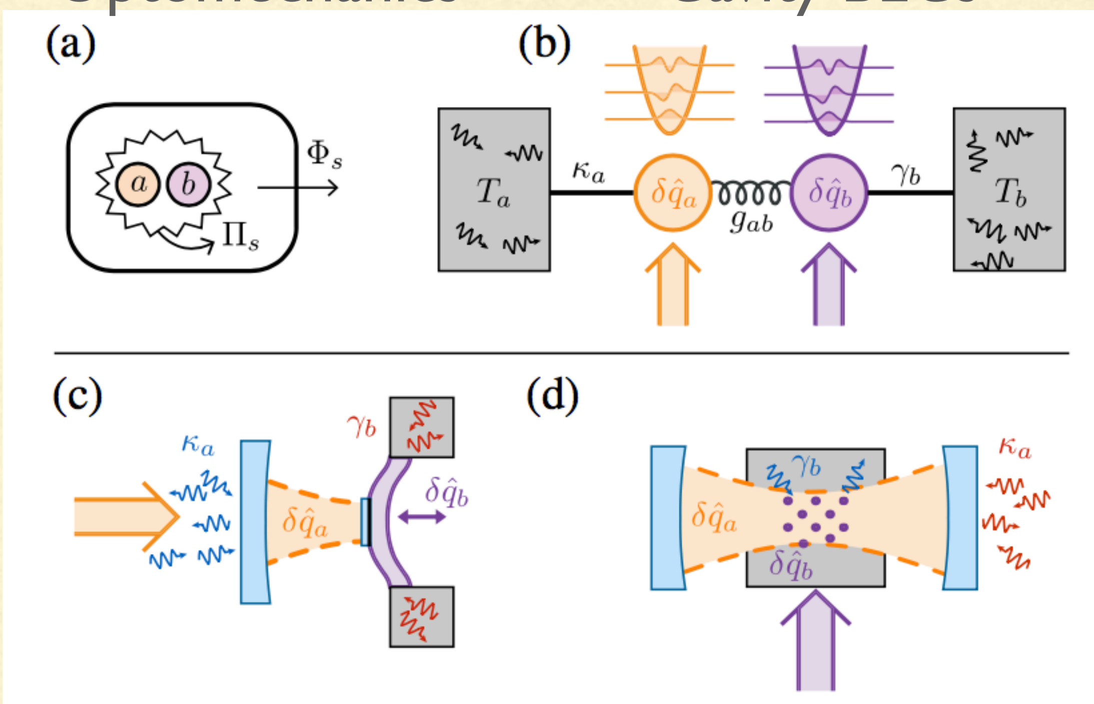
- Its a steady-state: no time-dependence.
  - But it is not in equilibrium: *constant flow of heat from hot to cold.*
- NESSs are the closest we have to thermal equilibrium.
- But for equilibrium we know the basic properties of the state: $\rho = \frac{e^{-\beta H}}{Z}$
- For the NESS, no such statements can be made.
  - One needs to know the full dynamical equations.



- Clean and natural platform for studying non-equilibrium physics:
  - Fourier's law: diffusive vs. ballistic.
  - Steady-state fluctuation theorems.
  - Transport of excitations (squeezing, magnetization, etc).
- Connections with quantum information still poorly explored for NESSs:
  - Information transport and quantum state transfer.
  - Entanglement dynamics in quench dynamics.
- Quantum features of the NESS are now becoming accessible in controlled quantum platforms.

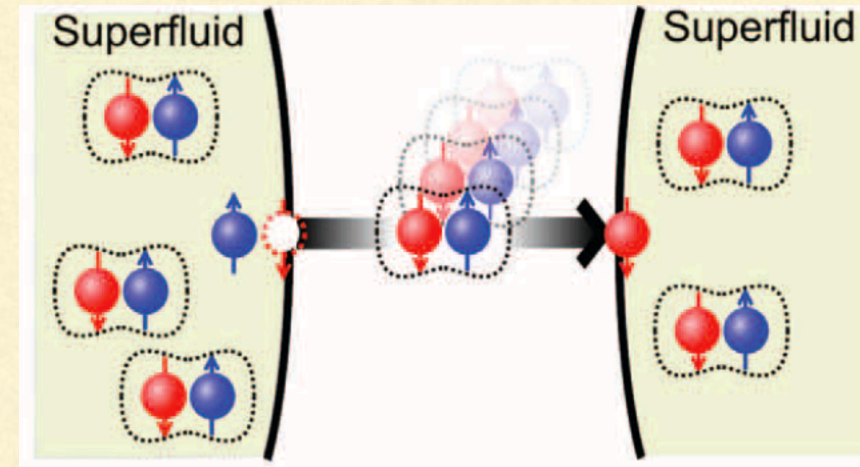
CONTROLLED QUANTUM PLATFORMS

Optomechanics Cavity BECs



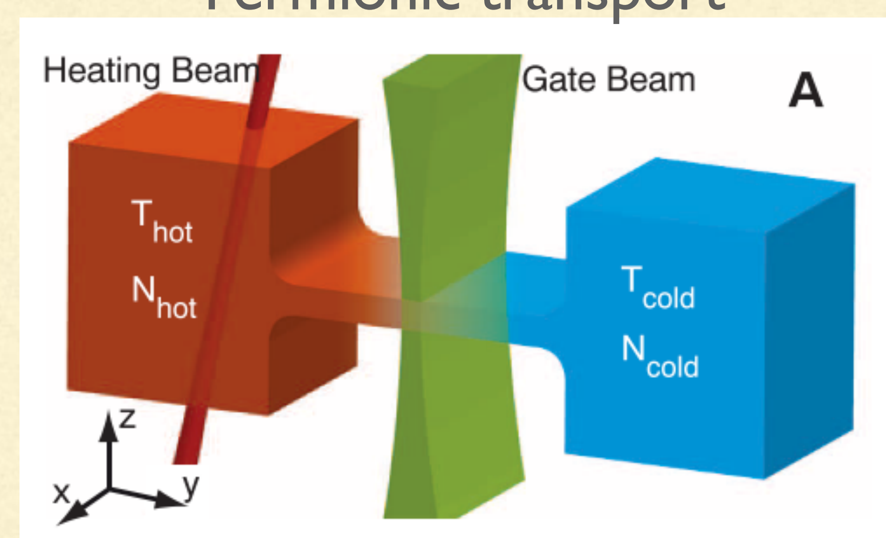
M. Brunelli, *et. al.*, *Phys. Rev. Lett.*., **121**, 160604 (2018)

Bosonic transport in optical lattices



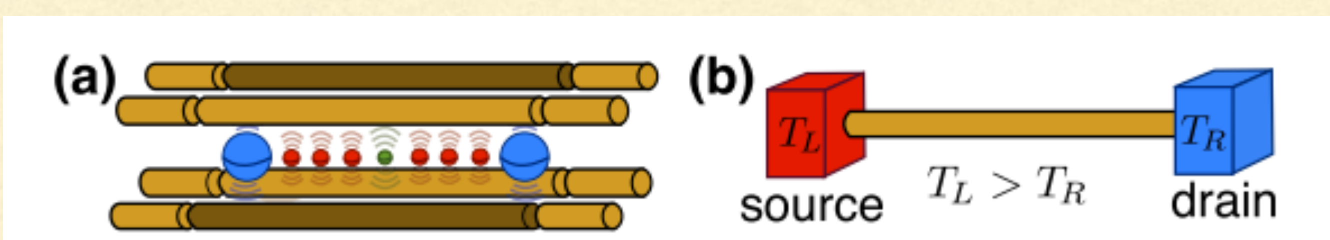
D. Husmann, *et. al.*, *Science*, **350**, 1498 (2015).

