

# **Finite-time quantum heat engines and implementations using trapped ions**

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## **Project summary**

There is currently a considerable effort from the quantum physics community in building microscopic heat engines operating in the quantum regime. The main goal, in this context, is to use genuine quantum mechanical effects, such as coherence and entanglement, to improve the engine's efficiency or reduce the time of operation. Motivated by these recent advances, in this project we propose to carry out a theoretical analysis of quantum heat engines operating in finite time. The first step will be to advance on the formal mathematical framework required for describing time-dependent open quantum systems, which is the basis for the description of finite-time cycles. Secondly, we shall apply this framework to model heat engines implemented in trapped ion systems. Trapped ions consists in one of the most versatile platforms for quantum information processing and has recently become one of the central tools for quantum thermodynamic experiments. For this reason, and in light of the booming interest in quantum heat engines, we believe that this project is timely and could result in a significant contribution to the quantum thermodynamics community.

# 1 Introduction

Recent experimental and theoretical progress has shown that quantum effects such as entanglement and coherence can be used to provide an exponential advantage to certain informational tasks, such as quantum computation, metrology, communications, etc. Key to these developments is the notion of *controlled experimental platforms*. In most systems in nature, properties such as entanglement are easily lost due to the contact of a system with the environment. However, several platforms now exist in which the shielding from the environment is sufficiently good so as to allow one to manipulate and experiment with these quantum features. For instance, in superconducting qubits the coherence time (the time it takes for the system to lose its quantum features) has increased by 9 orders of magnitude in 15 years, from a few nanoseconds to more than a second in present state-of-the-art implementations. These advances represent the main drive behind what is now being referred to as the Second Quantum Revolution.<sup>1</sup>

In light of this remarkable progress, a natural question from the theoretical side is to try to understand which physical applications could potentially benefit from these quantum resources. In this project we shall be interested in answering this question in the context of quantum thermodynamics or, more precisely, the operation of quantum heat engines. There is a growing amount of theoretical [1–19] and experimental [6, 20–25] evidence showing that quantum features could potentially benefit the operation of heat engines at the microscopic scale. For instance, in Refs [26–28] it was shown that in quantum systems one may employ *shortcuts to adiabaticity* which enable an engine to run finite time cycles with quasi-static efficiency. Similarly, the authors of Ref. [29] showed that entanglement between bosons can be used to enhance the information-to-work conversion in a Szilard engine.

Despite these recent advances, however, several aspects of this problem still remain largely unexplored. This is largely due to the lack of a consistent theoretical framework for modeling heat engines operating at finite time. By construction, heat engines require two main ingredients: contact with multiple environments for heat exchange and time-dependent drives for work extraction. Combining these two features into a single framework, however, is not trivial. Prof. Landi has been working directly on this problem over the past year [30–32]. This project is a direct continuation of this work. Our focus here will be on the description of stroke-based finite-time heat engines using the method of repeated interactions [30, 33–35]. In this method the environment is modeled as a set of independent and identically prepared ancillas (usually a single qubit or a single bosonic mode) which can interact with the system at most once. This therefore produces a stroboscopic dynamics, where at each stroke the system interacts with a fresh new ancilla, which is discarded afterwards. The method of repeated interactions therefore naturally fits into the stroke-based formalism of finite-time engines.

In this project the student will advance these ideas by studying specific implementations of Carnot and Otto cycles within the repeated interactions framework. The goal will be to construct general measures for the efficiency of the cycle and the output power, in terms of both thermodynamic as well as information-theoretic quantities concerning the correlations between system and environment. Afterwards, we shall investigate in detail the effects of introducing a dephasing stroke. That is, an intermediate stroke in the cycle whose sole effect is to dampen the coherences in the system, without affecting any energetics. In this way, one can analyze the role of the transition from quantum to classical in the cycle's efficiency.

