LANDAUER'S PRINCIPLE FOR GAUSSIAN QUANTUM SYSTEMS

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Quantum thermodynamics
quantum transport
IFUSP

Caxambu d'Aju
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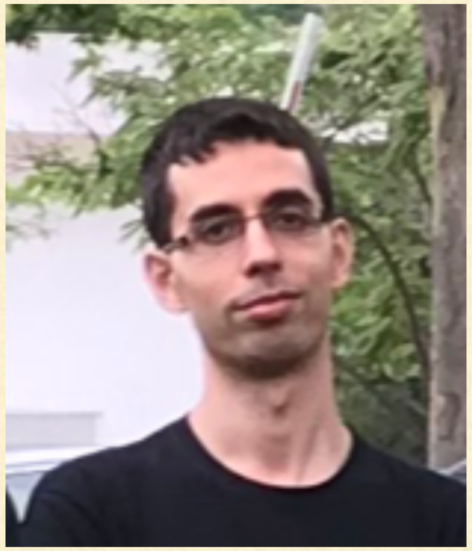
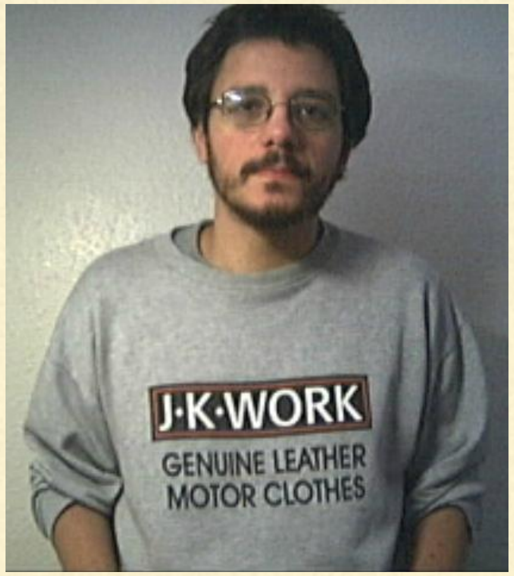
summary





I. Landauer's principle.

II. Entropy production.

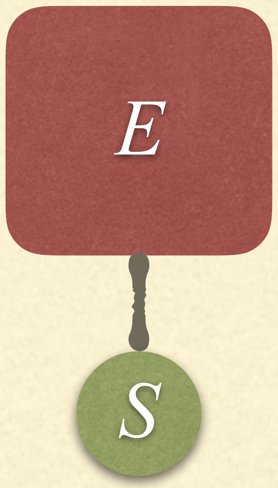
III. Quantum phase space.

IV. Landauer for Gaussian states.

J. P. Santos, GTL and Mauro Paternostro, *Phys. Rev. Lett,* **118**, 220601 (2017)
M. Brunelli, *et. al., Phys. Rev. Lett.*, **121**, 160604 (2018)
J. P. Santos, A. M. Timpanaro, M. Paternostro and GTL, *in preparation* (2019)

Landauer's principle: *information is physical*

- Fundamental **heat cost** for erasing information:



$$\Delta Q_E \geq -T\Delta S_S$$

$\Delta Q_E =$ heat dissipated in the reservoir.

$\Delta S_S =$ entropy change in the system/memory.

Mixed $\rightarrow$ Pure: $\quad \Delta S_S \leq 0 \quad \rightarrow \quad \Delta Q_E \geq 0.$

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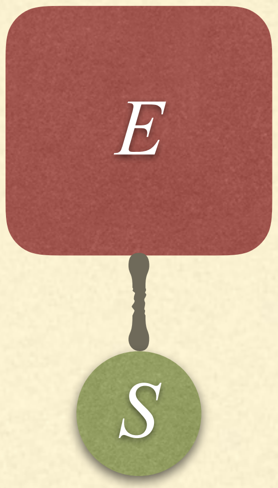

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- Would it be possible to have an erasure @ $T = 0$ with zero heat cost?

