THERMODYNAMICS OF PRECISION IN QUANTUM NON-EQUILIBRIUM SYSTEMS

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Caxambu d'Aju
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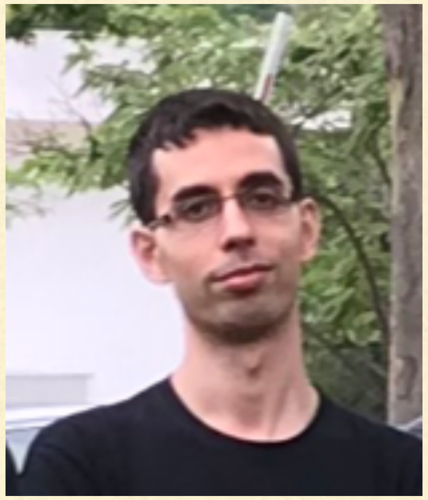
SUMMARY

I. Thermodynamic uncertainty relations (TURs)

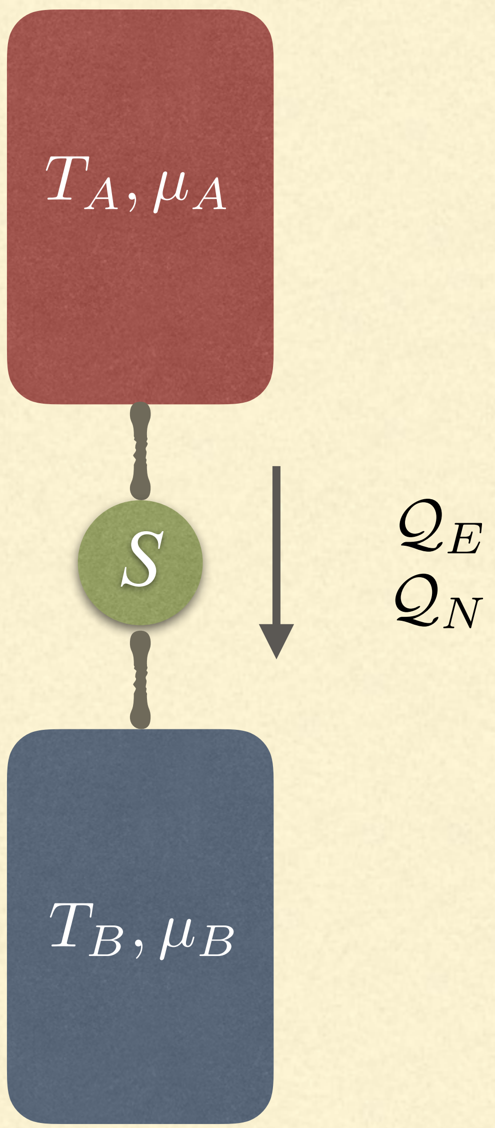
II. TURS and Zubarev Ensemble.

III. TURs and fluctuation theorems.

IV. Applications to quantum heat engines.

arXiv 1902.10428
arXiv 1904.07574

TURs - Thermodynamics of Precision



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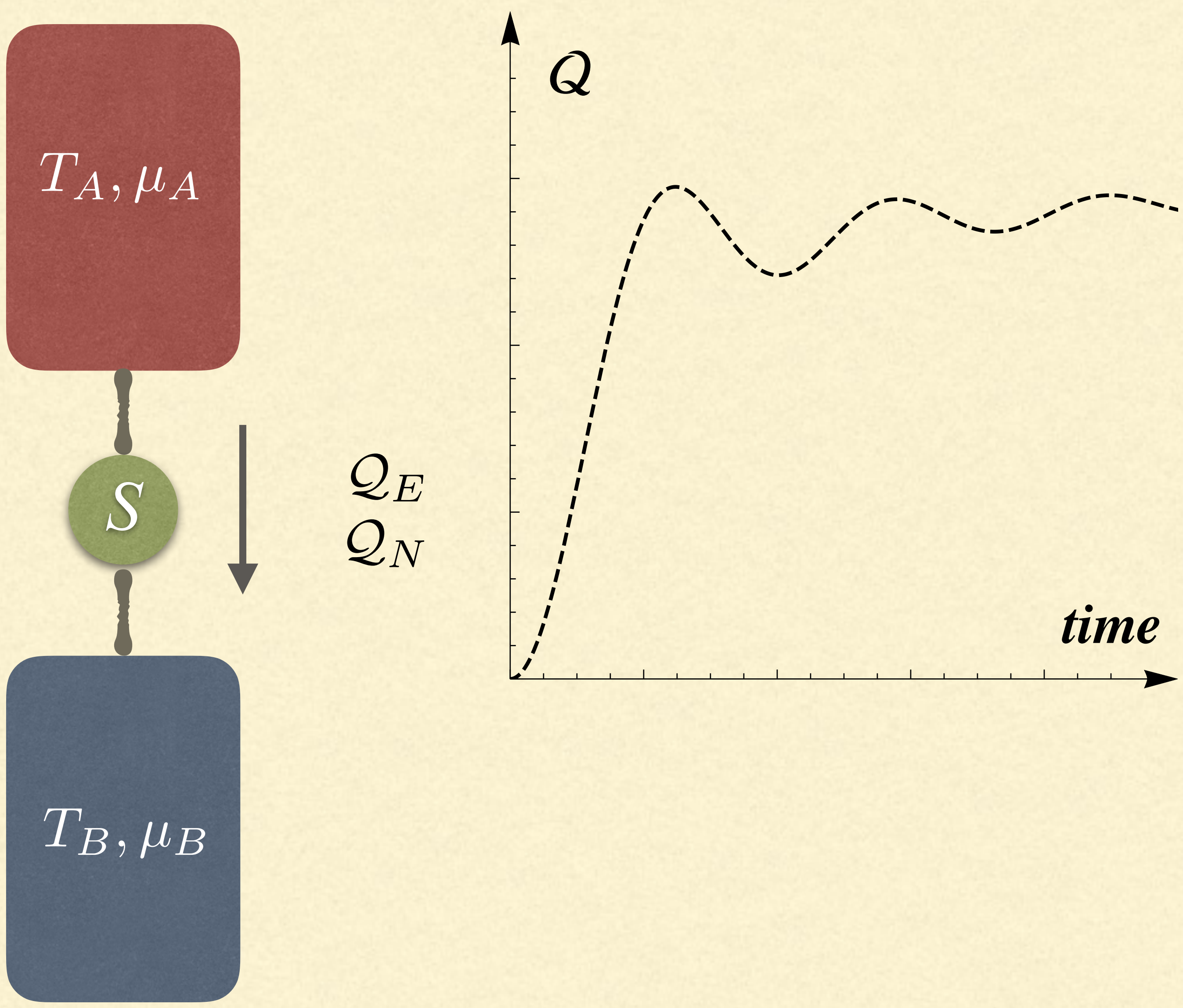

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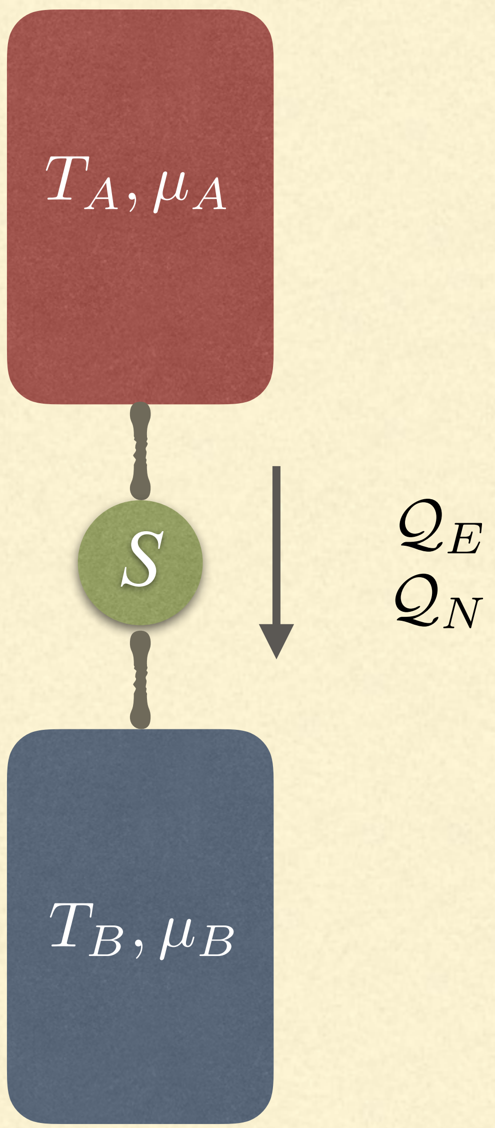

