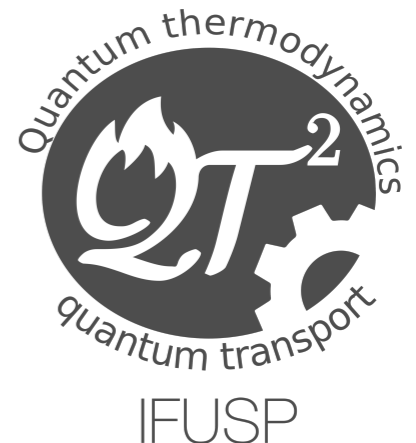


Landauer's principle at zero temperature

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June 19th, 2020.



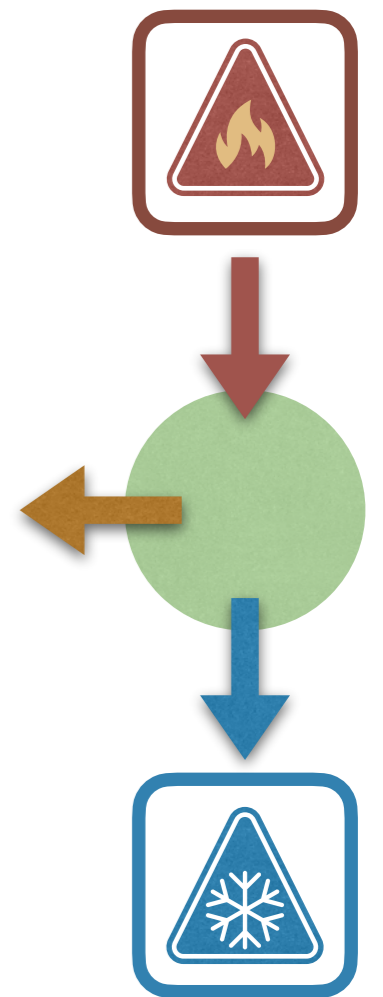
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The second law

- The 1st law puts heat and work on similar footing and says that, in principle, one can be interconverted into the other.
- For a system coupled to two baths, for instance, we have:

$$\frac{dU}{dt} = \dot{Q}_h + \dot{Q}_c + \dot{W}$$

- Not all such processes are possible, however:
 - This is the purpose of the 2nd law.



- The 2nd law deals with entropy.
 - *Entropy, however, does not satisfy a continuity equation.*
- There can be a flow of entropy from the system to the environment, which is given by the famous Clausius expression \dot{Q}/T .
- But, in addition, there can also be some entropy which is spontaneously *produced* in the process. The entropy balance equation thus reads

$$\frac{dS}{dt} = \dot{\Sigma} + \frac{\dot{Q}_h}{T_h} + \frac{\dot{Q}_c}{T_c}$$

- The quantity $\dot{\Sigma}$ is called the **entropy production rate**.
- The second law can now be formulated mathematically as:

$$\dot{\Sigma} \geq 0$$

Why entropy production matters

- 1st and 2nd laws for a system coupled to two baths:

$$\frac{dU}{dt} = \dot{Q}_h + \dot{Q}_c + \dot{W} = 0$$

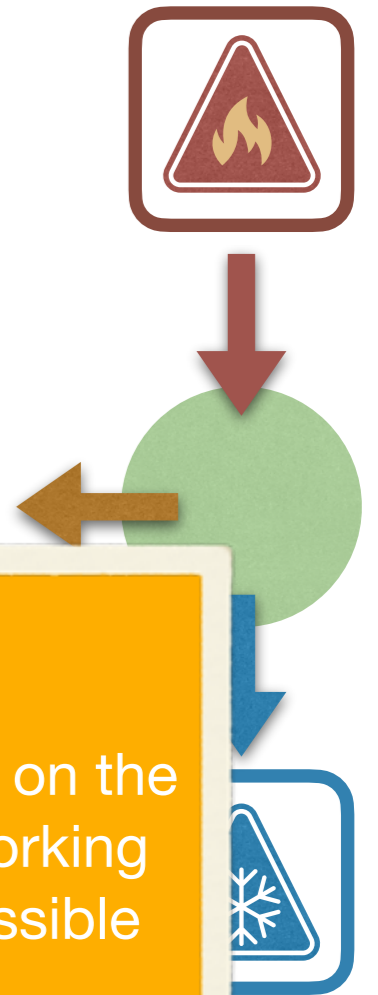
$$\frac{dS}{dt} = \dot{\Sigma} + \frac{\dot{Q}_h}{T_h} + \frac{\dot{Q}_c}{T_c} = 0$$