Fermionic transport



J. P. Brantut, *et. al.*, *Science*, **342**, 713 (2013).

Proposal for trapped ions



A. Bermudez, M. Bruderer and M. B. Plenio, Phys. Rev. Lett., 111, 040601 (2013)

OUTLINE

1. *Local vs. Global master equations*: thermodynamics of repeated interactions.

2. An analytically soluble model for a *Gaussian diffusive* NESS.

3. *Information transport* in the NESS.

Work done in collaboration with:

William Malouf (master student) and Jader P. Santos (USP)
Mauro Paternostro and Gabriele De Chiara (Queens University in Belfast)
John Goold (Trinity College Dublin)
Gerardo Adesso and Luis Correa (University of Nottingham)

arXiv
1808.10450
1809.09931

THERMODYNAMICS OF REPEATED INTERACTIONS

New J. Phys. **20** (2018) 113024 https://doi.org/10.1088/13

New Journal of Physics

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PAPER

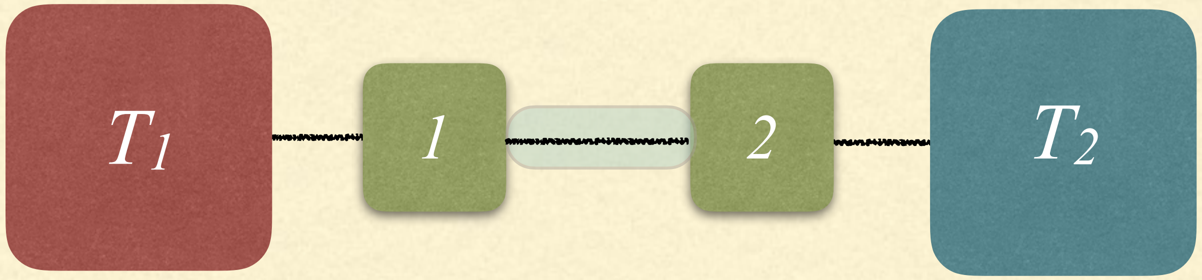
Reconciliation of quantum local master equations with thermodynamics

Gabriele De Chiara[1,2,7], Gabriel Landi[3], Adam Hewgill[1], Brendan Reid[1], Alessandro Ferraro[1], Augusto J Roncaglia[4] and Mauro Antezza[2,5,6]

arXiv
1808.10450

LOCAL VS. GLOBAL DILEMA

$$\frac{d\rho}{dt} = -i[H,\rho] + D_1(\rho) + D_2(\rho)$$



$$D_i(\rho) = \gamma_i(N_i+1)\left[a_i\rho a_i^\dagger - \frac{1}{2}\{a_i^\dagger a_i,\rho\}\right] \quad +\gamma_i N_i\left[a_i^\dagger\rho a_i - \frac{1}{2}\{a_i a_i^\dagger,\rho\}\right]$$

$$H = H_1 + H_2 + H_I$$
$$= \omega_1 a_1^\dagger a_1 + \omega_2 a_2^\dagger a_2 + \epsilon(a_1^\dagger a_2 + a_1 a_2^\dagger)$$

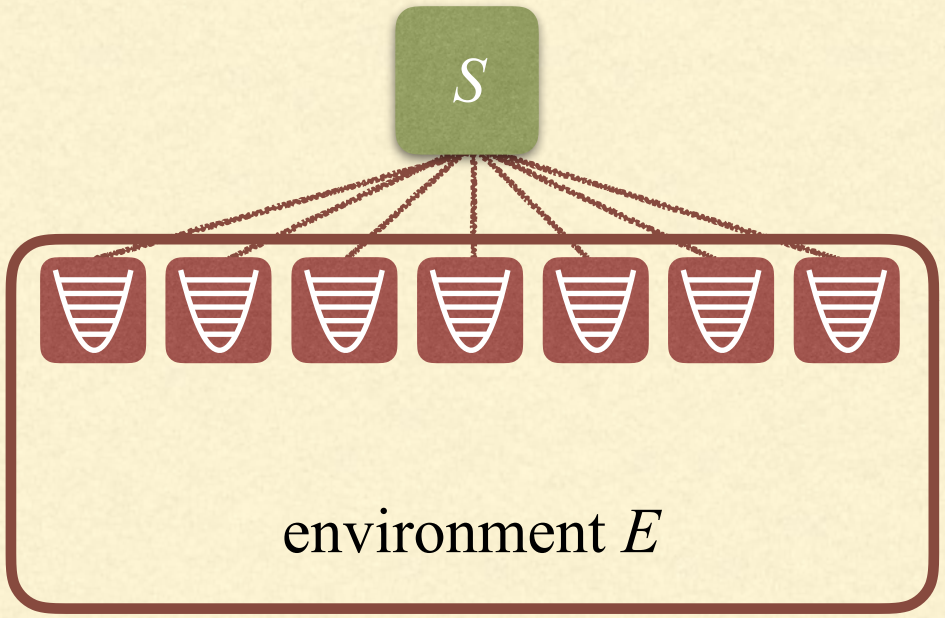
$$N_i = \frac{1}{e^{\beta_i\omega_i} - 1}$$

- Seem to violate the 2nd law.
- Microscopic derivation yields a global ME. But this also has unphysical properties.

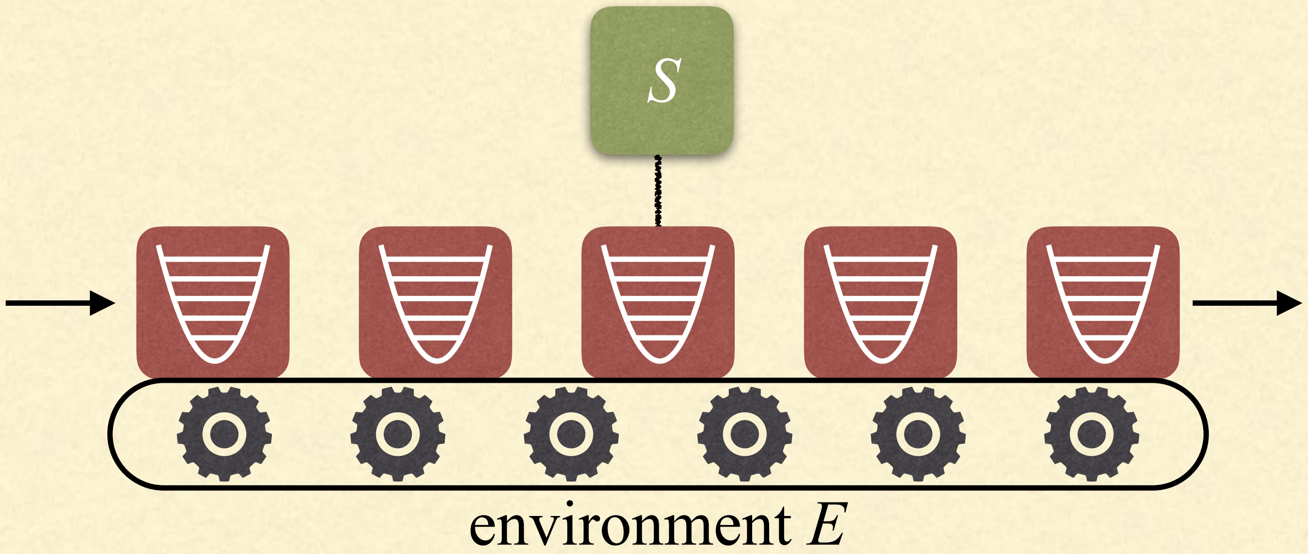
A. Levy, R. Kosloff, *Europhys.Lett.* **107**, 20004 (2014)
J. Onam González, L.A. Correa, G. Nocerino, J. P. Palao, D. Alonso, G. Adesso, *Open Systems & Inf. Dyn,* **24**, 1740010 (2017).
M.T. Mitchison and M. B. Plenio, *NJP,* **20**, 033005 (2018).