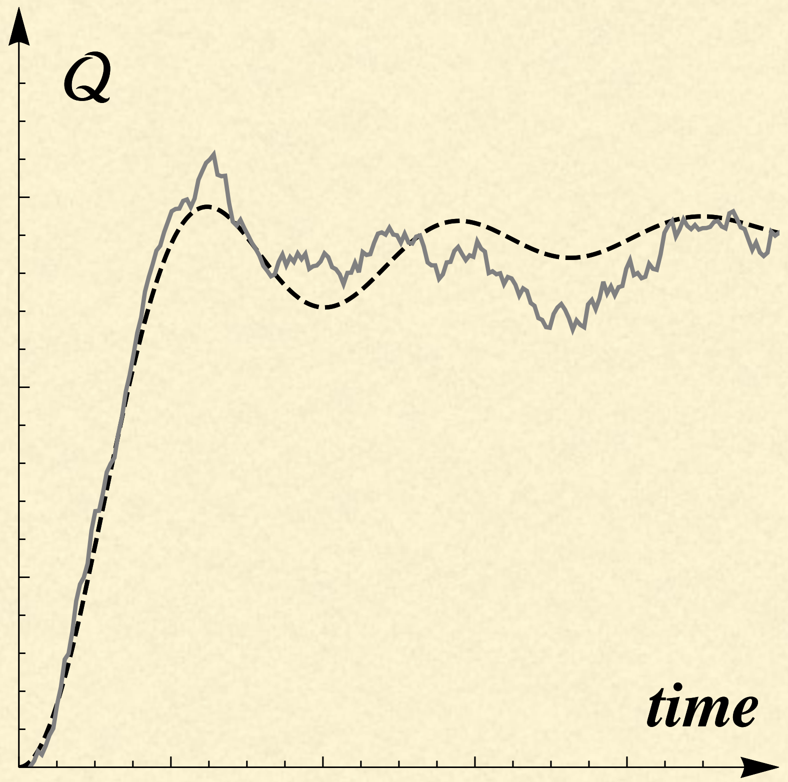

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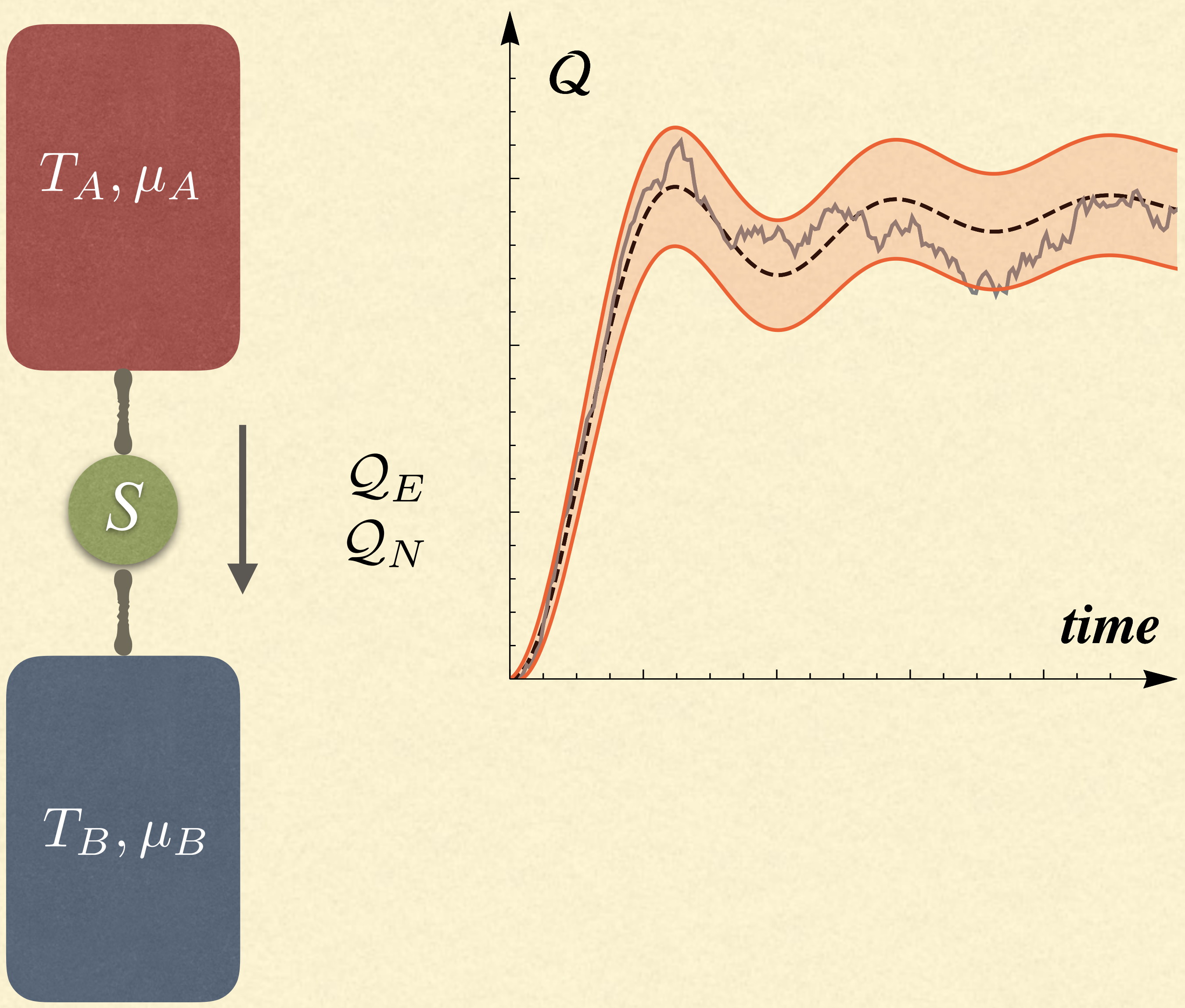

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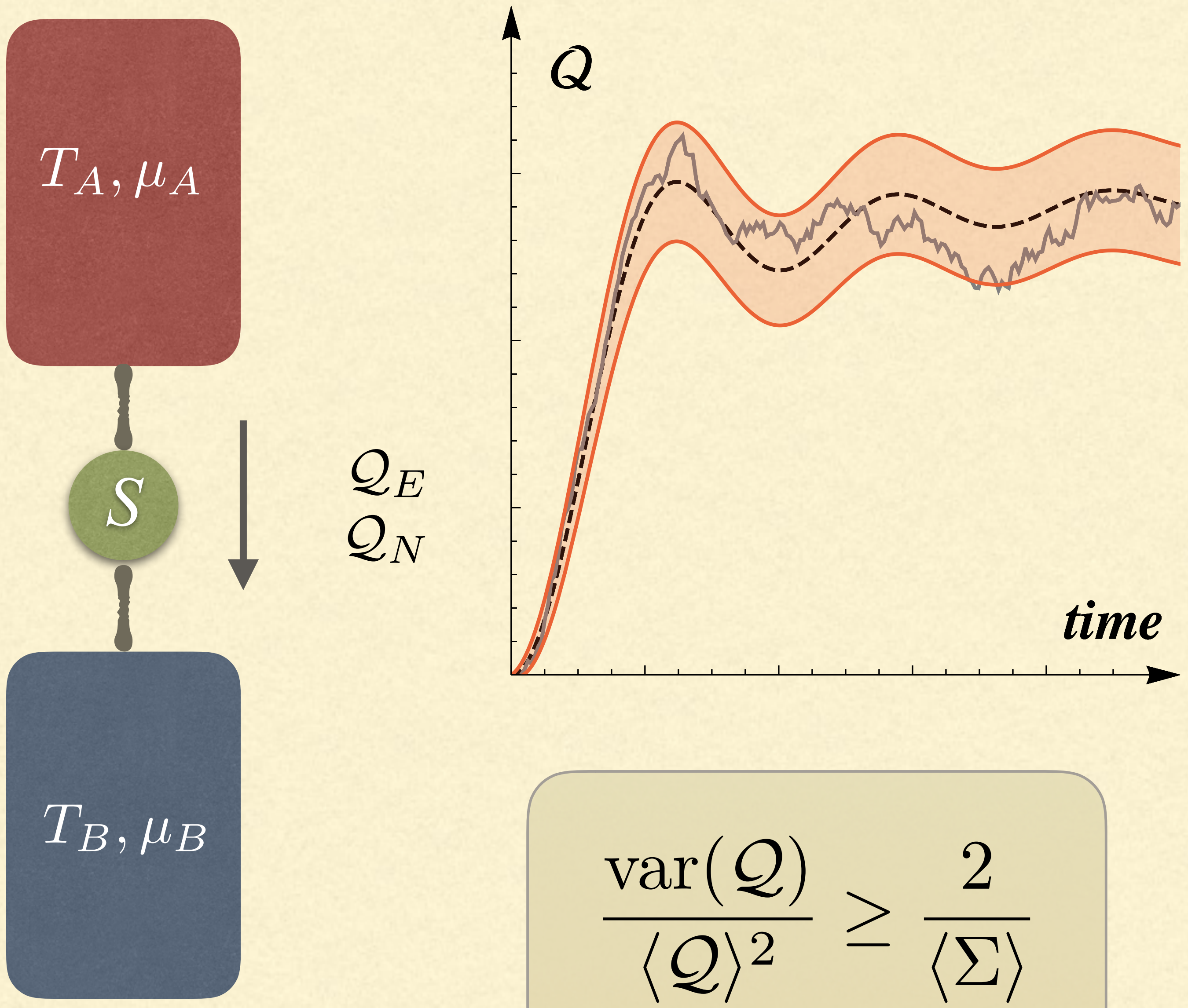

$$\frac{\mathrm{var}(\mathcal{Q})}{\langle \mathcal{Q} \rangle^2} \geq \frac{2}{\langle \Sigma \rangle}$$

A. C. Barato, U. Seifert, "Thermodynamic Uncertainty Relation for Biomolecular Processes", *Physical Review Letters*, **114**, 158101 (2015)

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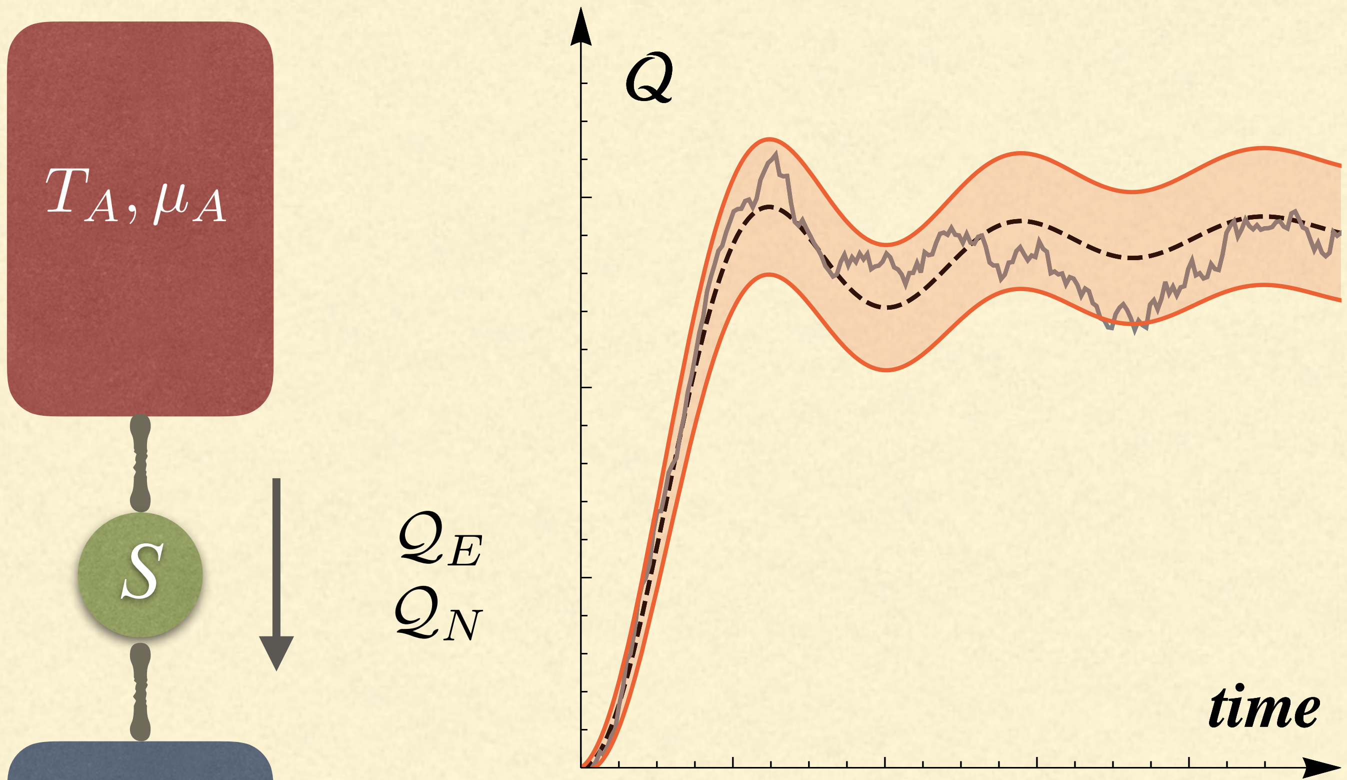