Carnot's statement of the 2nd law

- T “The efficiency of a quasi-static or reversible Carnot cycle depends only on the temperatures of the two heat reservoirs, and is the same, whatever the working substance. A Carnot engine operated in this way is the most efficient possible heat engine using those two temperatures.”

- Entropy production is therefore the reason the efficiency is smaller than Carnot:*

$$\eta = \eta_C - \frac{T_c}{\dot{Q}_h} \dot{\Sigma}$$



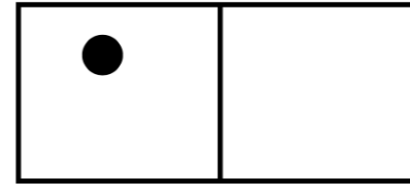
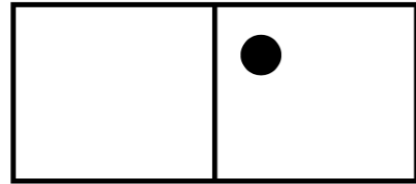
Entropy and information

- In information theory, *entropy* acquires a different interpretation.
 - S = amount of ignorance (lack of information) one has about the system.
- If the system is described by a set of states $n = 1, 2, 3, \dots$, each with probability p_n , the Shannon entropy is defined as

$$S = - \sum_n p_n \ln p_n$$

- The quantity $-\ln p_n$ is called the “surprise” of the state:
 - It measures how surprised we are to observe the system in state n .
 - If $p_n \sim 1 \rightarrow -\ln p_n \sim 0$ (no surprise at all)
 - If $p_n \sim 0 \rightarrow -\ln p_n \gg 1$ (huge surprise)
- Entropy is thus the “average surprise”. 🤔

- To have a concrete example, consider a particle which can be found in 1 of 2 sides of a box:



- Suppose we have some *ignorance* about the system:
 - We do not know which side the particle is.
- Let (p_R, p_L) be the probabilities of finding it on the left or on the right.
- Consequently, the “state” of the particle (i.e., left or right) has some entropy associated to it:

