Applying the Duan criteria for problem 3.3 (d)

Hi guys. There was a mistake in my notes on the expression for the Duan criteria. I'm really sorry! The correct criteria is:

define

$$q_1 = \frac{a_1^\dagger + a_1}{\sqrt{2}} \qquad P_1 = \frac{i}{\sqrt{2}}(a_1^\dagger - a_1) \qquad \text{(as usual)} \tag{1}$$

but

$$q_2 = \frac{1}{\sqrt{2}}\left(e^{i\theta}a_2^\dagger + e^{-i\theta}a_2\right) \qquad P_2 = \frac{i}{\sqrt{2}}\left(e^{i\theta}a_2^\dagger - e^{-i\theta}a_2\right) \tag{2}$$

Note that $[q_2, P_2] = i$, so q_2, P_2 are still quadrature operators, but they are rotated in same direction. Then define

$$Q_+ = \frac{q_1 + q_2}{\sqrt{2}} \tag{3}$$

$$P_- = \frac{P_1 - P_2}{\sqrt{2}}$$

The Duan criteria then reads

$$\boxed{\langle Q_+^2\rangle + \langle P_-^2\rangle \geq 1} \tag{4}$$

More explicitly

$$\langle Q_+^2\rangle + \langle P_-^2\rangle = \frac{1}{2}\left\{\langle q_1^2\rangle + \langle p_1^2\rangle + \langle q_2^2\rangle + \langle p_2^2\rangle + 2\langle q_1 q_2\rangle - \langle p_1 p_2\rangle\right\}$$

$$= \langle a_1^\dagger a_1\rangle + 1/2 + \langle a_2^\dagger a_2\rangle + 1/2 + e^{i\theta}\langle a_1^\dagger a_2^\dagger\rangle + e^{-i\theta}\langle a_1 a_2\rangle \tag{5}$$

For problem 3.3 you guys probably got

$$\textcircled{H} = \frac{1}{2(\kappa^2+\omega^2-\lambda^2)}\begin{pmatrix} \kappa^2+\omega^2 & 0 & 0 & \lambda(\kappa-i\omega) \\ 0 & \kappa^2+\omega^2 & \lambda(\kappa+i\omega) & 0 \\ 0 & (\kappa-i\omega) & \kappa^2+\omega^2 & 0 \\ \kappa+i\omega & 0 & 0 & \kappa^2+\omega^2 \end{pmatrix} \tag{6}$$

Thus

$$\langle Q_+^2\rangle + \langle P_-^2\rangle = \frac{1}{2(\kappa^2+\omega^2-\lambda^2)}\left\{2(\kappa^2+\omega^2) + e^{i\theta}\lambda(\kappa+i\omega) + e^{-i\theta}\lambda(\kappa-i\omega)\right\}$$

\lambda\omega

$$\boxed{\langle Q_+^2\rangle + \langle P_-^2\rangle = \frac{\kappa^2+\omega^2 + \kappa\lambda\cos\theta - \kappa\omega\sin\theta}{\kappa^2+\omega^2-\lambda^2}} \tag{7}$$

The minimum of this quantity occurs for $\tan\theta = -\omega/\kappa$. This leads to two results

$$\langle Q_+^2\rangle + \langle P_-^2\rangle\Big|_{min} = \frac{1}{1 \pm \frac{\lambda}{\sqrt{\kappa^2+\omega^2}}} \tag{8}$$

The "−" solution gives nothing. But the "+" solution gives a violation for <u>any</u> $\lambda \neq 0$