$$\frac{\mathrm{var}(\mathcal{Q})}{\langle \mathcal{Q} \rangle^2} \geq \frac{2}{\langle \Sigma \rangle}$$

- Implications for quantum heat engines:

$$\mathrm{var}(P) \geq 2T_H \langle P \rangle \frac{\eta}{\eta_C - \eta}$$

arXiv 1705.05817

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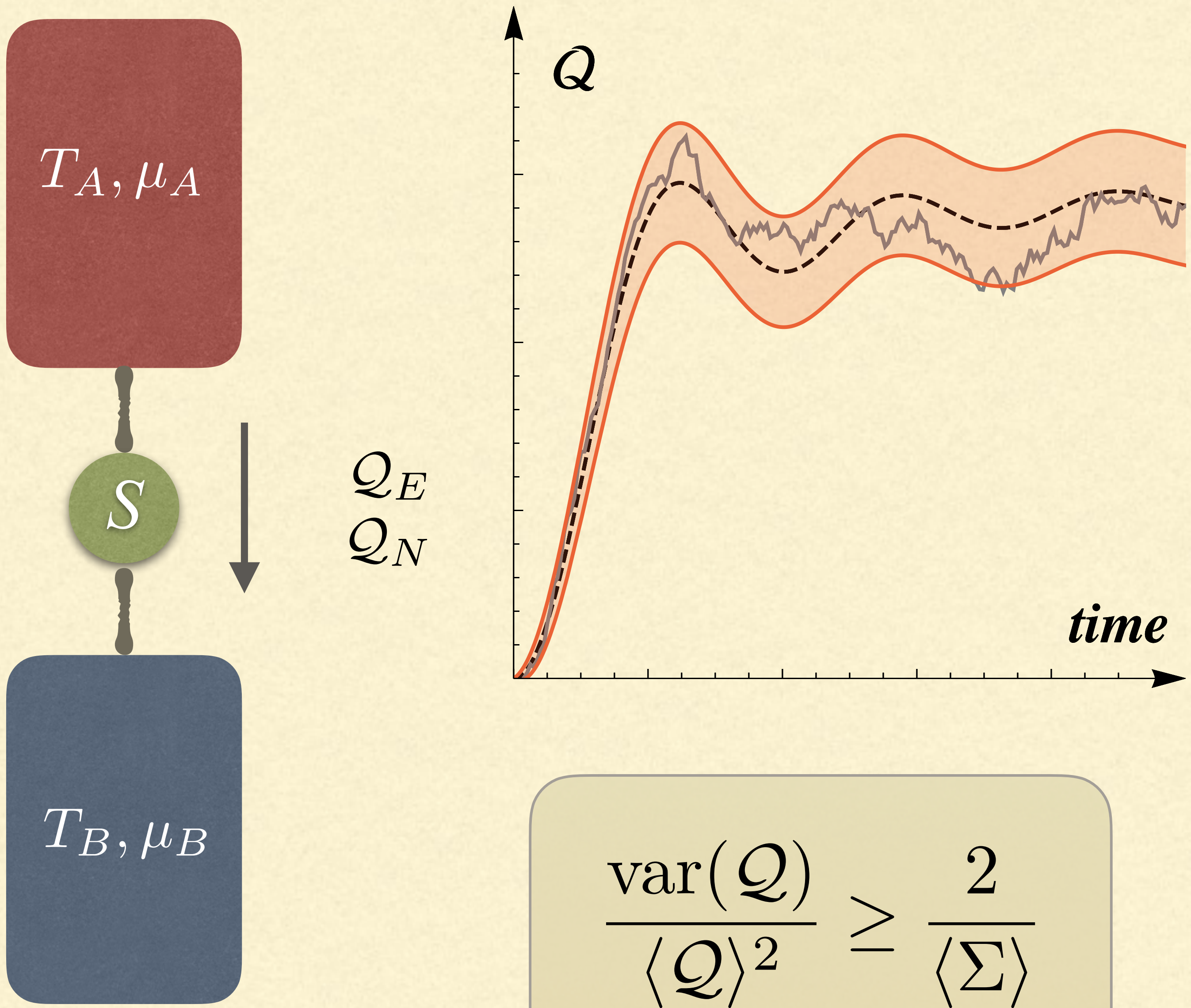

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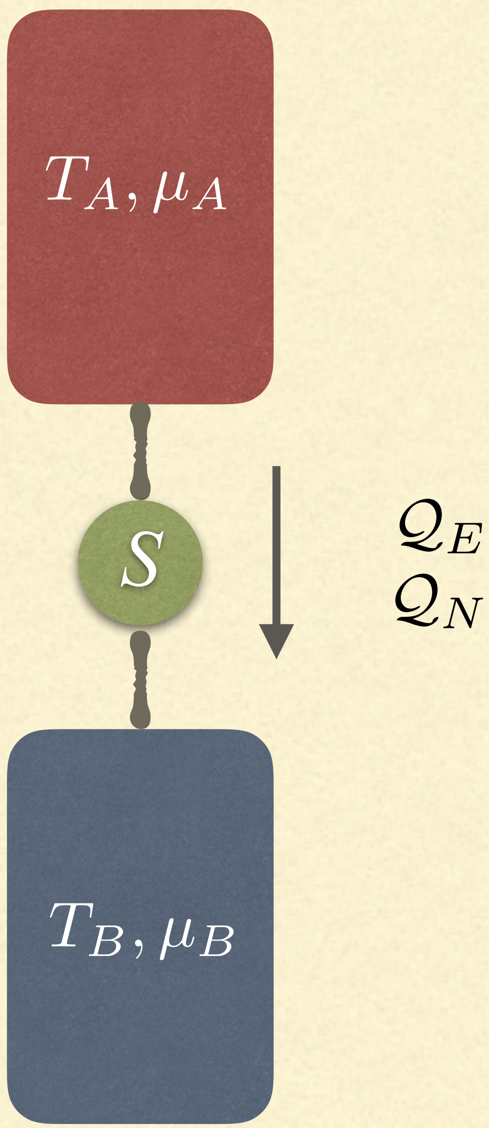