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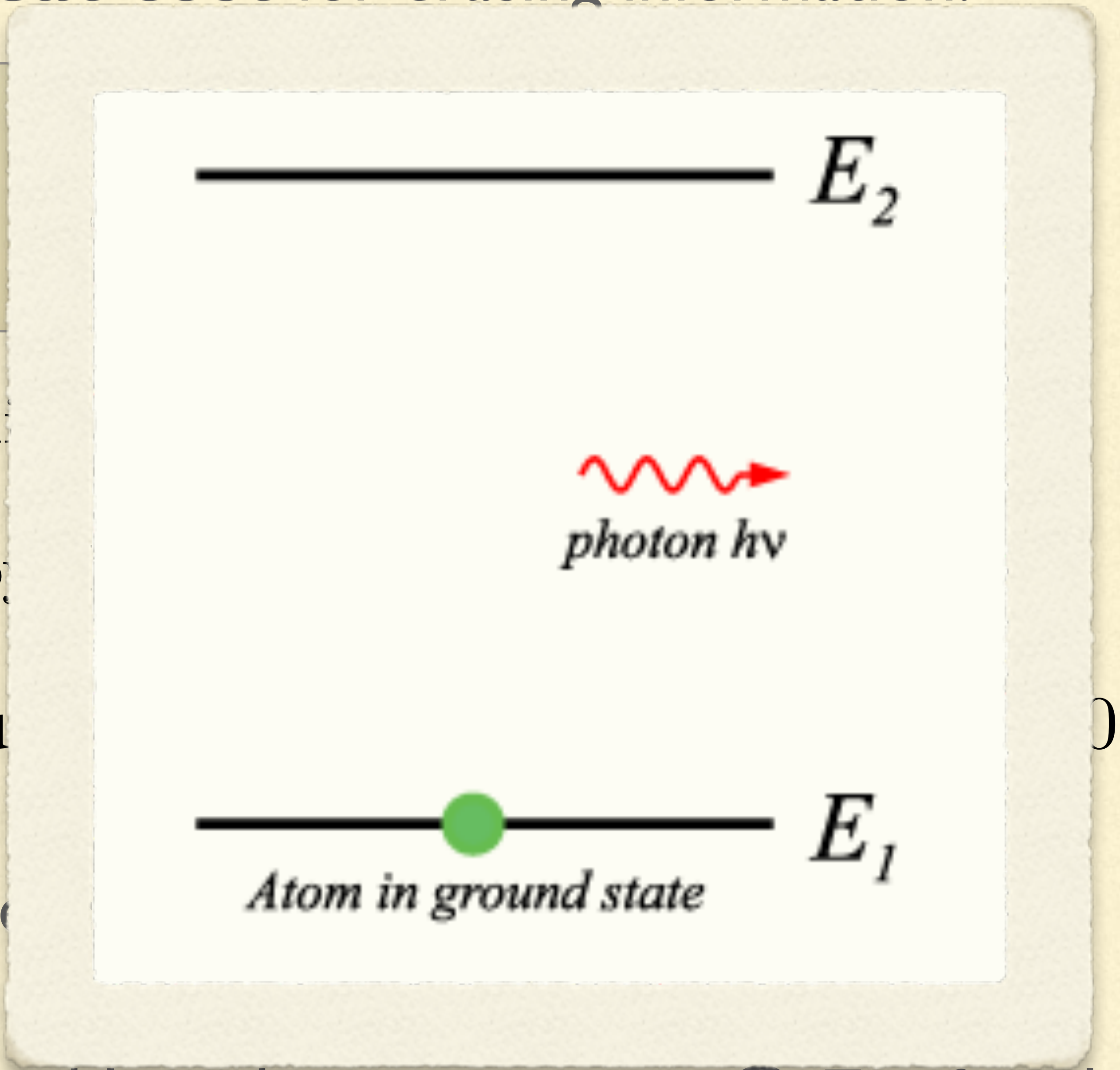

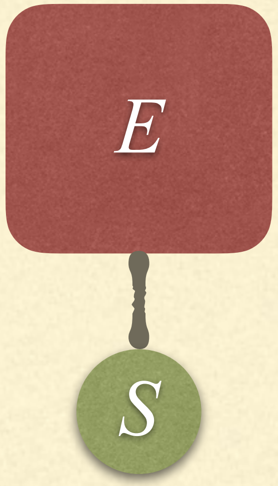

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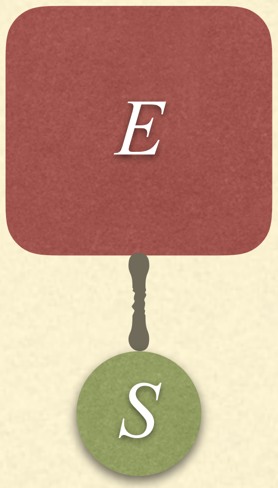
Open Quantum Dynamics

- System-environment interaction:

$$\rho'_{SE} = U(\rho_S \otimes \rho_E^{\mathrm{th}})U^\dagger \qquad \rightarrow \qquad \rho'_S = \mathrm{tr}_E \rho'_{SE}$$



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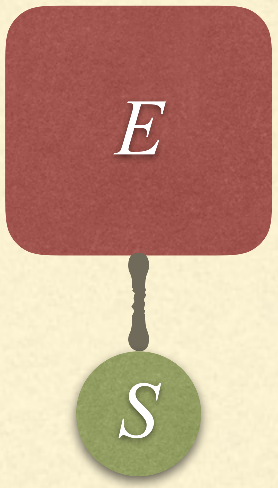

