Simple applications of 2^{nd} quantization

Ground-states of non-interacting systems

Consider a gas of Bosons in a box. The Hamiltonian will be

$$\mathcal{H} = \sum_{k} E_k \, a_k^{\dagger} a_k \tag{1}$$

where

$$E_k = \frac{k^2}{2m} \qquad k_i = \frac{2\pi l_i}{L}, \quad l_i = 0, \pm 1, \pm 2, \ldots \tag{2}$$

The eigenstates of (1) are the Fock states

$$|\vec{n}\rangle = |n_1, n_2, \ldots, n_k, \ldots\rangle \tag{3}$$

with corresponding energies

$$\mathcal{E}(\vec{n}) = \sum_{k} E_k \, n_k \tag{4}$$

The ground state is the configuration with the smallest possible energy. Thus, if we have N particles, the ground-state will be simply the Fock space with N particles in the $k=0$ state. We can construct this state from the vacuum as

$$|gs\rangle = \frac{(a_0^{\dagger})^N}{\sqrt{N!}} |0\rangle \tag{5}$$

where $1/\sqrt{N!}$ is just a normalization factor.