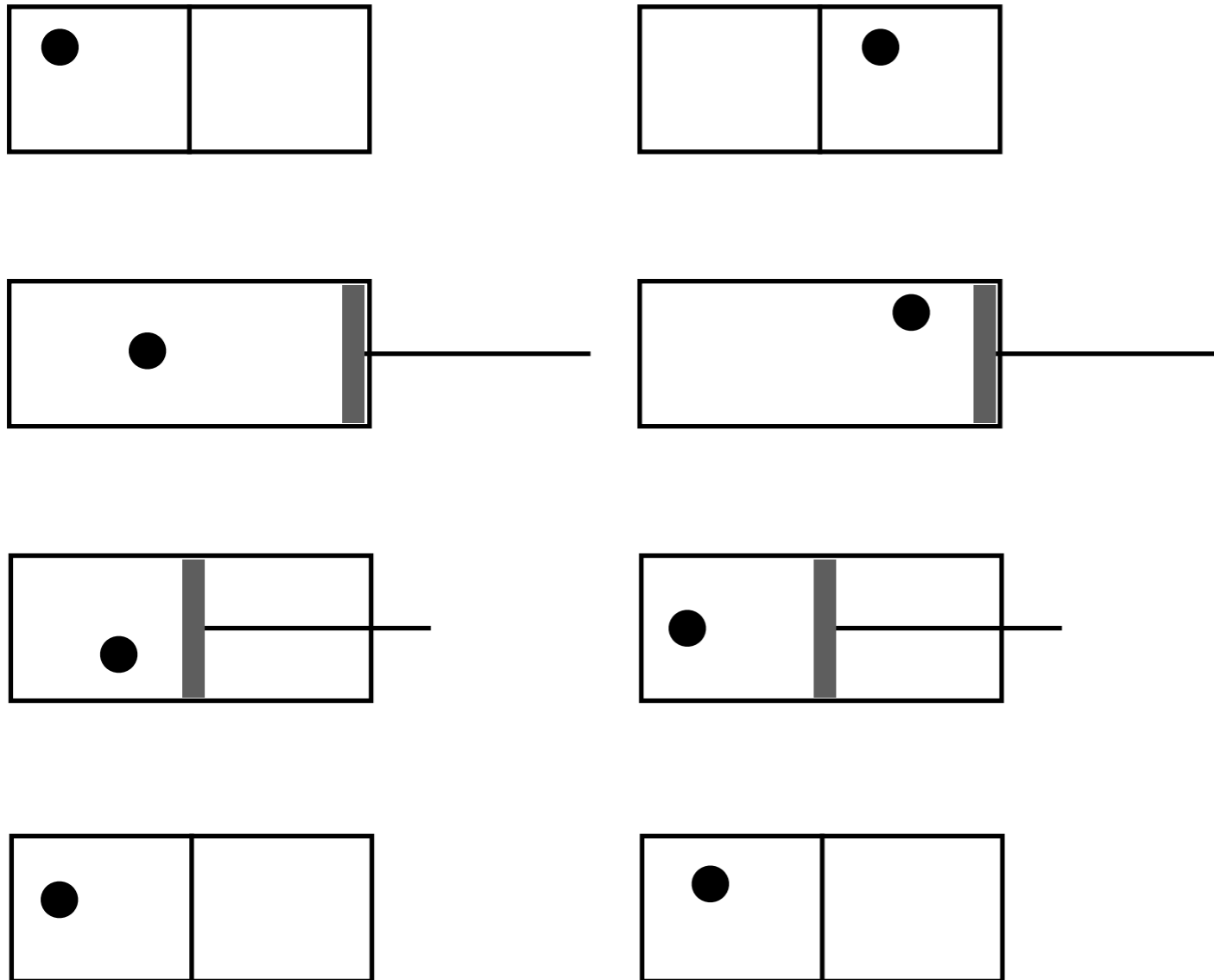
$$S = -p_R \ln p_R - p_L \ln p_L$$

- e.g. maximum ignorance: $p_R = p_L = 1/2$ and thus

$$S = \ln 2$$

Information erasure

- Consider now the following procedure:



- Irrespective of the initial state, the final state is always “left”:

$$(p_R, p_L) = (0, 1)$$

- Hence the final entropy is zero:

$$S = 0$$

- We call this *information erasure* because any initial information about where the particle was is now forever lost.

Landauer's principle

- Landauer's principle states that there is a **fundamental heat cost associated with information erasure**.
 - To erase information one must pay an energy bill:

$$\Delta Q_E \geq -T \Delta S_S$$

- ΔS_S is the change in entropy of the system
 - ΔQ_E is the amount of heat flowing to the *environment* ($\Delta Q_E > 0$ when energy leaves the system).
- **Landauer's bound:** minimum heat cost for erasing information.
- For instance, if $\Delta S_S = 0 - \ln 2$, we find

$$\Delta Q_E \geq T \ln 2$$

- Landauer worked for IBM. In computing terms, this is the energy cost required to erase one bit of information.

