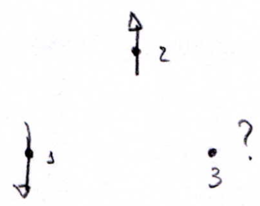
Magnetic frustration

Consider 3 spins in a triangle, interacting anti-ferromagnetically:



Suppose we minimize the 1-2 energy by making them anti-parallel. then what should be the orientation of spin 3. If it is anti-parallel to 1 it will be parallel to two, and vice-versa. thus, there is no way we can satisfy all 3 bonds.

this is a typical example of magnetic frustration, an idea which underlies many current topics of research, such as spin ices, spin glasses, spin liquids, high temperature superconductivity and so on.

the hallmark of magnetic frustration is a macroscopic degeneracy of the ground state. In these notes we will learn how to spot these frustrated configurations by making a classical analysis of the Heisenberg model.

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Consider the Heisenberg model on an arbitrary lattice

$$\mathcal{H} = - \sum_{i,j} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j \qquad (1)$$

where J_{ij} is the interaction between spins i and j (not necessarily nearest neighbors). Due to translation invariance, J_{ij} may depend only on the distance $R_i - R_j$ between the sites. Thus we write

$$\boxed{\mathcal{H} = - \sum_{i,j} J(\mathbf{R}_i - \mathbf{R}_j) \, \mathbf{S}_i \cdot \mathbf{S}_j} \qquad (2)$$

Our goal is to find a method to determine the ground-state approximately for different lattices and interactions. We will do that by replacing the spin operators $\mathbf{S}_i$ with classical vectors. This will be reasonable when the magnitude S of the spin is large.

We can see that by looking at the commutation relations

$$[S^x, S^y] = i S^z \qquad (3)$$

Dividing both sides by S^2 we get

$$\left[\frac{S^x}{S}, \frac{S^y}{S}\right] = i \frac{S^z}{S^2} \qquad (4)$$

The commutation relations determine the degree of non-classicality of an algebra. In classical physics everything commutes. Looking at (4) we see that when $S \to \infty$ the operators S^x/S and S^y/S commute. Thus, when S is large, we may treat the $\mathbf{S}_i$ as classical vectors.

From quantum mechanics we know that the eigenvalues of $\mathbf{S}^2$ are $S(S+1) \simeq S^2$. Thus, each $\mathbf{S}_i$ will be a vector of magnitude S. Thus, we can parametrize a classical spin $\mathbf{S}$ by a point in a sphere of radius S

$$
\begin{aligned}
S^x &= S \sin\theta \cos\phi \\
S^y &= S \sin\theta \sin\phi \qquad (5) \\
S^z &= S \cos\theta
\end{aligned}
$$

Now let us go back to the Heisenberg Hamiltonian (2). We define the correlation function as

$$\boxed{C(\mathbf{R}_i - \mathbf{R}_j) = \langle \mathbf{S}_i \cdot \mathbf{S}_j \rangle} \qquad (6)$$

This measures the degree of statistical correlation (either quantum or classical) between spins i and j. If this function tends to a finite value when $\mathbf{R}_i - \mathbf{R}_j$, it means the system has long-range order.