Statistical Mechanics - Problem set 1

Professor: Gabriel T. Landi

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The problems in this problem set are *open ended*. This means I will only specify which system you should study. But you will have to figure out for yourself which quantities to compute (of course, following the notes may help). The reason to do this is because research is open ended, so it is useful to practice a bit.

I also want many pretty plots, but feel free to choose how to present them: you can just sketch them on paper, you can print and paste, or you can simply e-mail me the Mathematica notebook. Up to you.

1. **A degenerate 3-level system:** Consider a 3-level system (qutrit) with energy levels given by

$$E_0 = 0, \qquad E_1 = \epsilon + \Delta, \qquad E_2 = \epsilon - \Delta. \tag{1}$$

   Study the equilibrium properties of this system. And, in particular, compare the cases $\Delta \neq 0$ with $\Delta = 0$.

2. **Lipkin-Meshkov-Glick model:** Study the equilibrium properties of a spin $S = 1$ particle subject to the Hamiltonian

$$H = kS_x^2 + \lambda S_z, \tag{2}$$

   where

$$S_z = \text{diag}(1, 0, -1), \qquad S_x = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

   This is called the Lipkin-Meshkov-Glick model, which has seen a revival of popularity in recent years. The spin S version of this model, for $S \to \infty$, has a quantum phase transition. In fact, this model can be viewed as a mean-field version of the transverse field Ising model (this sentence probably didn't make any sense now, but I promise until the end of the semester it will!). We will come back to this model, but if you want to learn more about it, a paper which I like is `arXiv 0805.4078`.