BE WISE, DISCRETIZE

- The LME generates a valid CPTP-semigroup evolution.
  - Is there any physical scenario who is described by a LME?



Usual approach



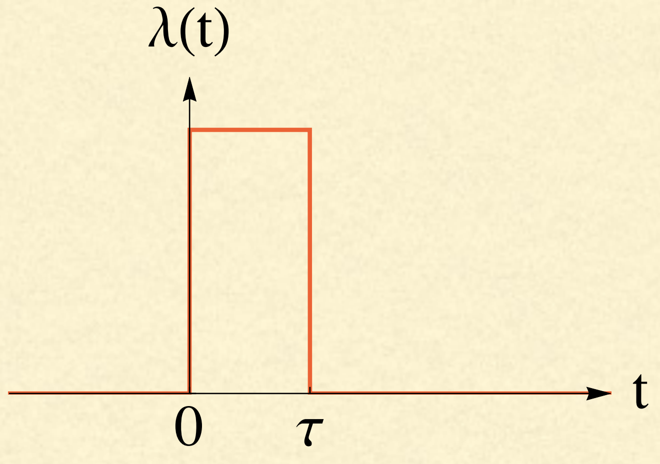
Repeated interactions (collision model/conveyor belt)

- RI continuous-time limit always generates *local* dissipators.
- This is a *physically realization scenario*: LMEs have physical realizations!

P. Strasberg, G. Schaller, T. Brandes, M. Esposito, *Phys. Rev. X*, **7**, 021003 (2017)

FUNDAMENTAL WORK COST

- In general there will be a *work cost* associated with turning the interactions on and off:
  - External agent has to pull S and E apart.



$$H_{SE_n}(t) = H_S + H_{E_n} + \lambda(t) V_{SE_n}$$

$$\delta W = \langle V \rangle - \langle V \rangle'$$

- The exception is when S+E is a *thermal operation.*

$$[H_S + H_E, V] = 0$$

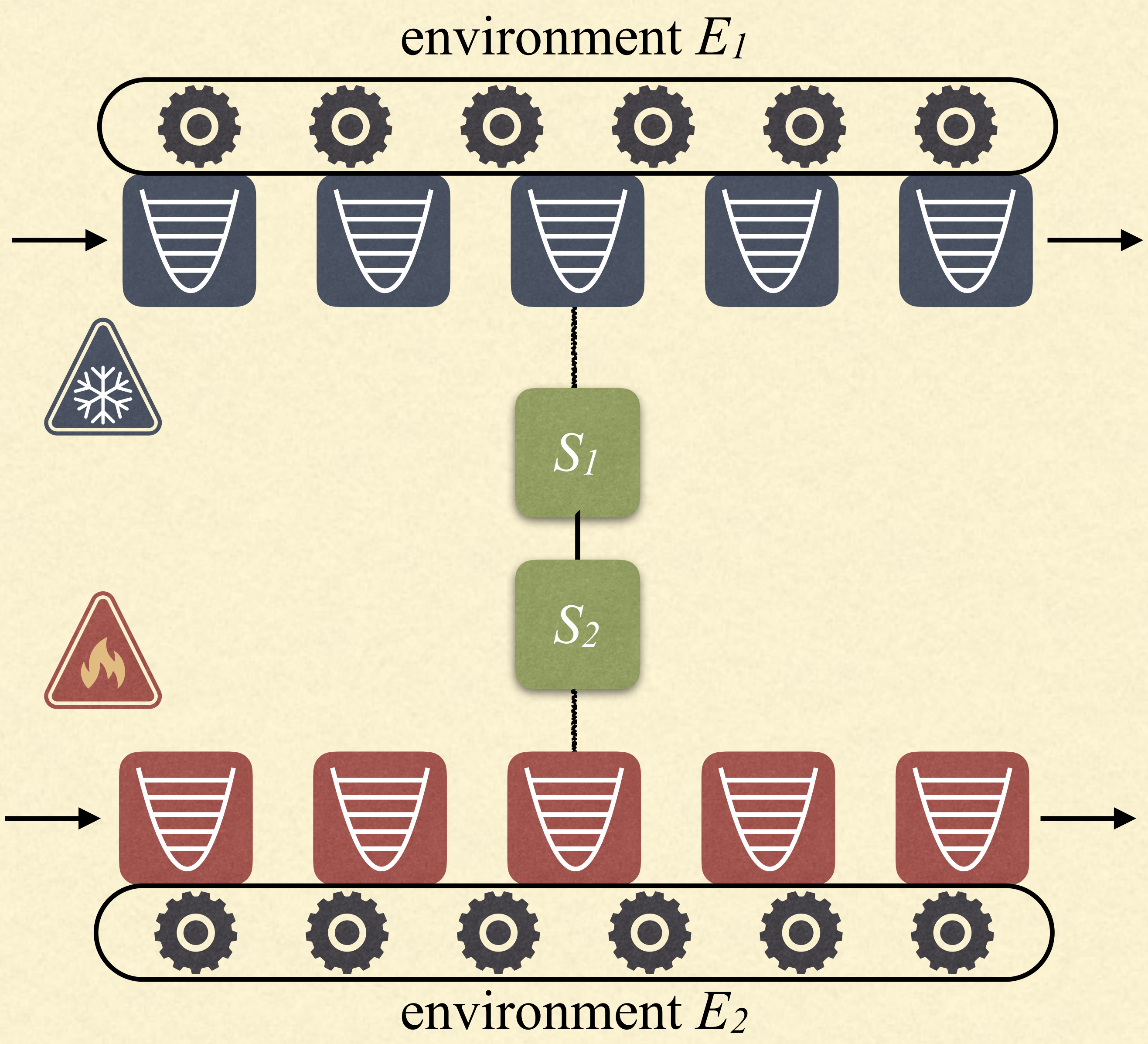
- Operator version of detailed balance.
  - Can be neatly formulated in terms of *eigenoperators*.

F. Barra, *Scientific Reports*, **5**, 14873 (2015)

F. Brandão, M. Horodecki, J. Oppenheim, J. Renes, R. Spekkens, *Phys. Rev. Lett.*, **111**,250404 (2013).

RECONCILIATION WITH THERMODYNAMICS

$$H_{SE}(t) = \sum_{i=1}^{N} H_{S_i} + H^{\text{int}}_{S_1,\ldots,S_N} + \sum_{i=1}^{N} H_{E_i} + \lambda(t) \sum_{i=1}^{N} V_{SE_i}$$



Local detailed balance is satisfied:

$$[H_{S_i} + H_{E_i}, V_i] = 0$$

But global detailed balance is broken due to interactions.

$$[H_I, V_i] \neq 0$$

In this case there will be a work cost.