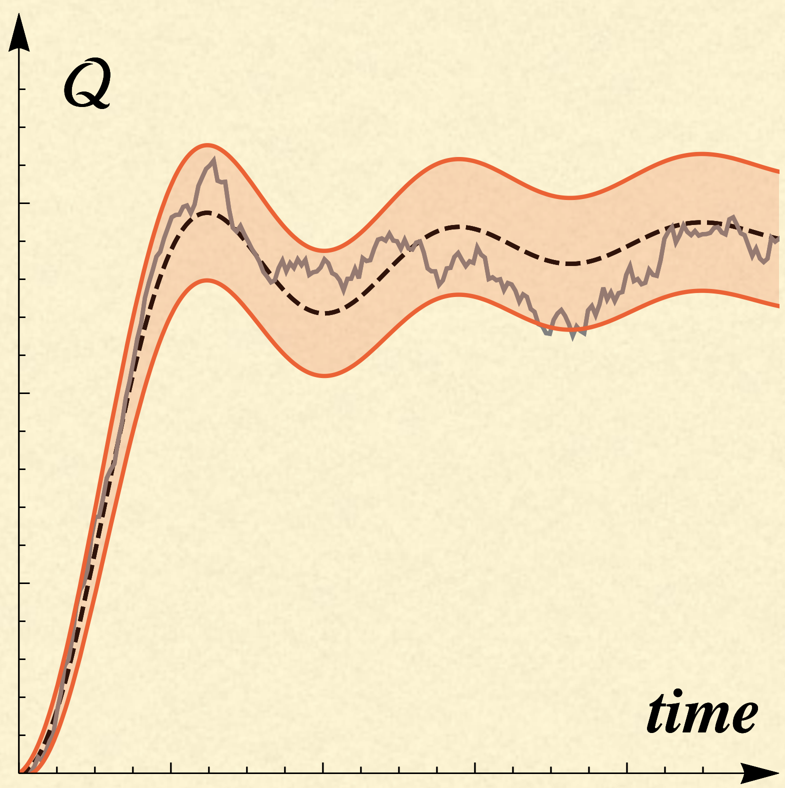

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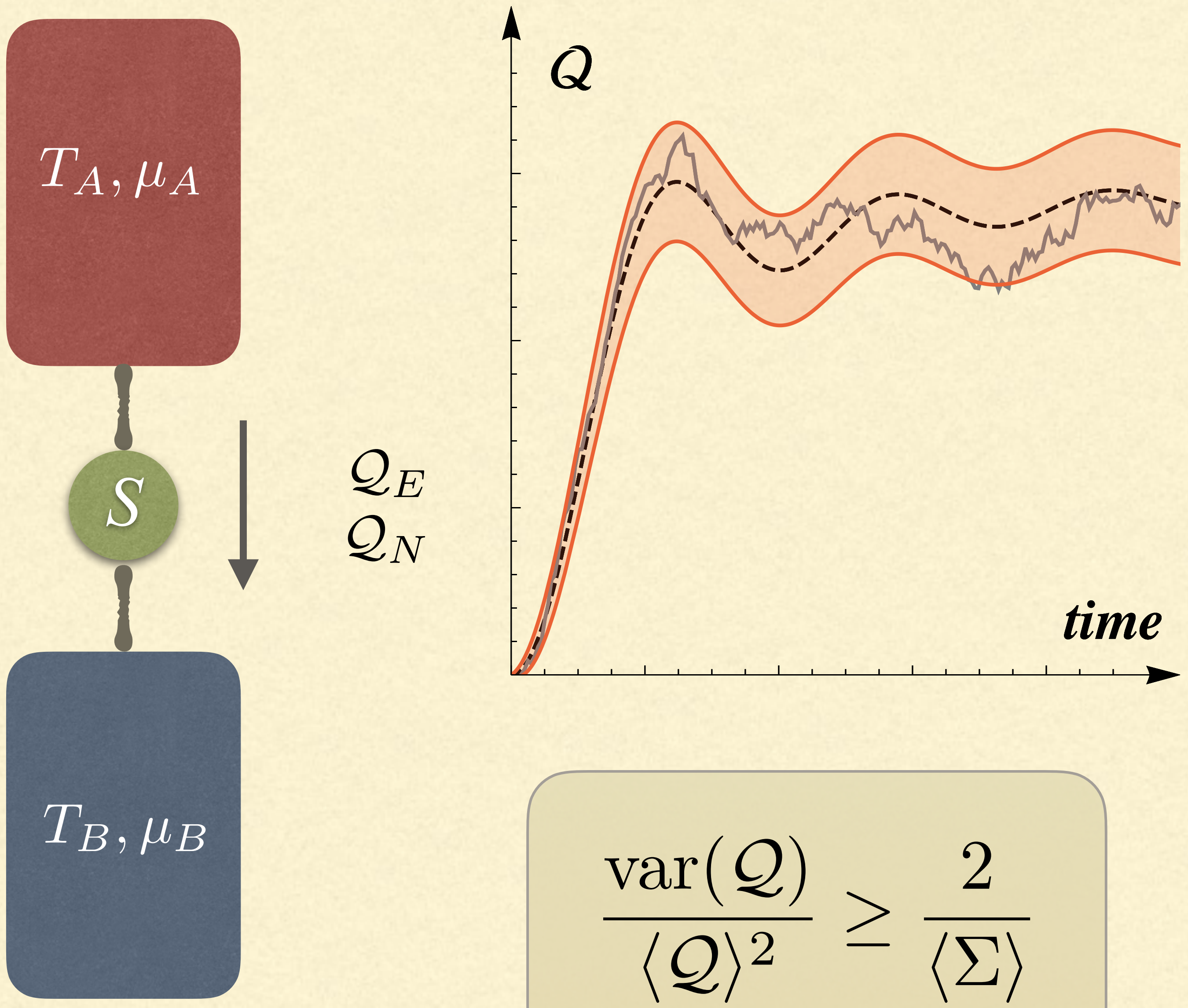

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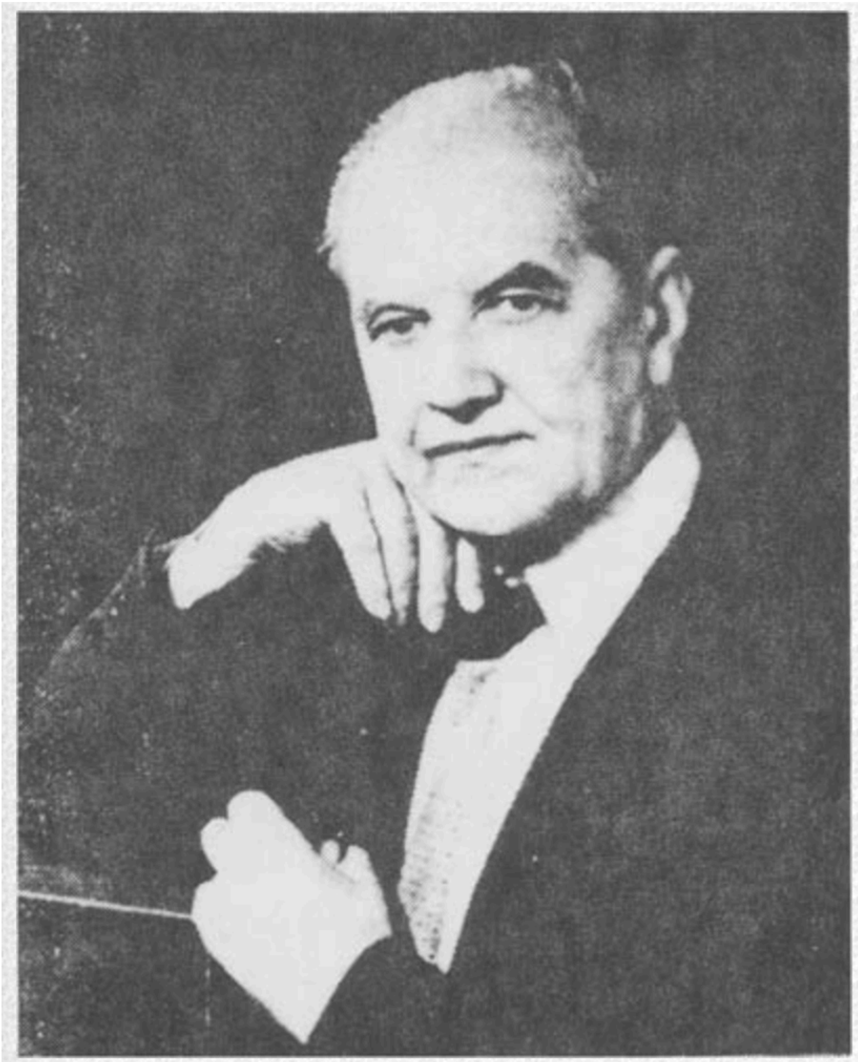

UDK 536.75, 536-12.01; PACS 05.70.L, 47.70

NONEQUILIBRIUM STATISTICAL OPERATOR AS A GENERALIZATION OF GIBBS DISTRIBUTION FOR NONEQUILIBRIUM CASE

D. N. Zubarev

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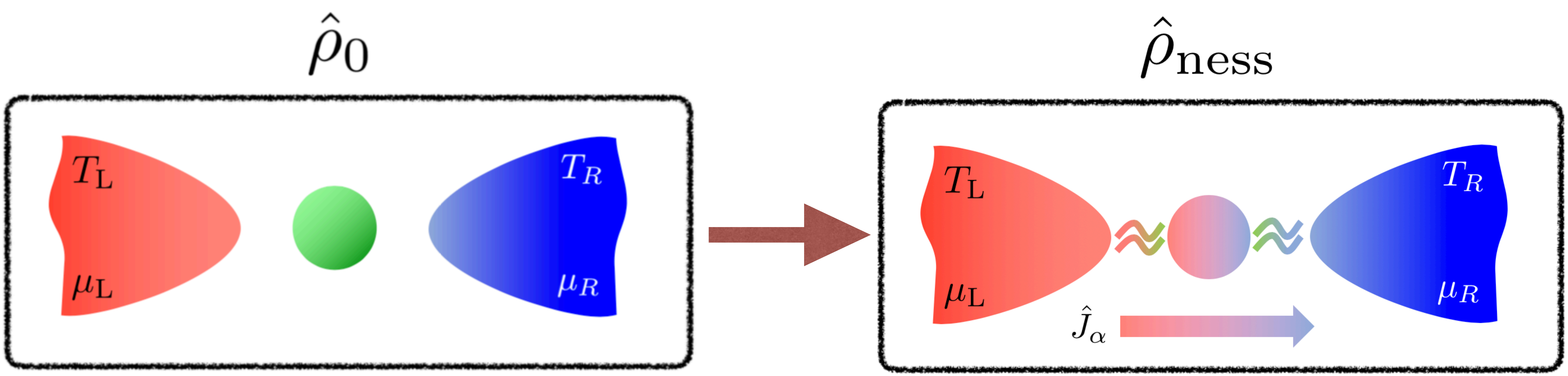
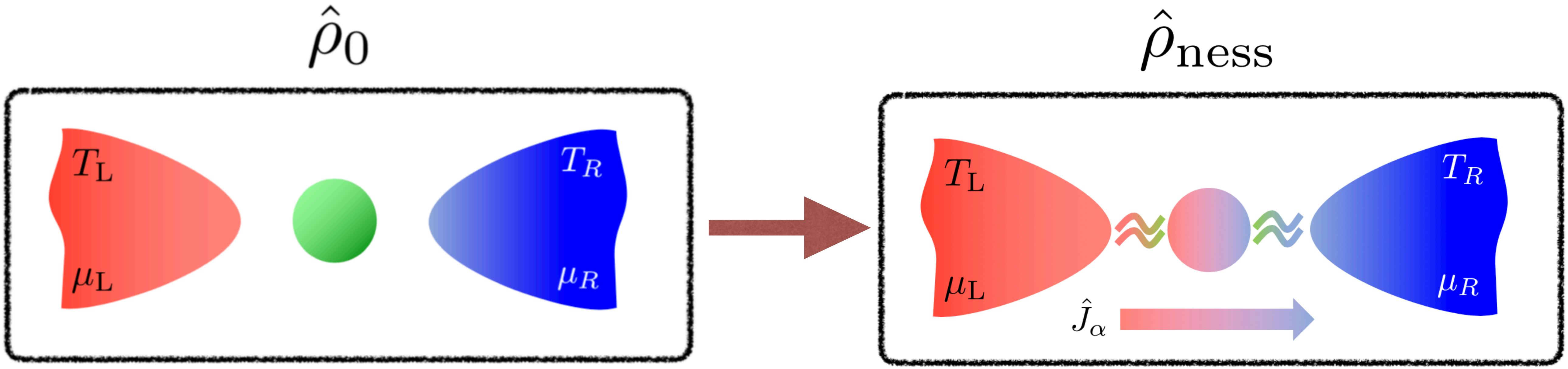
Russian Academy of Sciences
Moskow, Russia