Finally, the student shall apply this framework in the description of heat engines implemented using trapped ions. This is largely motivated by the experimental results published recently in Ref. [23], which uses a harmonic oscillator as a flywheel in order to extract work from a qubit system. The experiment allows for the state of the flywheel to be reconstructed (in the form of the Husimi Q function) which opens up, for the first time, the possibility of directly monitoring the stochastic work extraction at the quantum level. One particular feature of this experiment, however, is that the decoherence rate of the harmonic oscillator is much larger than the cycle time, so that the work extraction part behaves essentially in a classical manner. Extending the results beyond this regime corresponds to an ongoing effort in the community. And, in this sense, the aforementioned study using a dephasing stroke will be particularly useful.

This project will count with the collaboration of several researchers from Brazil and abroad. In Brazil, we mention the collaboration of Prof. Roberto Serra, from UFABC, and his PhD student Ivan Henao, who are also investigating heat engines in collaboration with Prof. Landi. The work

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<sup>1</sup>This name first appeared in a report by the British Defence Science and Technology Laboratory.

will also count with the collaboration of Prof. Cecilia Cormick, from the Universidad de Cordoba, in Argentina, who is an expert in trapped ions. We also mention the ongoing collaboration with Profs. Mauro Paternostro and Gabriele De Chiara, from Queen's University in Belfast, in the context of the FAPESP SPRINT project 2017/50304-7. Together with Prof. Mauro Paternostro, we have already begun discussing possible future experimental implementations with the group from Universität Mainz who carried out the study in Ref. [23]. For this reason, there is the concrete possibility that the results developed in this project may eventually be put to test in an experimental platform.

## 2 Methodology

In this section we describe the basic goals that the candidate will have to pursue in order to complete this project, as well as the methodology for doing so.

### A. Bibliographical review and basic tools

#### A.1 Quasi-static quantum thermodynamic cycles:

In order to become familiar with the concepts of heat engines, Mr. Molitor will first carry out a review on the implementations of quasi-static engines using quantum working fluids. The focus will be on the Carnot and Otto cycles, operated using either a harmonic oscillator or a single qubit as working fluids. The goal of this part of the work is to explore the new features brought about by the use of these microscopic working fluids, such as the concept of quasi-static irreversibility [36], which does not exist in classical thermodynamics.

#### A.2 Finite-time quantum heat engines described by master equations:

As the second preparatory step, Mr. Molitor will carry out a review of papers which use time-dependent Lindblad equations as a tool to model finite-time heat engines. The case of Otto cycles offer a particular simple platform, as it separates heat and work strokes. During the work strokes the dynamics is unitary and therefore easily modeled by the Schrödinger equation. Conversely, during the heat strokes no work is done, so that the relaxation process can be well described by standard Lindblad master equations. The case of a Carnot cycle is more complicated, as it requires work to be performed simultaneously with the contact with the baths. As a consequence, one enters into the difficult question on how to model Lindblad equations when the Hamiltonian is time-dependent. Tools for dealing with this problem have been developed in Refs [37, 38] as well as in two recent publications by the group of Prof. Landi [31, 32].

#### A.3 The method of repeated interactions:

Next Mr. Molitor will study the basics of the method of repeated interactions. Initially this will be based on Ref. [35], which contains a nice bibliographical review of the method and also an important discussion on the connection between thermodynamics and information. Secondly, Mr. Molitor will focus on a recently published paper by Prof. Landi's group [30], which discusses the thermodynamic aspects of the repeated interactions method, as well as the short-time expansions that are required to move from a stroke-based to a continuous-time description.

#### A.4 Bibliographical review of the latest literature in Quantum Heat engines:

During the course of the master's project, Mr. Molitor will be continuously studying the latest literature in the field. It is important to mention that this area of research is extremely new and active, so that most papers are from the last 5 years, with new papers appearing frequently. For this reason, it is paramount that Mr. Molitor learns to stay in touch with the latest developments. Among the topics, we mention shortcuts to adiabaticity [1, 26, 27, 39–43], information-based engines [29, 44–49], engineered reservoirs [6, 8, 21], measurement based engines [50–53] and so on.

#### A.5 Bibliographical review of experimental realizations of Quantum Heat engines:

Particularly important in this bibliographical review, will be the experimental realizations of quantum heat engines. In the last two years there has been a boom of interest in these

experimental implementations, as we are now reaching the point in which we have sufficient control to experiment with quantum features in a full thermodynamic cycle. Examples of implementations include a single-ion heat engine [20], a vibrating membrane under squeezed baths [21], a spin heat engine using magnetic resonance [22], a nitrogen-vacancy based engine [25], a continually operated absorption refrigerator using trapped ions [24] and the aforementioned spin heat engine using a harmonic oscillator flywheel [23].