THE HARMONIC CHAIN WITH SELF-CONSISTENT RESERVOIRS

QUANTUM HARMONIC CHAIN

$$\frac{d\rho}{dt} = -i[H,\rho] + D_1(\rho) + D_L(\rho) \qquad H = i\lambda \sum_i (a_i^\dagger a_{i+1} - a_{i+1}^\dagger a_i),$$

$$D_i(\rho) = \gamma_i(N_i+1)\left[a_i \rho a_i^\dagger - \frac{1}{2}\{a_i^\dagger a_i, \rho\}\right] \quad + \gamma_i N_i \left[a_i^\dagger \rho a_i - \frac{1}{2}\{a_i a_i^\dagger, \rho\}\right]$$

$$J_{i,i+1} = 2\lambda \langle a_i a_{i+1}^\dagger + a_{i+1}^\dagger a_i \rangle$$

This model can be solved analytically for arbitrary *L:*

Z. Rieder, J. L. Lebowitz and E. Lieb, *J.. Math. Phys.*, **8**, 1073 (1967)
M. Žnidarič, *J. Stat. Mech*, L05002 (2010)
A. Asadian, D. Manzano, M. Tiersch and H. J. Briegel, *Phys. Rev. E.*, **87**, 012109 (2013)
F. Nicacio, A. Ferraro, A. Imparato, M. Paternostro, F. L. Semião, *PRE*, **91**, 042116 (2015)

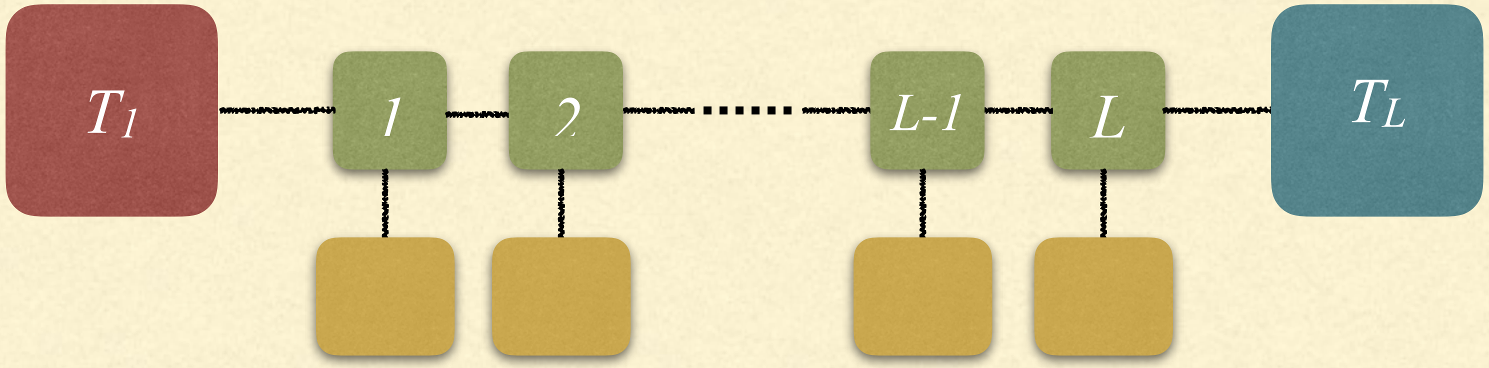
SELF-CONSISTENT BATHS

- We consider instead a modified MEq. of the form

$$\frac{d\rho}{dt} = -i[H,\rho] + D_1(\rho) + D_L(\rho) + \sum_{i=1}^{L} \tilde{D}_i(\rho)$$

$$\tilde{D}_i(\rho) = \Gamma(\tilde{N}_i + 1)\mathcal{D}[a_i] + \Gamma \tilde{N}_i \mathcal{D}[a_i^\dagger] \qquad \tilde{N}_i = \langle a_i^\dagger a_i \rangle_{\text{NESS}}$$

- Gives the same 2nd moments as a dephasing model, but has a Gaussian NESS.



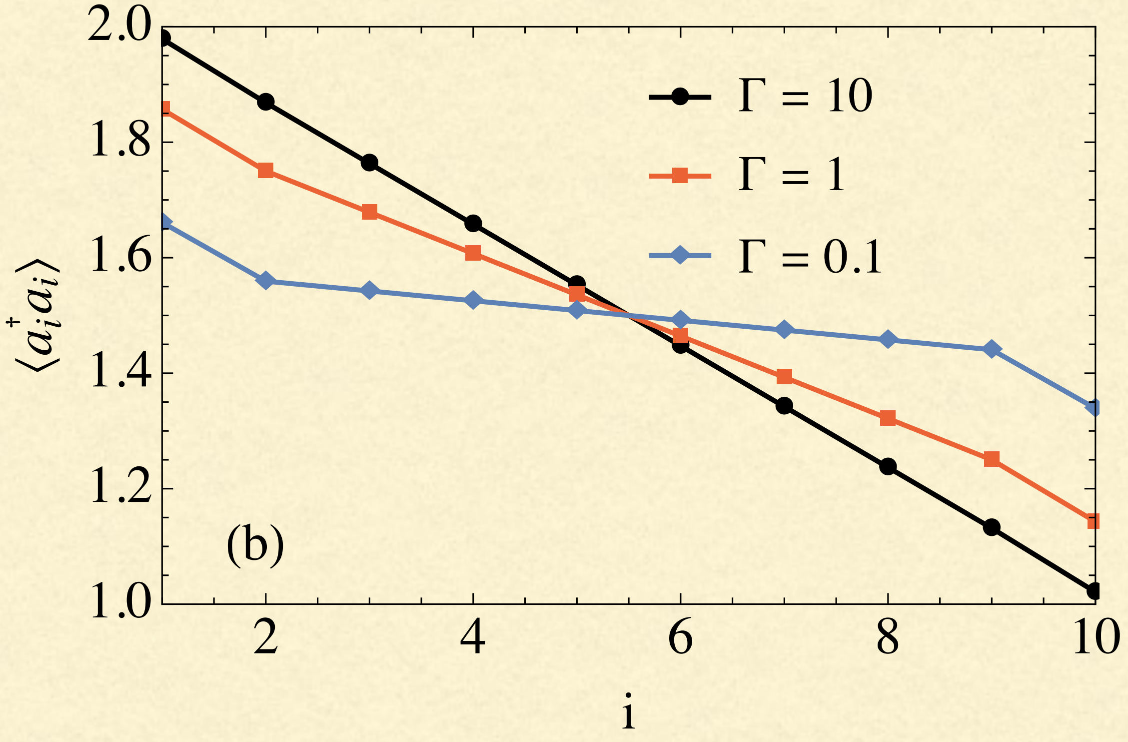
A. Asadian, D. Manzano, M. Tiersch and H. J. Briegel, *Phys. Rev. E.*, **87**, 012109 (2013)
M. Bolsterli, M. Rich, W. M. Visscher, *Phys. Rev. A.*, **1**, 1086 (1970)

NOISE SOURCES ARE *RELEVANT*

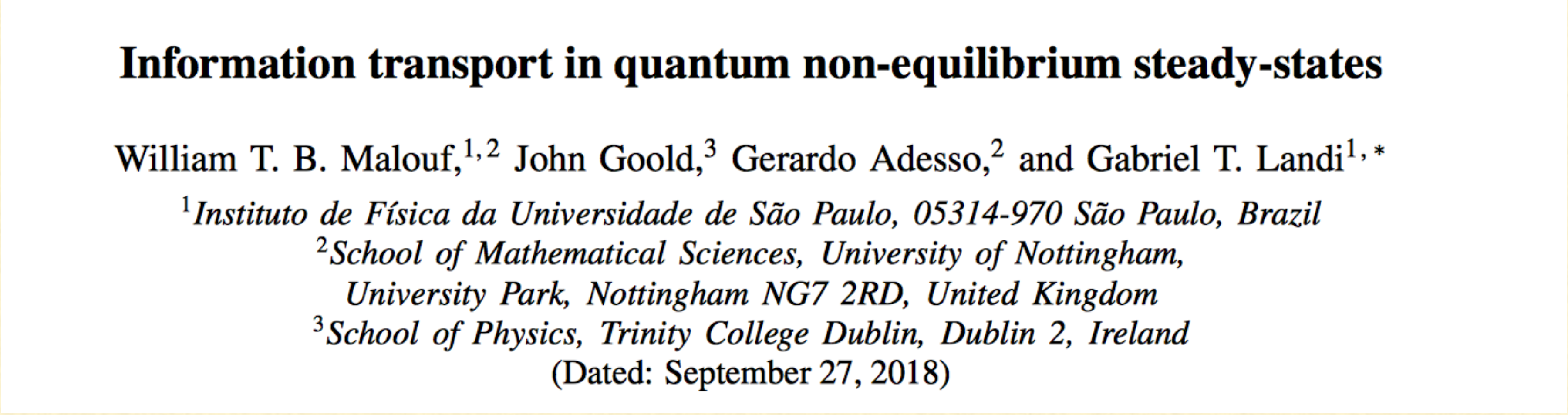
$$\langle a_i^\dagger a_i \rangle = \frac{N_1 + N_L}{2} + \frac{1}{2}\frac{\gamma(N_1 - N_L)}{4\lambda^2 + \gamma^2 + \gamma\Gamma(L-1)}\Gamma(L - 2i + 1)$$

$$J = \frac{2\gamma\lambda^2}{4\lambda^2 + \gamma^2 + \gamma\Gamma(L-1)}(N_L - N_1).$$