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  - There is an associated entropy production:

$$\Sigma = \mathcal{I}'(S:E) + D(\rho'_E || \rho_E^{\mathrm{th}})$$

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$$S(\rho) = -\text{tr}\rho \log \rho$$

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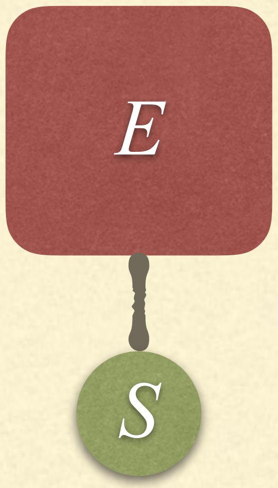

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$$\Sigma \geq 0$$

2nd law

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Quantum Landauer's Principle

- It is possible to show that if the environment is thermal, then

$$\Sigma = \mathcal{I}'(S:E) + D(\rho'_E || \rho_E^{\mathrm{th}}) = \beta \Delta Q_E - \Delta S_S \geq 0$$

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- Landauer's principle is thus a *direct* consequence of the 2nd law.
- But this continues to be problematic @ T = 0.
  - Note that in this case the relative entropy diverges because the target state becomes pure.

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Wigner entropy production

- Focus on continuous variable systems with Gaussian states (q-optics, BECs, phonons, trapped ions, &c.).

- Instead of using the von Neumann entropy, use the entropy of the *Wigner function*

$$S_W = -\int d^2\alpha \, W \log W$$

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- For Gaussian states, the Wigner function is always non-negative.

- Moreover, it is related to the Rényi-2 entropy:

$$S_2 = -\log \mathrm{tr} \rho^2$$

J. P. Santos, GTL and Mauro Paternostro, *Phys. Rev. Lett,* **118**, 220601 (2017)
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Adesso, Girolami, Serafini, *PRL,* **109**, 190502 (2012)

Wigner-Landauer principle for CVs

- We assume that *S* and *E* are both bosonic and Gaussian.
    - Moreover, their unitary is Gaussian preserving.
- Define the Wigner entropy production:

$$\Sigma_W = \mathcal{I}'_W(S\!:\!E) + D_W(W'_E||W_E^{\text{th}}) \geq 0$$

- where all quantities are now defined in terms of the Wigner function. e.g.,

$$D_W(W_1||W_2) = \int d^2\alpha\, W_1 \log W_1/W_2$$

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- For Gaussian states, the Wigner entropy satisfies the strong subadditivity inequality. All entropic quantities are well behaved.

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- This is not yet very useful:
    - What makes Landauer useful is the connection to **heat**:

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- Playing with a chain of inequalities leads to:

$$\Delta Q_E \geq \Gamma_{\rm min}\Delta S_{W,S}, \qquad \Gamma_{\rm min} = \min_k \Omega_k(\langle b_k^\dagger b_k\rangle_{\rm th} + 1/2)$$

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- For high temperatures $(T \gg \Omega_k, \quad \forall k)$

$$\Omega_k (\langle b_k^\dagger b_k \rangle_{\text{th}} + {}^1\!/_2) \simeq T$$

- We then recover the usual Landauer principle.

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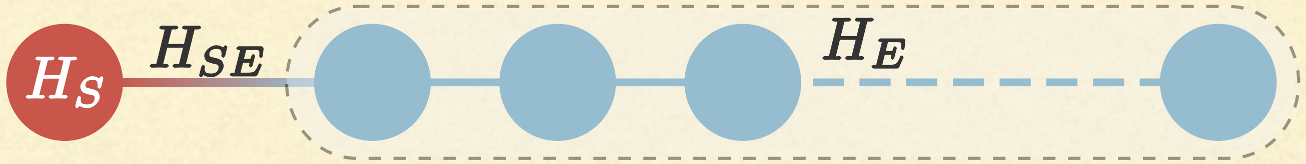
- But now if $T = 0$

$$\Gamma_{\min} = \min_k \Omega_k/2 = \text{infrared cutoff}$$

- The infrared cutoff establishes a fundamental lower bound at zero temperature

Example: linear chain



$$H_{SE} = \omega a^\dagger a + \sum_{i=1}^{N_E} \omega_0 c_i^\dagger c_i - g \sum_{i=1}^{N_E - 1} (c_i^\dagger c_{i+1} + c_{i+1}^\dagger c_i) - \lambda (a^\dagger c_1 + a c_1^\dagger)$$