Total Hamiltonian: $\hat{\mathcal{H}} = \hat{\mathcal{H}}_0 + \hat{\mathcal{H}}_{\mathrm{int}}$

$$\hat{\mathcal{H}}_0 = \hat{\mathcal{H}}_C + \hat{\mathcal{H}}_R + \hat{\mathcal{H}}_L$$

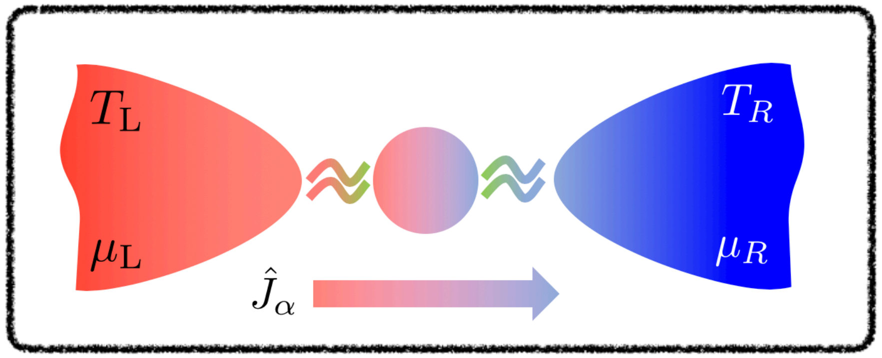
$$\hat{\mathcal{H}}_{\mathrm{int}} = \hat{V}_{LC} + \hat{V}_{RC}$$

Initial factorized condition: $\hat{\rho}_0 = Z_L^{-1} e^{-\beta_L\left(\hat{\mathcal{H}}_L - \mu_L \hat{N}_L\right)} \otimes \hat{\rho_C} \otimes Z_R^{-1} e^{-\beta_R\left(\hat{\mathcal{H}}_R - \mu_R \hat{N}_R\right)}$

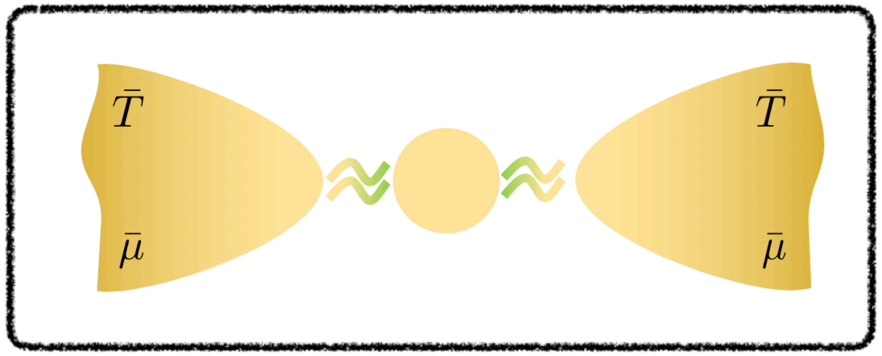
$$\hat{\rho}_{\mathrm{ness}} = Z_{\mathrm{ness}}^{-1} e^{-\overline{\beta}\left(\hat{\mathcal{H}} - \overline{\mu}\hat{N}\right) - \hat{\Sigma}} \qquad \hat{\Sigma} = -\delta_\beta \mathcal{Q}_E + \delta_{\beta\mu} \mathcal{Q}_\mu$$

TUR stems from the geometry of quantum states

$$\hat{\rho}_{\text{ness}} = Z_{\text{ness}}^{-1} e^{-\overline{\beta}(\hat{\mathcal{H}} - \overline{\mu}\hat{N}) - \hat{\Sigma}}$$



$$\hat{\rho}_{\text{les}} \equiv Z_{\text{les}}^{-1} e^{-\overline{\beta}(\hat{\mathcal{H}} - \overline{\mu}\hat{N})}$$



$$\langle \hat{\Sigma} \rangle = D(\hat{\rho}_{\text{ness}} || \hat{\rho}_{\text{les}}) + \Delta\psi$$

$$\Delta\psi = \ln\left(\frac{Z_{\text{ness}}}{Z_{\text{les}}}\right)$$

$$D(\rho||\sigma) = \text{tr}\left\{\rho \log \rho - \rho \log \sigma\right\}$$

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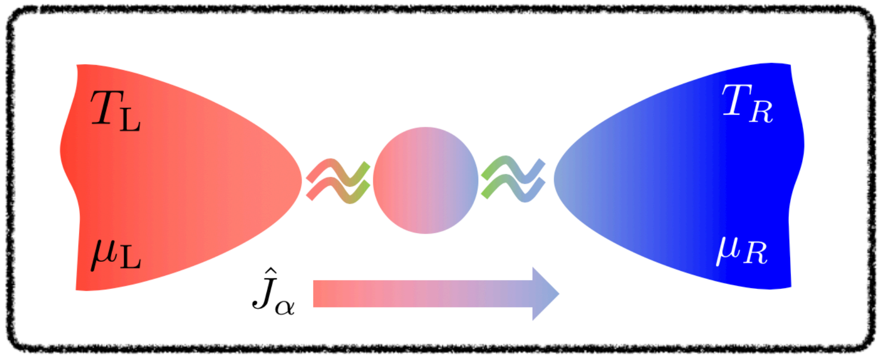

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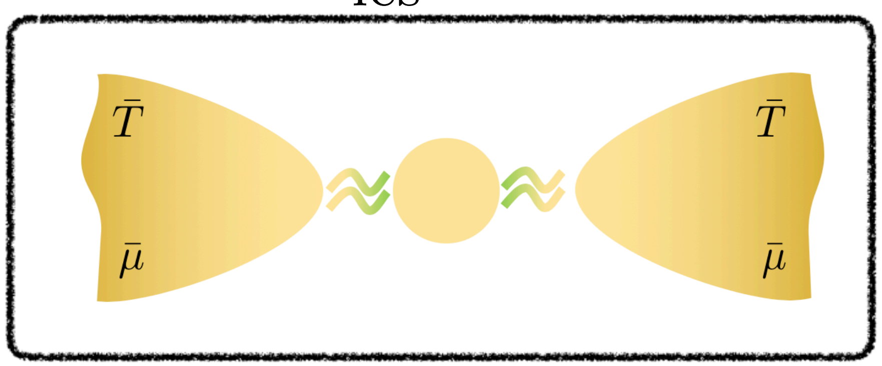

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- Fisher Information metric.
- Quantum Cramer-Rao bound.

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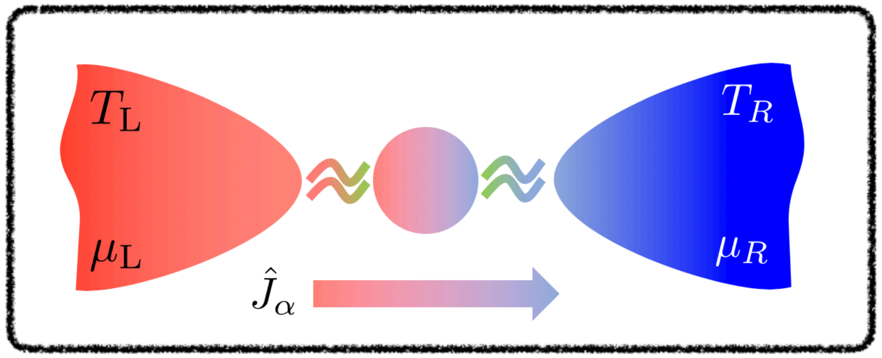

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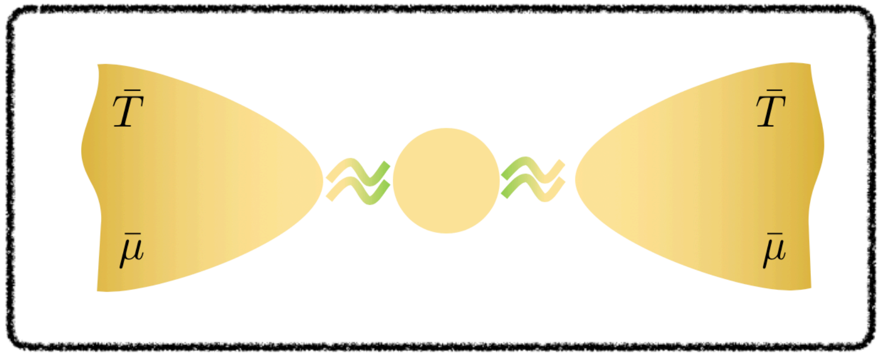

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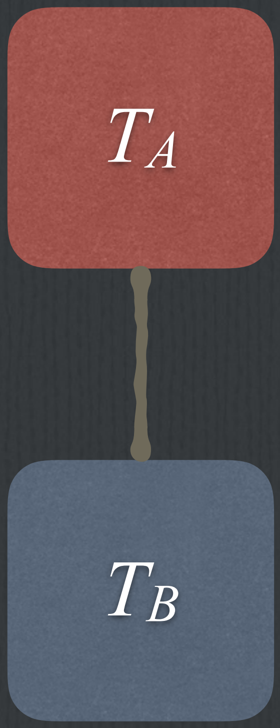
G. Guarnieri, G.T. Landi, S. R. Clark, J. Goold, “Thermodynamics of precision in quantum non equilibrium steady states”, arXiv 1902.10428

TURs from Fluctuation Theorems

- Jarzynski-Wójcik Fluctuation Theorem
  (*Phys. Rev. Lett.* **92**, 230602 (2004)

$$\frac{P(\mathcal{Q})}{P(-\mathcal{Q})} = e^{\delta\beta\mathcal{Q}} \quad \longrightarrow \quad \delta\beta\langle Q\rangle \geq 0$$

- Stronger than regular FTs since there is only the forward distribution.