- We see that for *any* non-zero Γ the current always scales as *1/L*.
  - Diffusive behavior.



INFORMATION TRANSPORT IN THE NESS


Information transport in quantum non-equilibrium steady-states

William T. B. Malouf,[1,2] John Goold,[3] Gerardo Adesso,[2] and Gabriel T. Landi[1,*]

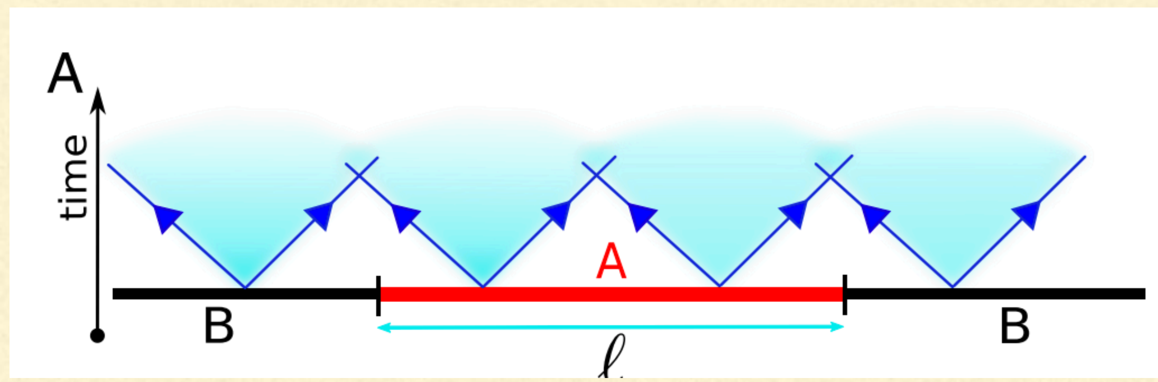
[1] *Instituto de Física da Universidade de São Paulo, 05314-970 São Paulo, Brazil*
[2] *School of Mathematical Sciences, University of Nottingham, University Park, Nottingham NG7 2RD, United Kingdom*
[3] *School of Physics, Trinity College Dublin, Dublin 2, Ireland*

(Dated: September 27, 2018)

arXiv
1809.09931

ENTANGLEMENT DYNAMICS AND QUANTUM STATE TRANSFER

- When excitations propagate through a chain (e.g. after a quench), they carry information from one part to the other.



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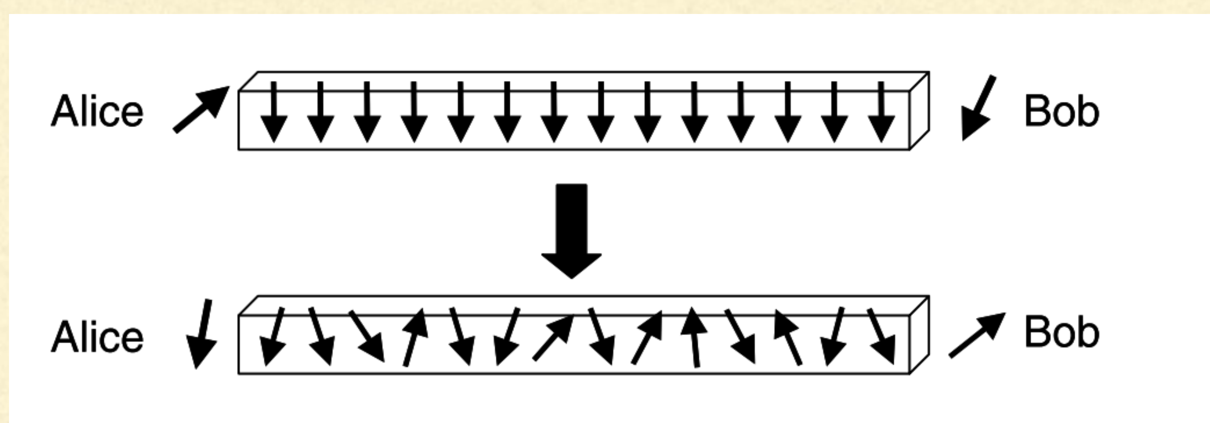


V. Alba and P. Calabrese, *PNAS*, **114**, 7947 (2017)

S. Bose, *PRL*, **91**, 207901 (2003)

INFORMATION TRANSPORT IN THE NESS

- Entanglement dynamics and quantum state transfer are both instances of *information transport.*

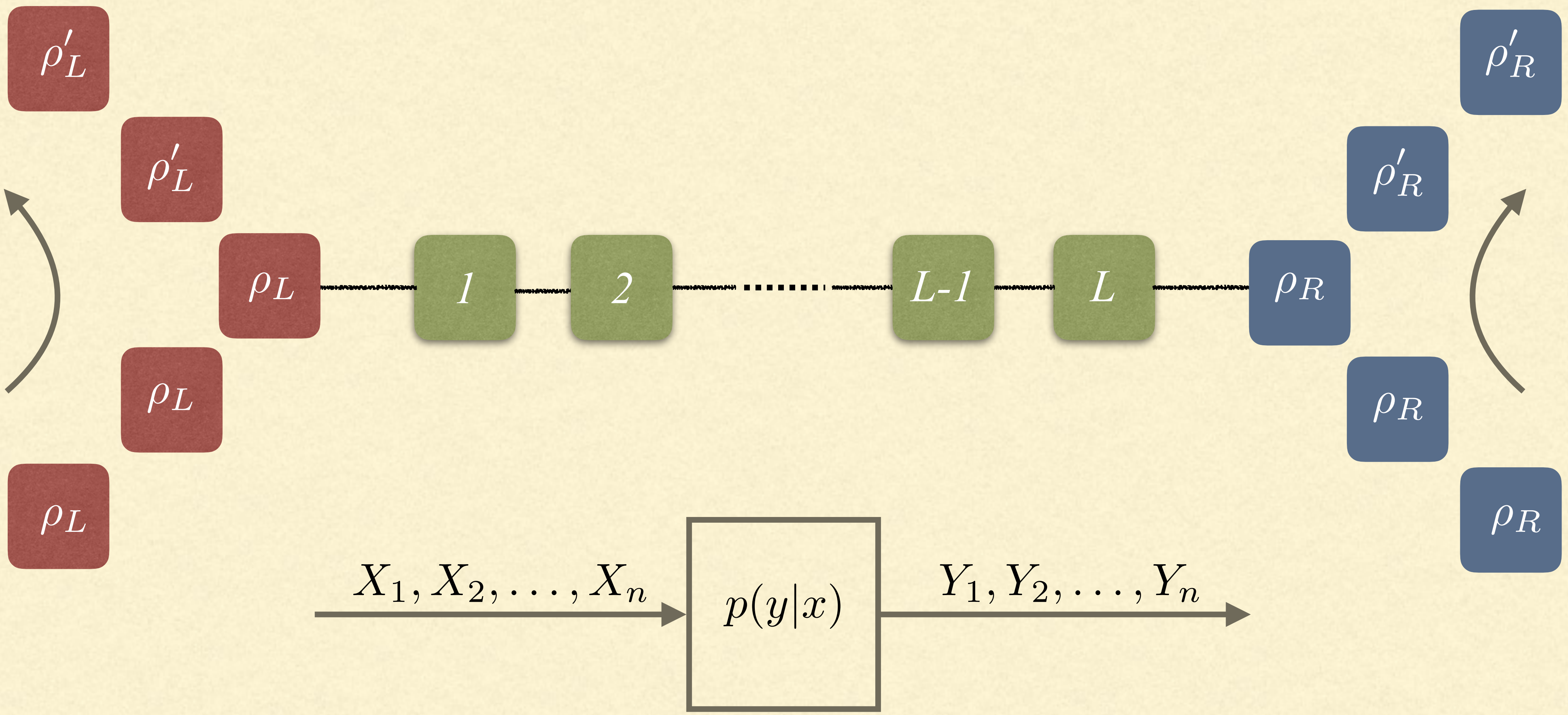
- It should be possible to formulate an analogous question for NESSs.

