Magmans

The Heisenberg interaction

In these notes we are going to consider the interaction among spins in a crystal. We assume the electrons are localized in the atomic sites, as in an insulator. This means we'll be concerned with localized magnetism, in contrast to the itinerant magnetism that occurs in metals, which we will discuss later.

Each atom may have a spin contribution from several of its electrons. To account for this, we will assume each atom can be described by a spin S, not necessarily $1/2$. To each atom we therefore attribute 3 spin operators S_i^x, S_i^y, S_i^z, where $i = 1, \dots, N$ labels the atoms in the crystal. Spins of different sites commute and spins of the same site satisfy the canonical algebra

$$[S_i^x, S_j^y] = i \delta_{ij} S_i^x \qquad (1)$$

(plus cyclic permutations).

As a convenient basis, we choose the states which diagonalize each S_i^z,

$$|\vec{m}\rangle = |m_1, \dots, m_N\rangle \qquad (2)$$

where

$$m_i = -S, -S+1, \dots, S-1, S$$

Each spin Hilbert space has dimension $(2S+1)$, so the total space will have dimension $(2S+1)^N$.