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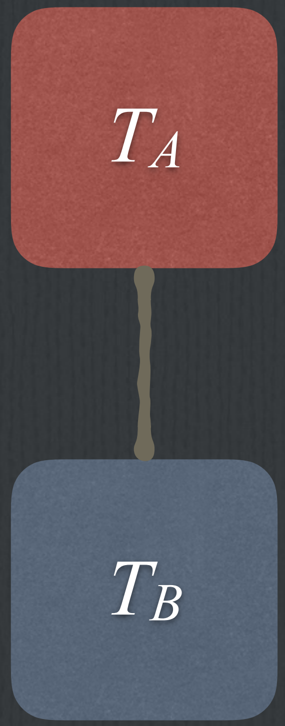

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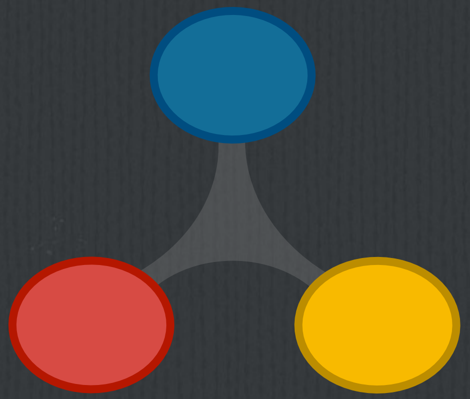
- Can be generalized to an arbitrary number of systems and an arbitrary number of currents:

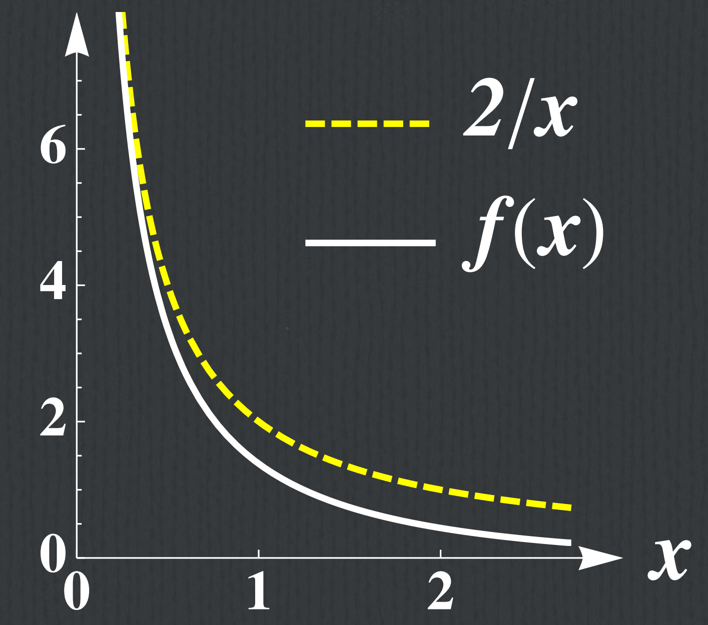
$$\frac{P(\mathcal{Q}_1, \ldots, \mathcal{Q}_n)}{P(-\mathcal{Q}_1, \ldots, -\mathcal{Q}_n)} = e^{\sum_i A_i \mathcal{Q}_i}$$



✓ TUR for the currents:

$$\frac{\text{var}(\mathcal{Q}_i)}{\langle \mathcal{Q}_i \rangle^2} \geq f(\langle \Sigma \rangle)$$

✓ This is the *tighest* (saturable) bound possible for this scenario.



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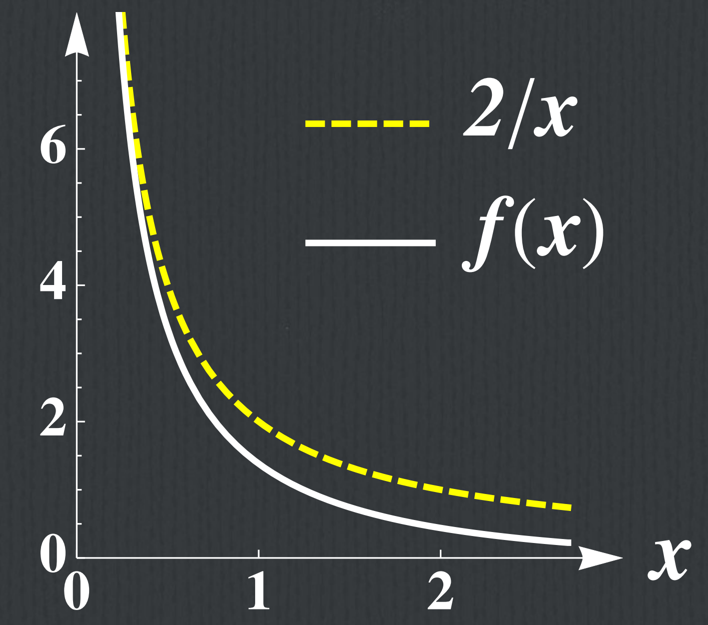

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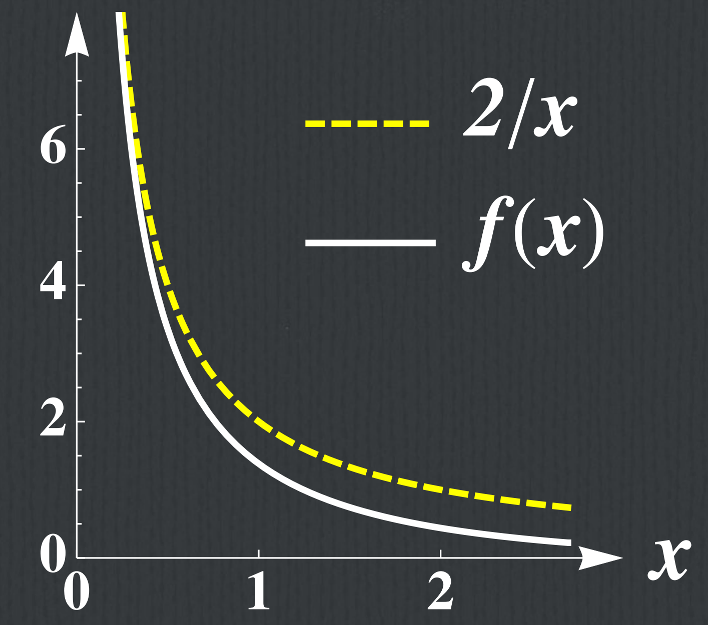

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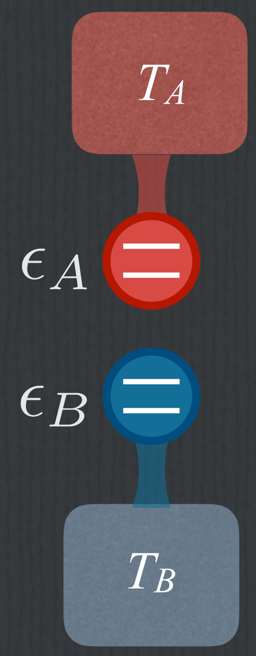
✓ Conditions on the *signs* of the covariances:

$$\frac{q_i^2}{\mathrm{var}(\mathcal{Q}_i)} + \frac{q_j^2}{\mathrm{var}(\mathcal{Q}_j)} \geq \frac{1}{f(\langle \Sigma \rangle)} \quad \longrightarrow \quad \mathrm{sign}\,\mathrm{cov}(\mathcal{Q}_i, \mathcal{Q}_j) = \mathrm{sign}\, q_i q_j$$

SWAP engine

(a)



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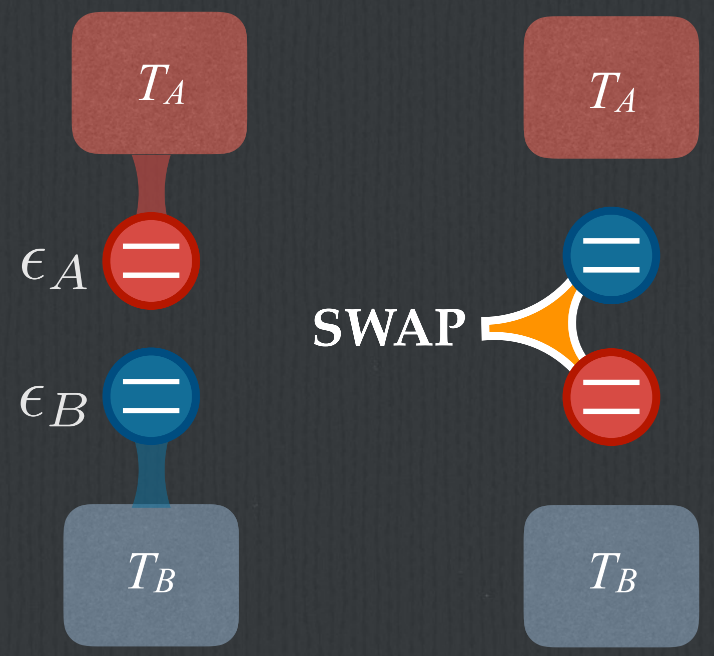

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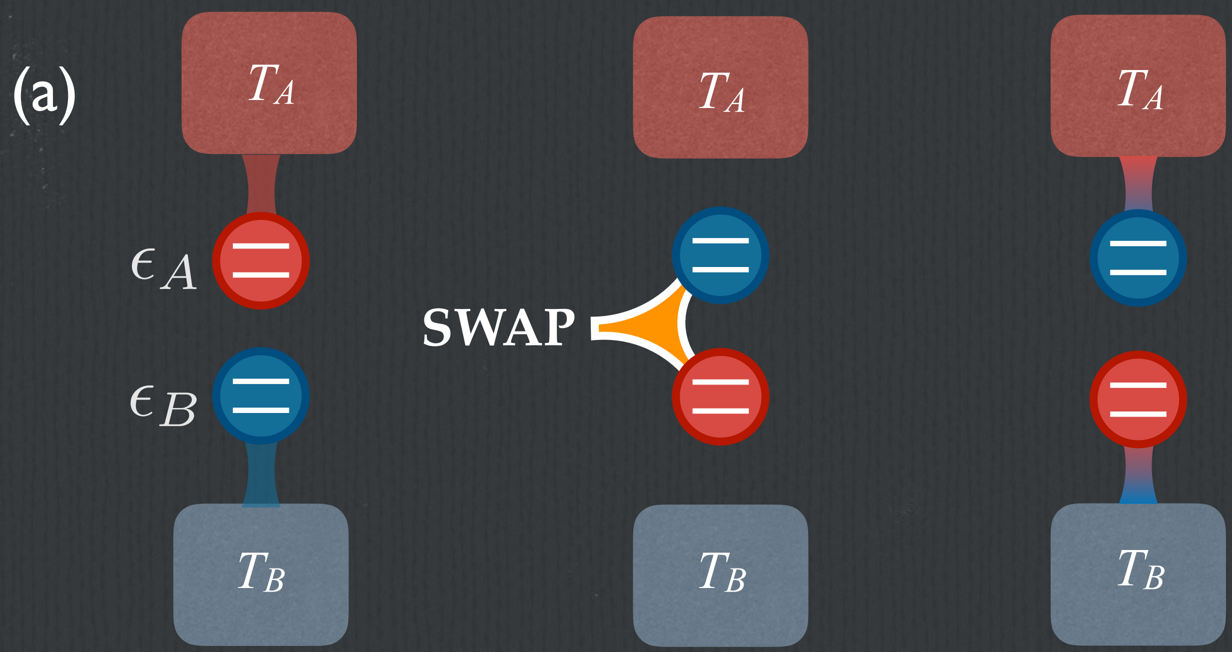