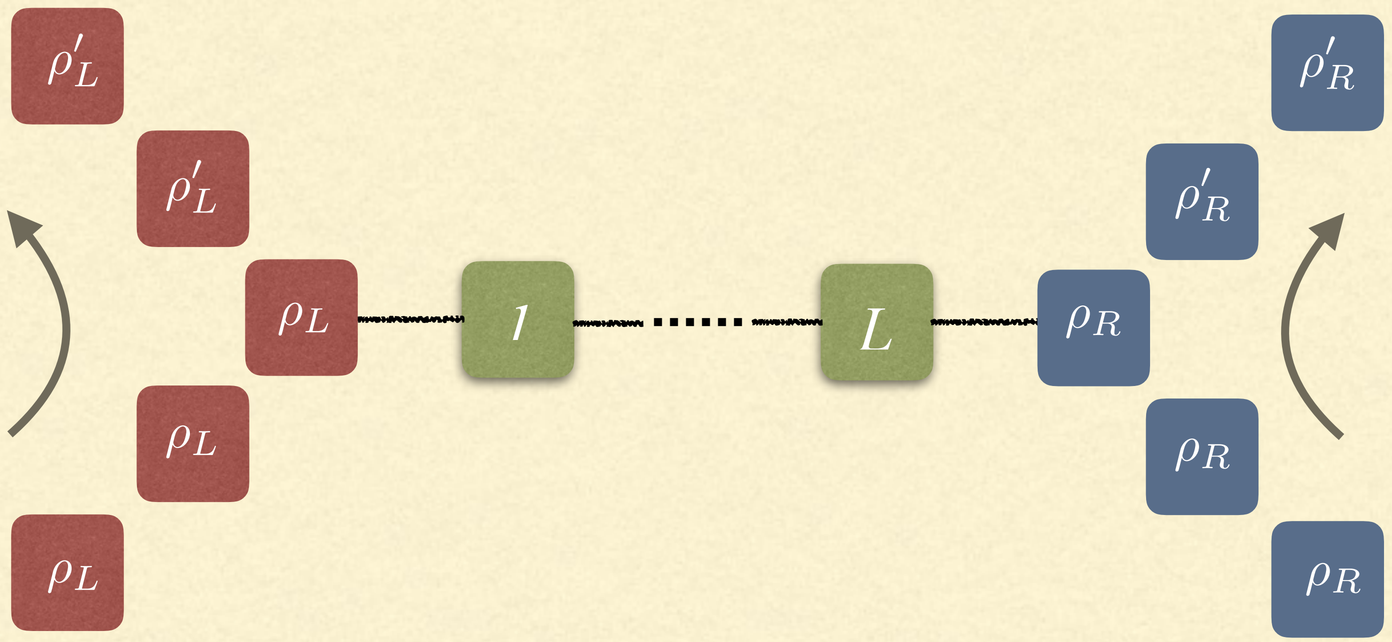
    - Heat currents also carry information.

- But the NESS is fundamentally different: its a steady-state; no dynamics.

REPEATED INTERACTIONS SCENARIO

- The repeated interactions scenario is remarkably similar to quantum state transfer.
- In fact, it is also remarkably similar to a noisy communications channel.





Joint unitary of each RI stroke:

$$\rho'_{LSR} = U(\rho_L \otimes \rho_S \otimes \rho_R)U^\dagger$$

In the NESS:

$$\mathrm{tr}_{LR}\Big\{U(\rho_L \otimes \rho_S \otimes \rho_R)U^\dagger\Big\} = \rho_S$$

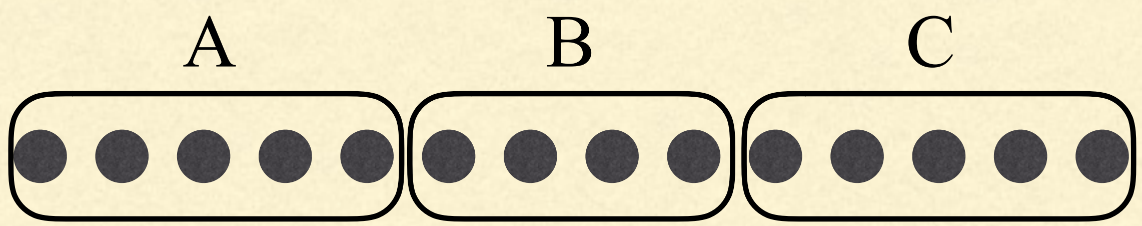
- We now define the NESS map:

$$\rho'_R = \Phi(\rho_L) = \mathrm{tr}_{LS}\Big\{U(\rho_L \otimes \rho_S \otimes \rho_R)U^\dagger\Big\}$$

- This is *exactly* like the noisy channel scenario.
- I don't think the thermodynamic/transport property of this type of map has ever been explored.

SHARED INFORMATION BETWEEN DISTANT PARTS

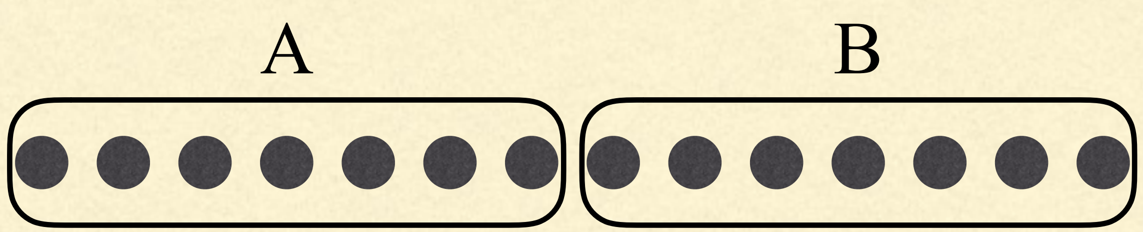
- The heat currents transmit information from one part of the chain to the other.



