Statistical Mechanics - Problem set 4

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Deadline: 05/11

1. **Fermionic double well:** Study a system containing only two sites, L and R, which can take fermionic particles. The Hamiltonian is given by

$$H = \sum_{i\in\{L,R\}} U\hat{n}_{i+}\hat{n}_{i-} - J\sum_{\sigma=\pm 1}(c^\dagger_{L\sigma}c_{R\sigma} + c^\dagger_{R\sigma}c_{L\sigma}), \tag{1}$$

   where $\hat{n}_{i\sigma} = c^\dagger_{i\sigma}c_{i\sigma}$. I want you to consider both the cases where the number of particles is fixed and when it can fluctuate. In the latter, you should introduce a chemical potential μ. You can imagine that these fluctuations of the particle number appear because the system is coupled to a particle bath.

2. **Bosonic ladder:** Consider two 1D tight-binding chains disposed in the form of a ladder, as in the figure below. We assume the system to be bosonic and described by a set of operators a_i for the upper ladder and b_i for the lower ladder, where $i = 1, 2, \ldots, L$.

$$H = \sum_{i=1}^{L} \epsilon_a a^\dagger_i a_i + \epsilon_b b^\dagger_i b_i - J\sum_{i=1}^{L}(a^\dagger_i a_{i+1} + a^\dagger_{i+1}a_i) - J\sum_{i=1}^{L}(b^\dagger_i b_{i+1} + b^\dagger_{i+1}b_i)$$
$$-J_{ab}\sum_{i=1}^{L}(a^\dagger_i b_i + b^\dagger_i a_i).$$

   Assume periodic boundary conditions. Find the dispersion relation for this Hamiltonian.