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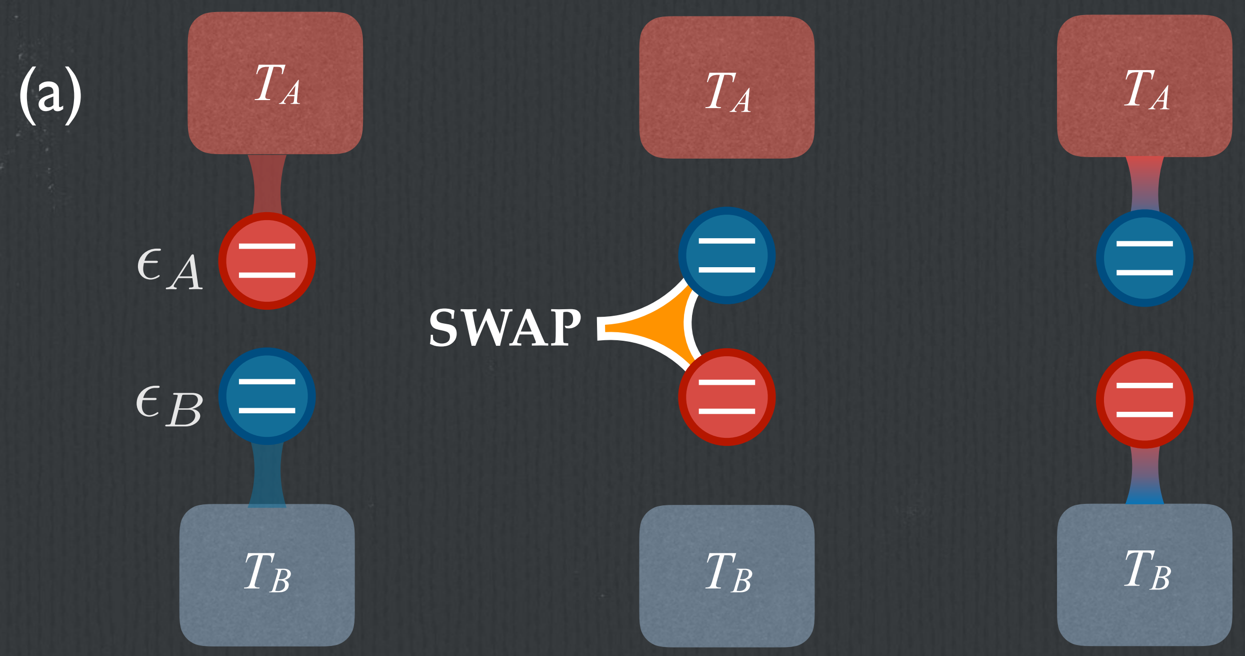
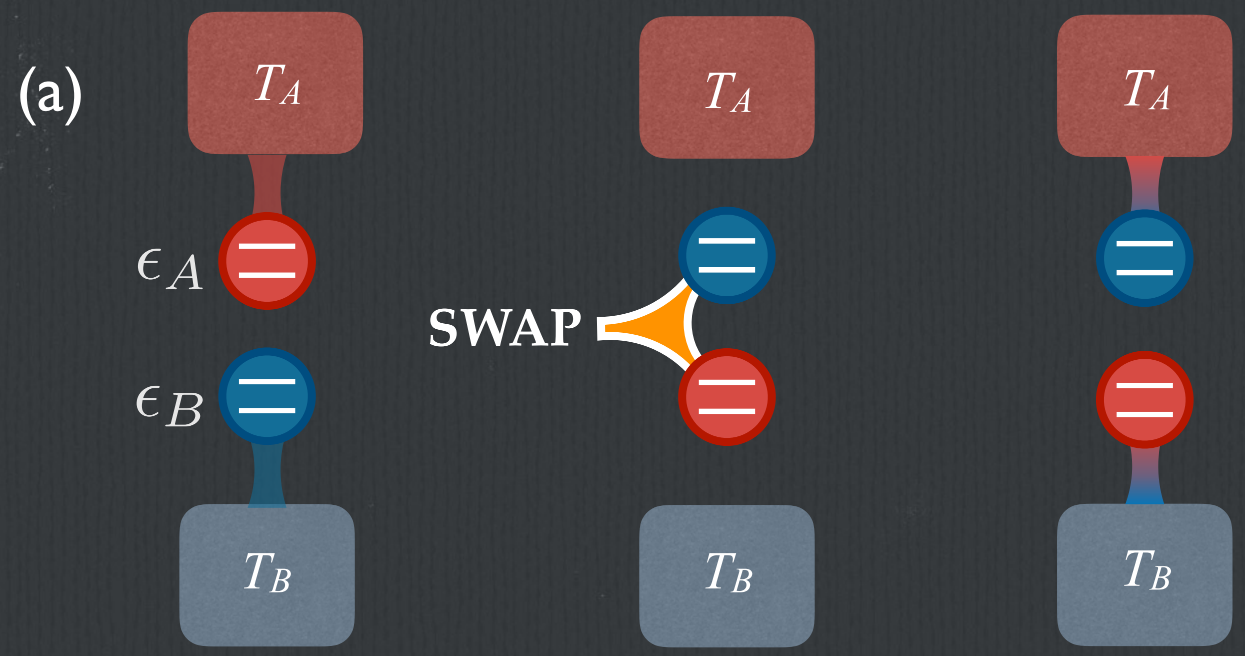


$$\frac{P(Q_H, W)}{P(-Q_H, -W)} = e^{(\beta_B - \beta_A)Q_H + \beta_B W}$$

SWAP engine



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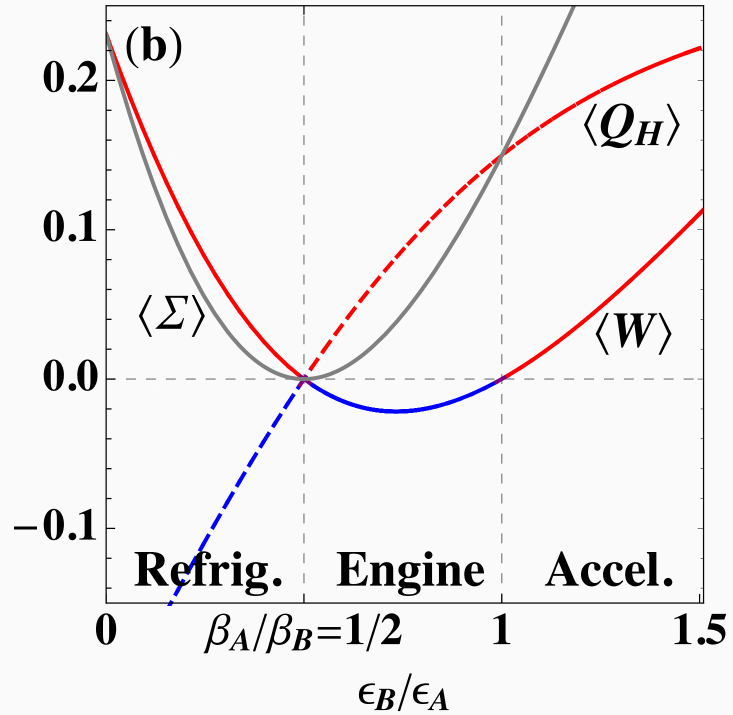

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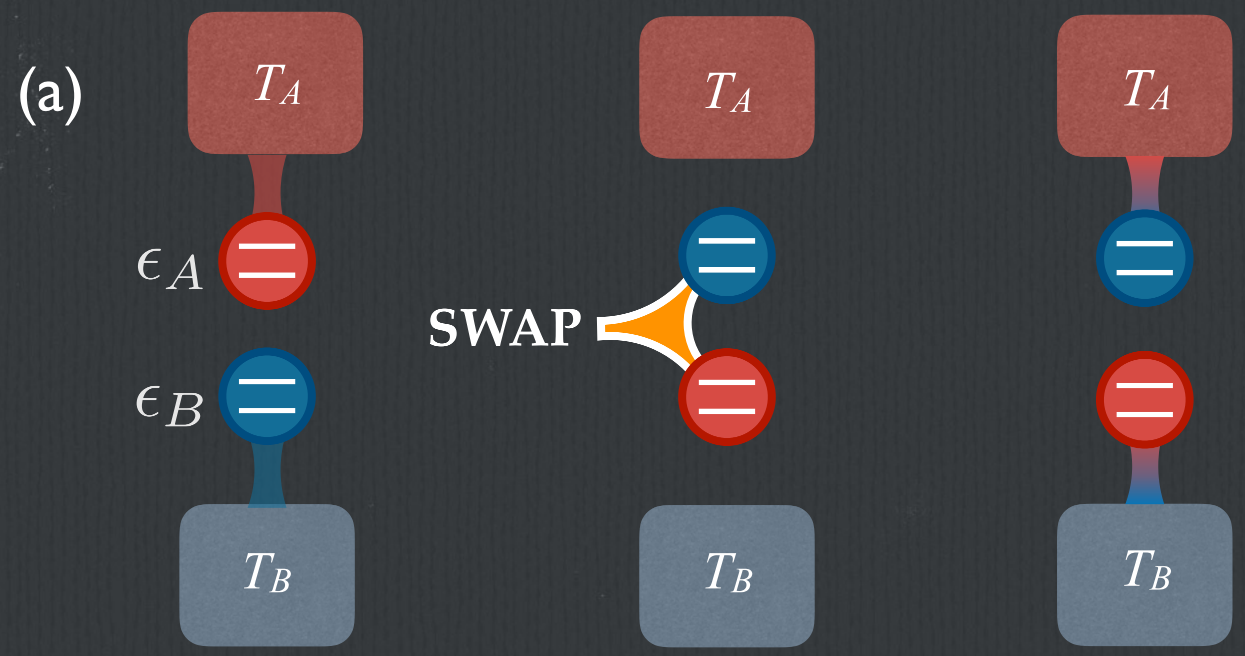