- The total information can be quantified by the total correlations:

$$\mathcal{T}(\rho) = \sum_{i=1}^{L} S(\rho_i) - S(\rho) = S(\rho || \rho_1 \ \ldots \rho_L)$$

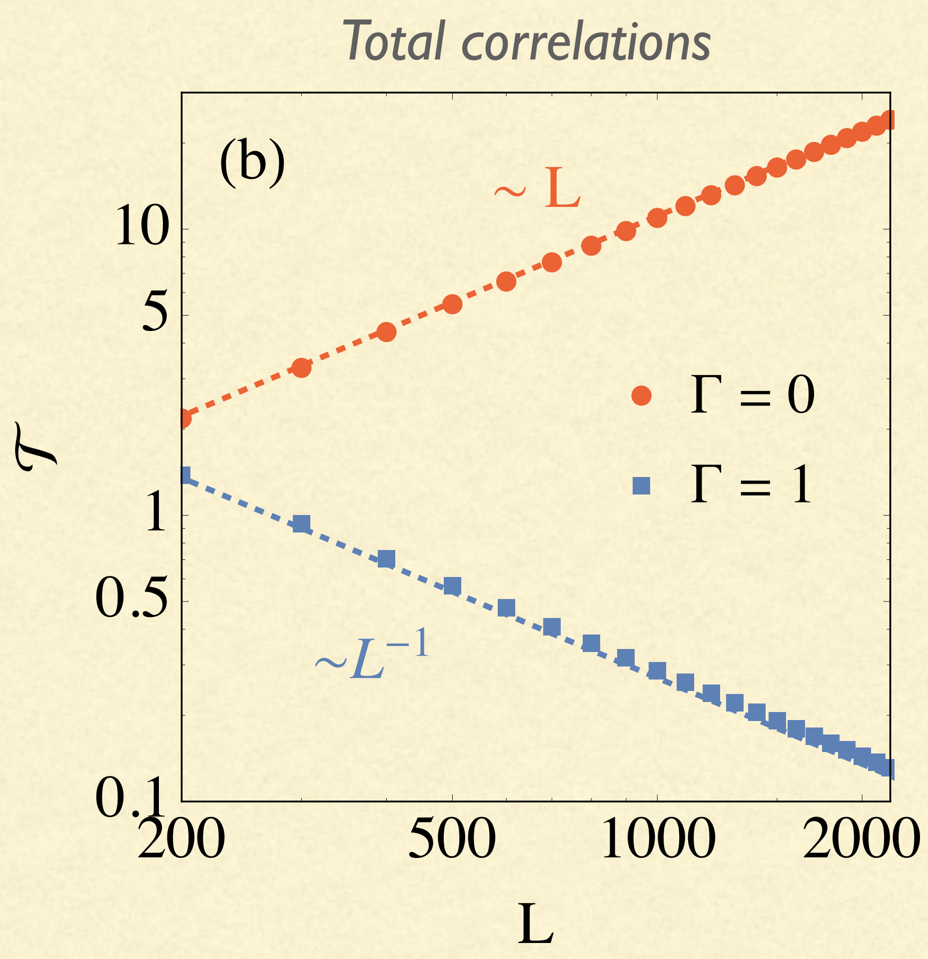
- For connected parts, shared information can also be quantified by the Mutual Information.

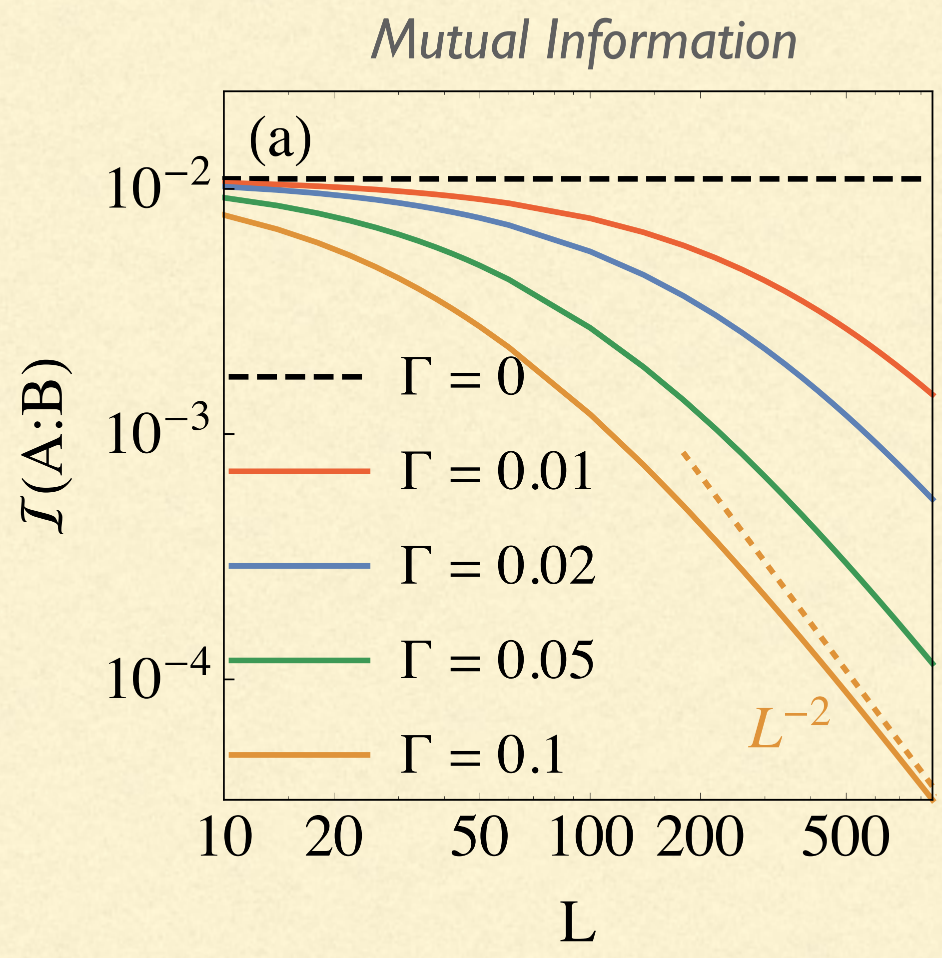


$$\mathcal{I}(A:B) = S(\rho_A) + S(\rho_B) - S(\rho_{AB})$$

J. Goold, C. Gogolin, S. R. Clark, J. Eisert, A. Scardicchio, and A. Silva, *Phys. Rev. B*, **92**, 180202 (2015)

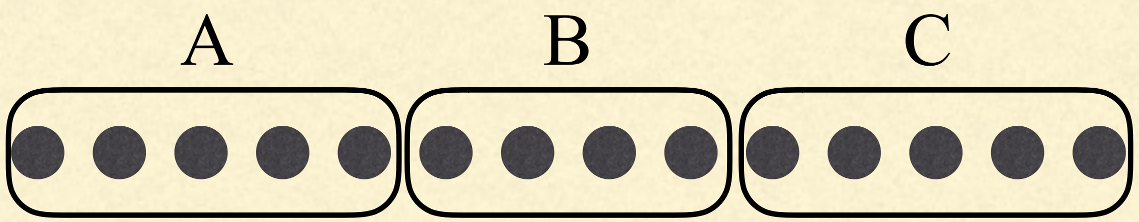
HARMONIC CHAIN WITH SELF-CONSISTENT BATHS





CONDITIONAL MUTUAL INFORMATION

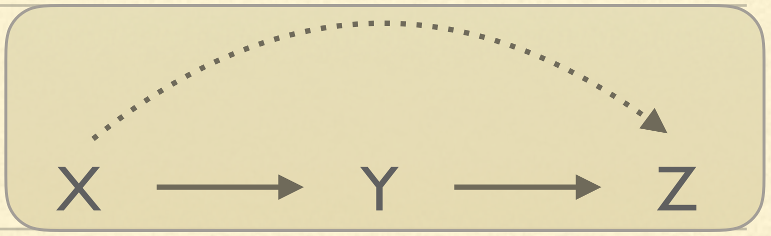
- For disconnected parts, however, the MI is not adequate.
- Instead, the correct quantifier of this is the Conditional Mutual Information (CMI):



$$\mathcal{I}(A\!:\!C|B) = S(\rho_{AB}) + S(\rho_{BC}) - S(\rho_B) - S(\rho_{ABC})$$

E. Lieb and Mary B. Ruskai, *J. Math. Phys.*, **14**, 1938-1941 (1973)

CLASSICAL MARKOV CHAIN



- We usually write the Markovianity condition as: $P(z|x,y) = P(z|y)$
- But this is equivalent to: $P(x,z|y) = P(z|y)P(x|y)$
- Markovianity: Past and future are independent, *given the present.*
- Non-Markovianity can thus be viewed as the ability of the past to communicate information to the future, *through* the present.
- But this does not mean that past and future are (unconditionally) independent:

$$P(x,z) = \sum_y P(z|y)P(x|y)P(y)$$

- Past and future are dependent because they share a *common ignorance* about the present.