## **B. Execution of the main objectives of the project**

### **B.1 Implementation of bosonic stroke-based finite-time cycle:**

As the first actual contribution to the master's project, Mr. Molitor will put forth a detailed analysis of a stroke-based heat engine which uses only bosonic modes for both the working fluid and the environmental ancillas. If one also assumes that the interactions are all Gaussian preserving, then one may benefit from the tools of Gaussian open quantum systems, in particular the notion of Lyapunov equations. This part will be based on Refs. [30, 54]. The results will also be compared with the analysis of Ref. [13], where the authors (which are collaborators in this project) studied an engine operating with finite-size environments, so that the operation efficiency is degraded after a certain number of cycles.

### **B.2 Comparison with a qubit engine:**

For the purpose of comparison, Mr. Molitor will then study the analogous problem formulated in terms solely of qubits, which can also be dealt with easily due to the small dimension of the Hilbert space. A comparison between the two platforms will be useful for understanding the basic features of quantum heat engines. Among other things, Mr. Molitor will compute the efficiency and output power, as well as study the continuous-time limit. The goal of this numerical investigation is to gain intuition that should prove useful in developing the more general formalism.

### **B.3 Entropy production and bounds on the efficiency and output power:**

As the main topic of this dissertation, Mr. Molitor will work towards the development of a theory for the entropy production in a stroke-based finite-time engine using the method of repeated interactions. The goal will be to obtain expressions that relate the efficiency and output power with the entropy production. In turn, the entropy production will be related to quantifiers of the system-environment correlations. Hence, in this way one may formulate bounds for the efficiency and output power which are expressible in terms of information-theoretic quantities, highlighting the growing relationship between thermodynamics and information.

### **B.4 Dephasing strokes:**

Next Mr. Molitor will consider the consequences of introducing intermediate strokes which do not change the energetics of the system, but only cause decoherence. The idea is that these dephasing strokes are to be intercalated with the usual strokes of the engine. The effect of varying the positions, within the cycle, at which they are introduced, will be investigated. By introducing these effects as an extra stroke, we will be able to separate the contributions of the entropy production from population changes and those due solely to decoherence, in the same spirit as Ref. [55]. The main goal will be to understand how decoherence affects the efficiency and output power of the cycle. We are unaware of any papers in the literature making use of this idea.

### **B.5 Theoretical analysis of the trapped ion experiment of Ref. [23]:**

The experiment reported in Ref. [23] uses a spin heat engine coupled to a harmonic oscillator flywheel, used to store work. In their implementation, the authors can reconstruct the Husimi Q function of the harmonic oscillator, therefore allowing one to reconstruct the stochastic features of the work extraction. Mr. Molitor will carry out a detailed study of this experiment. As the first step, he will reproduce the results of the experiment. Next, once he is familiar with the basic models, he will try to go beyond and consider more general situations aimed at formulating interesting experimental proposals that could in the future be implemented by the Mainz group.

## B.6 Assessment of the entropy production in the trapped ion experiment:

The entropy production is not an observable and must therefore be associated with an observable in order to be experimentally assessed. Quantifying it is essential to understand the origins of the irreversibility which bound the engine's efficiency. However, doing so experimentally is usually challenging. Recently, Prof. Landi and collaborators have put forth [58] an experimental determination of the entropy production in two controlled platforms: an optomechanical system and a Bose-Einstein condensate confined in an optical lattice. In this project we propose to use similar ideas to assess the entropy production of the experiment performed in Ref. [23]. As shown recently in [59, 60], these phase space distributions can be used to reconstruct alternative formulations of the entropy production, which can be more easily assessed experimentally (this was the novel feature used in Ref. [58]). As the final part of the master's project, Mr. Molitor will therefore attempt to propose a similar theoretical framework for the experiments in [23].

## C. Optional routes of investigation

In this section we list optional routes of investigation that may eventually prove interesting in the course of the dissertation.