Information is physical

- Landauer's principle is often used to argue that “information is physical”.
 - Information is physical because information is *stored* in physical systems and *communicated* using physical systems.
- Information theory is thus not purely mathematical.
- Landauer's principle also highlights a fundamental irreversibility of physical processes:
 - If $\Delta S_S \geq 0$ then $\Delta Q_E \geq -T\Delta S_S$ does not impose any restrictions.
 - There is no energy cost to acquire information.
 - But there is an energy cost to erase it.

**Landauer's principle is a
consequence of the 2nd law**

Landauer from the 2nd law

$$\Delta Q_E \geq -T\Delta S_S$$

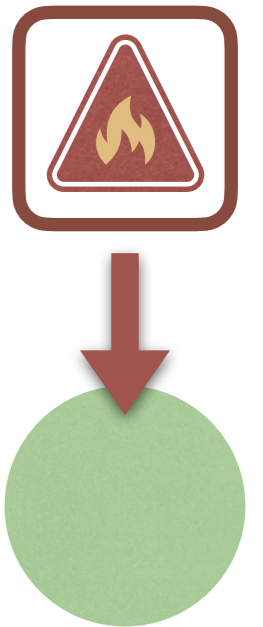
- Recall the 2nd law for a system coupled to a single bath:

$$\frac{dS_S}{dt} = \dot{\Sigma} - \frac{\dot{Q}_E}{T}, \quad \dot{\Sigma} \geq 0$$

- I changed the sign of \dot{Q}/T here because $\dot{Q}_S = -\dot{Q}_E$
- We integrate over some interval of time, leading to

$$\Sigma = \Delta S_S + \frac{\Delta Q_E}{T} \geq 0$$

- This looks exactly like Landauer's principle:
 - It is a direct consequence of the 2nd law $\Sigma \geq 0$.



Microscopic formulation of Landauer's principle

- But this is tricky because:
 - Landauer's bound is defined for the Shannon entropy and for systems of arbitrary size, such as a single bit.
 - The second law uses the thermodynamic entropy and holds only for macroscopic bodies.
- To connect the two universes we must construct a microscopic theory of thermodynamics that is capable of extending the 2nd law to the microscopic domain.
- In this talk I will focus on the quantum version, as it encompasses the classical theory as a particular case.
 - This is the theory we call Quantum Thermodynamics.

Entropy production in quantum systems

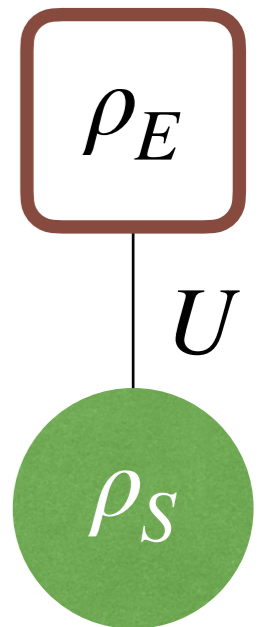
- All information about a quantum system is contained in its density matrix ρ .
- Entropy is now quantified by the von Neumann entropy:

$$S(\rho) = -\text{tr}(\rho \ln \rho)$$

- We consider two quantum systems, S and E (the “environment”) in arbitrary states ρ_S and ρ_E .
 - S and E can have any size: generalization of the bath concept.
- The two then interact with a unitary U , leading to

$$\rho'_{SE} = U(\rho_S \otimes \rho_E)U^\dagger$$

- This is the quantum version of a system interacting with a bath.
- What is the entropy production?



- The dynamics is unitary and so in principle one could say it is reversible.
 - Indeed, if all of ρ'_{SE} is accessible, everything would be reversible.
- “Irreversibility” depends on which degrees of freedom become inaccessible after the interaction.
- There are many possibilities:
 - System-bath correlations become, in practice, inaccessible.
 - Any changes we make in the bath may also not be recoverable.
 - If measurements are done in the system, quantum coherence may also be lost.
 - etc.
- Each of these features can be gauged using a certain information-theoretic quantifier.