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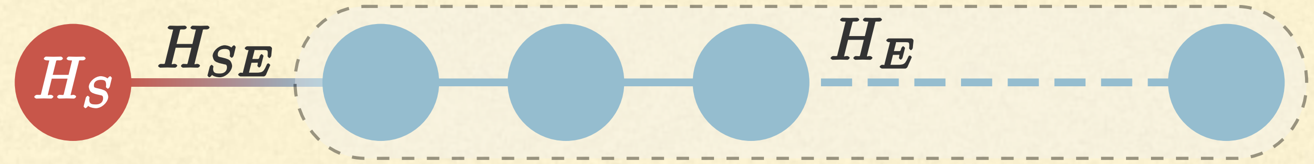

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- Diagonalizing the bath part leads to normal modes:

$$b_k = \sqrt{\frac{2}{N_E + 1}} \sum_{i=1}^{N_E} \sin(ik) c_i \qquad \Omega_k = \omega_0 - g \cos(k)$$

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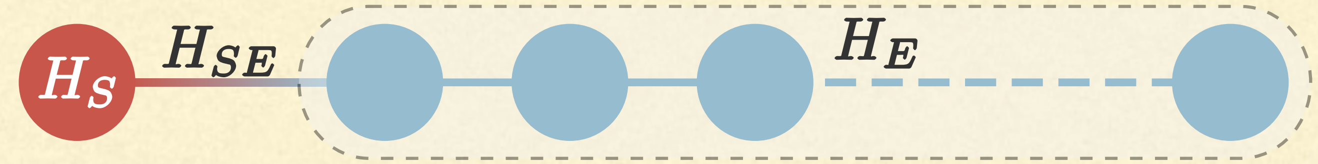

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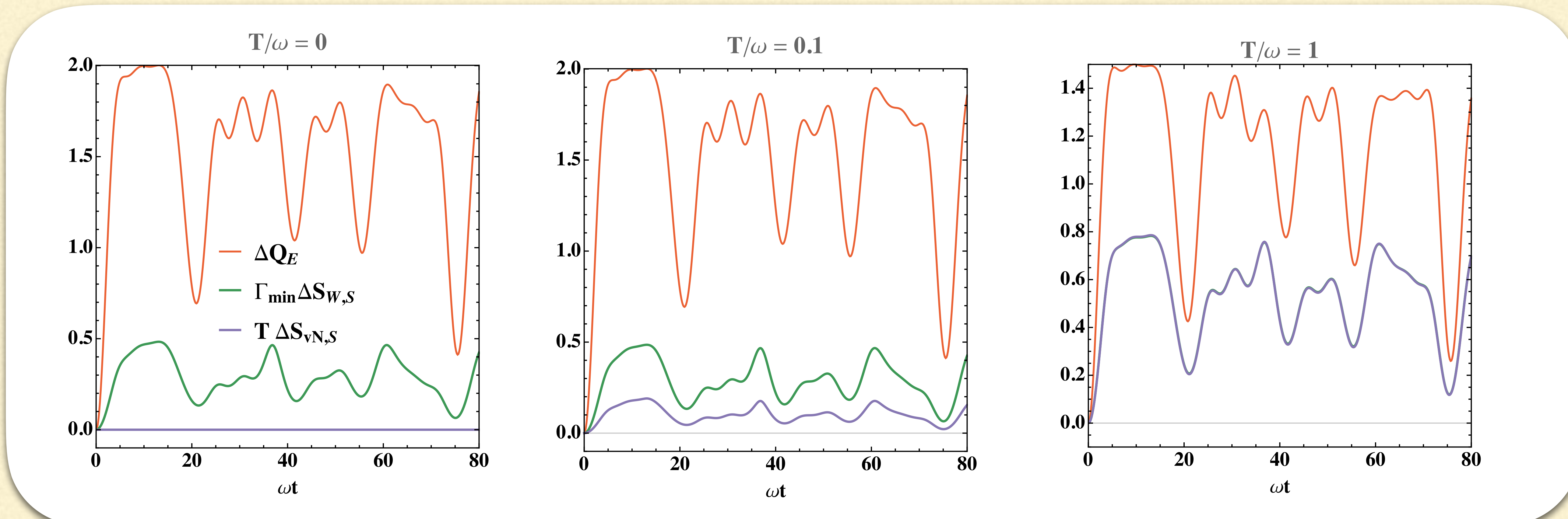

Conclusions

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IFUSP, FAPESP, CNPq

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Conclusions

- Landauer's principle: *heat up to erase information*
- Landauer is uninformative when $T = 0$.
- Formulating the problem in terms of phase space entropies reveals a minimum heat cost.

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Thank you!

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