### C.1 Stroke-based Ermakov-Lewis theory:

One of the main tools used in describing heat engines operating with harmonic oscillators, is the Ermakov-Lewis theory which provides an exact solution for the dynamics of a harmonic oscillator subject to a time-dependent frequency modulation [56, 57]. In Ref. [31], Prof. Landi and collaborators put forth an extension of this theory to the dynamics of open quantum systems described by a Lindblad master equation. A possibly interesting route of investigation is to try to extend this theory further to a stroke-based dynamics implemented by means of the method of repeated interactions. In view of the enormous success of the Ermakov-Lewis theory (which extends far beyond the applications in heat engines), we believe that this calculation could yield an important technical contribution to the quantum physics community.

### C.2 Finite-time cycles using non-equilibrium squeezed baths:

The framework put forth in Ref. [31] can be used to study the efficiency of engines operating under squeezed baths. An interesting perspective would be to apply these ideas to obtain analytical expressions for the efficiency, including the effects of the entropy production. An analysis of this problem was carried out in the case of the Otto cycle in Ref. [6]. Extending this to a Carnot cycle seems feasible and potentially interesting. The use of the repeated-interactions stroke-based formalism could also be of interest.

### C.3 Entropy production in shortcuts to adiabaticity:

The term shortcuts to adiabaticity refer to the use of a time-dependent Hamiltonian to steer the system through an adiabatic path without the need for a quasi-static evolution. This technique could therefore provide a method for constructing engines that operate at quasi-static efficiency, but at finite times. In the context of bosonic systems, shortcuts to adiabaticity are implemented using squeezing of the bosonic mode. One question which has thus far not been addressed is what is the associated entropy production of performing this shortcut. With the quantum phase-space formalism of Ref. [60], we believe it is possible to address this question analytically, at least in the case of a single mode working fluid. This, by itself, would also constitute an interesting contribution to the field. The difficulty with implementing this in practice is that, in order to obtain a faithful description of the model, one must also take into account the work cost of performing the shortcut. Consequently, similar considerations also have to be introduced in the entropy production.

### C.4 Bosonic Szilard engine using dichotomic measurements:

A Szilard engine uses information feedback from the system to operate the engine using a single reservoir [44, 61]. A relevant question, in the quantum setting, is what types of measurements can be used for this purpose. This is particularly relevant due to the fundamental measurement back-action that is present in quantum mechanical systems. An interesting perspective, in the context of bosonic systems, was recently put forth in Ref. [62]

in the context of the Leggett-Garg inequality. The authors considered a single harmonic oscillator subject to a dichotomic measurement (described in terms of a POVM) which only distinguishes whether the system has  $x < 0$  or  $x > 0$  (where  $x$  is the position of the oscillator). This type of measurement falls quite naturally under the original framework of Maxwell's demon, where a single particle is trapped in a box with a barrier in the middle. A study on the implementation of a Szilard engine using this type of measurement would therefore configure an interesting implementation of an informationally based engine.

### 3 Final considerations

In this master project, we propose a study of the physics of quantum heat engines, focusing on both the basic underlying theoretical framework, as well as potential experimental implementations in trapped ion platforms. The topic of Quantum Heat Engines has seen an explosion of interest in the last 5 years, culminating with several experimental demonstrations in 2017 and 2018. The possibility of using quantum resources, such as coherence and entanglement, to construct microscopic engines could configure as an important application in the context of the so-called Quantum Technologies 2.0. The project will count with the collaboration of several researchers that are specialists in related fields of knowledge. Moreover, it will count with the full support of Prof. Landi as well as his entire research group. In light of the booming interest in this field of research, we expect that this project will produce 2 to 3 scientific papers in high-quality research journals during the two years of execution.

For these reasons, we believe this project is both timely and relevant. Moreover, the knowledge and the tools that will be developed by the student during this project extend well beyond this specific area of research and will therefore prove useful for his general education in the field of Quantum Information. One of the key defining features of the Quantum Information community is the drive to seek a unified language and set of principles of broad applicability. In this sense, therefore, this project will contribute to put the student in contact with the current state-of-the-art in both theory and experiment.

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