Interacting Bosons

- Interaction operators in 2nd quantization
- The Bose-Hubbard model
- Superfluidity
  - ↳ Bogoliubov's approach
  - ↳ Landau-Ginzburg's approach.

Two-body operators

In these notes we are going to take a first look at interacting systems of identical particles. Interactions are the reason behind most of the remarkable effects in condensed matter. But they are also much harder to treat. There are very few exactly soluble interacting problems, so most of the progress is usually based on approximations. We will see some of these in these notes.

A typical interacting Hamiltonian may look like

$$H = \sum_i \left\{ \frac{p_i^2}{2m} + U(x_i) \right\} + \frac{1}{2} \sum_{i \neq j} V(x_i, x_j) \qquad (1)$$

the first term is the non-interacting (free) Hamiltonian, where $U(x)$ is an external potential. the second term is a sum of 2-body interactions. Examples include the <u>Coulomb repulsion</u>

$$V(x_i - x_j) = \frac{e^2}{|x_i - x_j|} \qquad (2)$$

or a <u>hard sphere interaction</u>

$$V(x_i - x_j) = \begin{cases} V_0 & \text{if } |x_i - x_j| < a \\ 0 & \text{otherwise} \end{cases} \qquad (3)$$

Oh! I forgot to say: the factor of $1/2$ in (1) is placed to avoid double counting.