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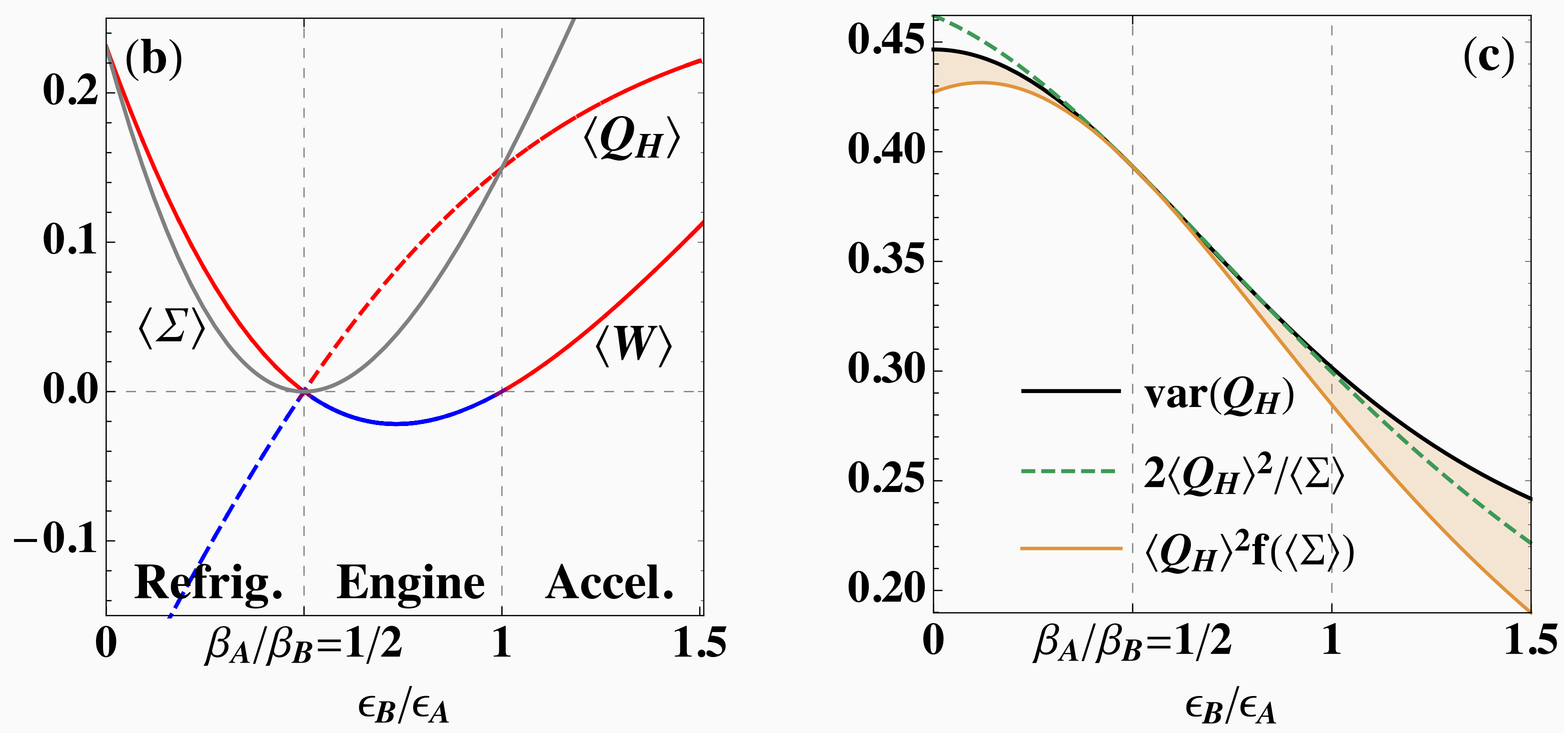

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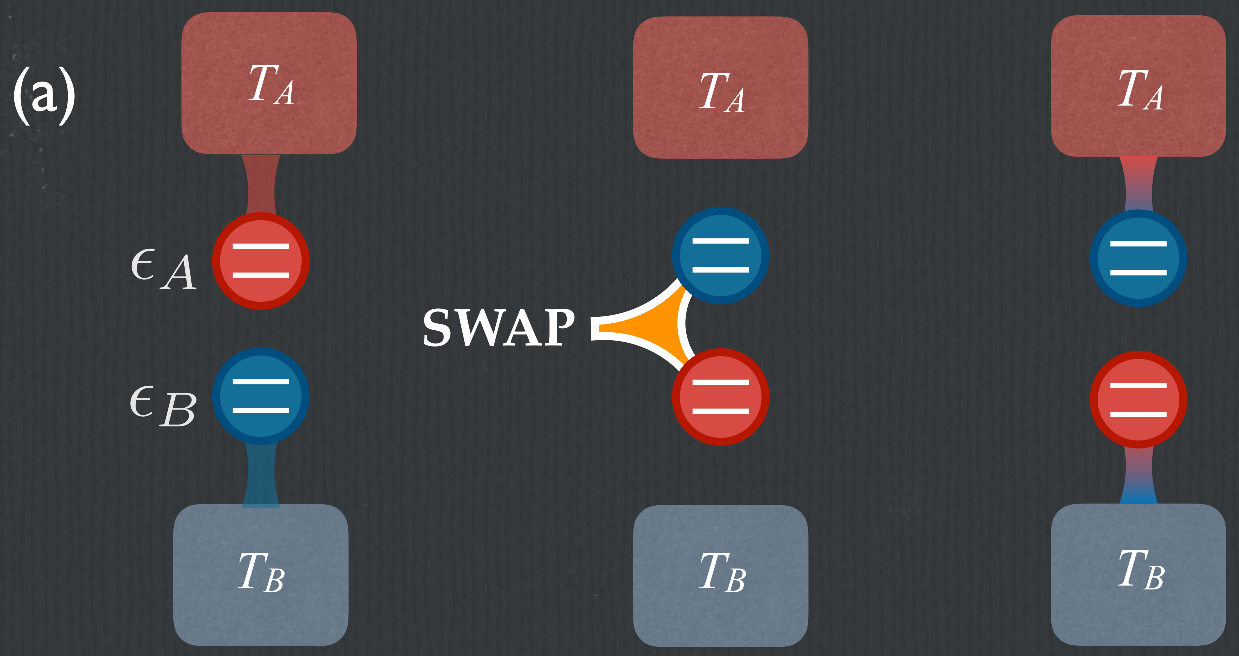

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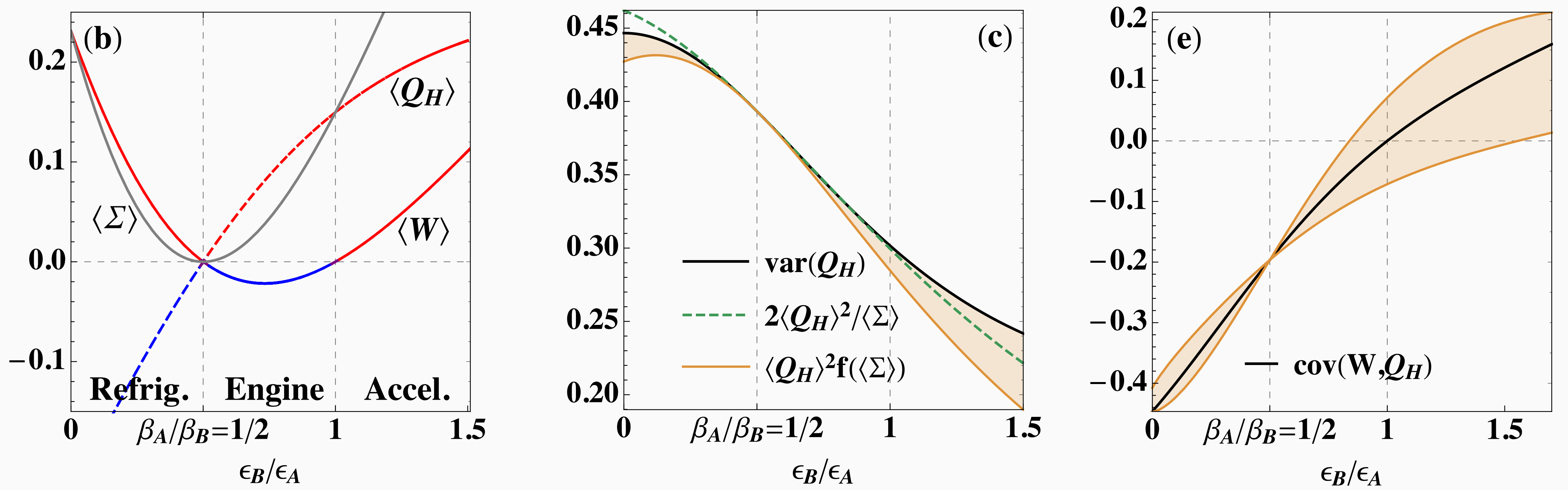

A. M. Timpanaro, G. Guarnieri, J. Goold, G, T. Landi "Thermodynamic uncertainty relations from exchange fluctuation theorems", arXiv 1904.07574

CONCLUSIONS

Quantum thermodynamics

QT^2

quantum transport

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Acknowledgements:
IFUSP, FAPESP, CNPq

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- A loose TURs exists *solely* as a consequence of the geometry of non-equilibrium steady-states.
- A dynamical TUR can be derived as a *consequence* of Fluctuation Theorems.

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