- In a Markovian chain the MI is not zero:

$$\mathcal{I}(X:Z) = \sum_{x,z} p(x,z) \ln \frac{p(x,z)}{p(x)p(z)} \neq 0$$

- But the CMI is zero:

$$\mathcal{I}(X:Z|Y) = \sum_{x,y,z} p(x,y,z) \ln \frac{p(x,y,z)p(y)}{p(x,y)p(y,z)}$$

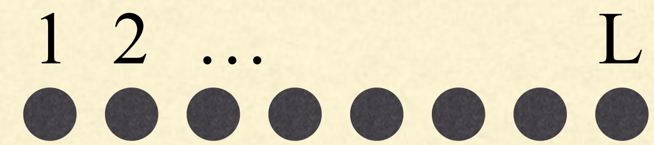
$$= \sum_{x,y,z} p(x,y,z) \ln \frac{p(x,z|y)}{p(x|y)p(z|y)} = 0$$

- In fact, a CMI = 0 can be taken as a definition of Markovianity.
- For a multi-time chain, the CMI as a function of the "size" of the present also quantifies the degree of non-Markovianity (*Markov Chain order*).

$$\mathcal{I}(X_1, \ldots, X_k : X_{k+\ell}, \ldots, X_n | X_{k+1}, \ldots, X_{k+\ell-1})$$

M. Papapetrou, D. Kugiumtzis, *Physica A,* **392**, 1593 (2013)

QUANTUM MARKOV CHAIN

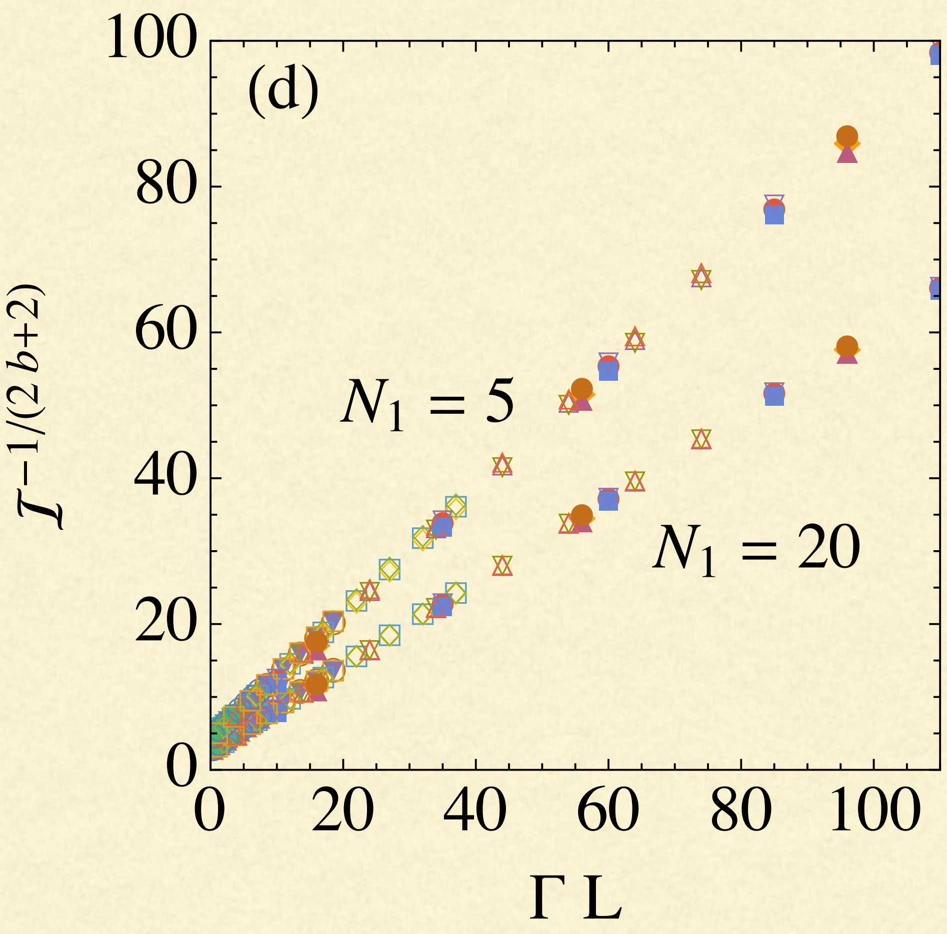


- A NESS density matrix can be viewed as a quantum Markov chain (this chain has no causal order).
- In the quantum case it can be shown that:

$$\mathcal{I}(A\!:\!C|B) = 0 \Longleftrightarrow \rho_{ABC} = \mathcal{E}_{BC}(\rho_{AB} \otimes |0\rangle\langle 0|_C)$$

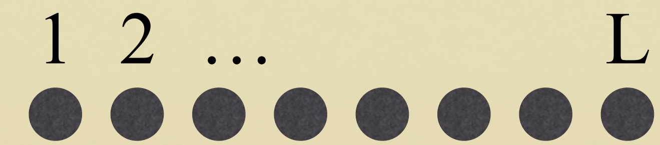
P. Hayden, R. Jozsa, D. Petz and A. Winter, *Comm. Math. Phys.*, **246**, 359 (2004)
O. Fawzi and R. Renner, *Comm. Math. Phys.*, **340**, 575 (2015)

HARMONIC CHAIN WITH SELF-CONSISTENT RESERVOIRS



$$\mathcal{I}(A\!:\!C|B) = \frac{u}{(v+\Gamma L)^{2b+2}}$$

LOCAL THERMALIZATION



$\rho_{1\dots L}$

- Kato-Brandão theorem on approximate Quantum Markov Chains:

$$\mathcal{I}(1,\dots,k-1 : k+1,\dots,L|k) \leq \epsilon, \qquad \forall k$$

- then

$$S\left(\rho \middle\| \frac{e^{-\sum_i h_{i,i+1}}}{Z}\right) \leq \epsilon L \qquad [h_{i,i+1}, h_{j,j+1}] \neq 0 \qquad j = i-1, i+1$$

- In the ballistic case: $\mathcal{I}(A:C|B=b) \sim L^0$
- Local states are therefore not necessarily thermal.
- Diffusive case: $\mathcal{I}(A:C|B=b) \sim L^{-4}$
- In the thermodynamic limit, the system becomes *locally thermal.*

$$\mathcal{I}(A:C|B) = \frac{u}{(v+\Gamma L)^{2b+2}}$$

K. Kato, F. Brandão, 1609.06636

CONCLUSIONS

- Plenty to explore in Quantum NESSs: *physics is still not fully understood.*
  - I'm particularly interested in this notion of information transport.
- Information transport offers a bridge between non-equilibrium physics and quantum information.

IFUSP

Thank you!

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Acknowledgements:
FAPESP Sprint