M. Esposito, K. Lindenberg, C. Van den Broeck, “*Entropy production as correlation between system and reservoir*”. New Journal of Physics, **12**, 013013 (2010).

G. Manzano, J. M. Horowitz, J. M. R. Parrondo, “*Quantum fluctuation theorems for arbitrary environments: adiabatic and non-adiabatic entropy production*”, Physical Review X, **8**, 031037 (2018).

- The choice of entropy production which is closest to the classical formulation is:

$$\Sigma = I'(S : E) + D(\rho'_E || \rho_E)$$

where

$$I'(S : E) = S(\rho'_S) + S(\rho'_E) - S(\rho'_{SE}) = \text{SE correlations}$$

$$D(\rho'_E || \rho_E) = \text{tr}(\rho'_E \ln \rho'_E - \rho'_E \ln \rho_E) = \text{change in the bath}$$

- This formula can be taken as a general *definition* of entropy production for an arbitrary system+bath interaction process.
 - The system and bath can have any size.
 - Their states are arbitrary, except that they start in any (product) state.
 - They interact with an arbitrary unitary U .
- Assuming that the bath is in a thermal state, one finds:

$$\Sigma = I'(S : E) + D(\rho'_E || \rho_E) = \Delta S_S + \beta \Delta Q_E \geq 0$$

- In the last 5 years there have been several papers which generalized/improved Landauer's original result:
 - J. Goold, M. Paternostro and K. Modi, *Phys. Rev. Lett.* **114**, 060602 (2015).
 - G. Guarnieri, S. Campbell, J. Goold, S. Pigeon, B. Vacchini and M. Paternostro, *NJP*, **19**, 103038 (2017).
 - S. Lorenzo, R. McCloskey, F. Ciccarello, M. Paternostro and G. M. Palma, *Phys. Rev. Lett.*, **115**, 120403 (2015).
 - P. Strasberg, G. Schaller, T. Brandes and M. Esposito, *Phys. Rev. X.*, **7**, 021003 (2017).
 - S. Campbell, G. Guarnieri, M. Paternostro and B. Vacchini, *Phys. Rev. A.*, **96**, 042109 (2017).

Experimental verification of Landauer's principle linking information and thermodynamics

Antoine Bérut¹, Artak Arakelyan¹, Artyom Petrosyan¹, Sergio Ciliberto¹, Raoul Dillenschneider² & Eric Lutz^{3†}

nature
physics

LETTERS

<https://doi.org/10.1038/s41567-018-0070-7>

Corrected: Publisher Correction

Quantum Landauer erasure with a molecular nanomagnet

R. Gaudenzi^{1*}, E. Burzurí¹, S. Maegawa², H. S. J. van der Zant¹ and F. Luis³

MATERIALS SCIENCE

Experimental test of Landauer's principle in single-bit operations on nanomagnetic memory bits

Jeongmin Hong,¹ Brian Lambson,² Scott Dhuey,³ Jeffrey Bokor^{1*}

PRL 113, 190601 (2014)

PHYSICAL REVIEW LETTERS

week ending
7 NOVEMBER 2014



High-Precision Test of Landauer's Principle in a Feedback Trap

Yonggun Jun,^{*} Momčilo Gavrilo, and John Bechhoefer[†]

Department of Physics, Simon Fraser University, Burnaby, British Columbia V5A 1S6, Canada

(Received 15 August 2014; published 4 November 2014)

Quantum Landauer erasure with a molecular nanomagnet

R. Gaudenzi ^{1*}, E. Burzuri¹, S. Maegawa², H. S. J. van der Zant¹ and F. Luis³

Experimental demonstration of information to energy conversion in a quantum system at the Landauer limit

J. P. S. Peterson¹, R. S. Sarthour¹, A. M. Souza¹,
I. S. Oliveira¹, J. Goold², K. Modi³, D. O. Soares-Pinto⁴
and L. C. Céleri⁵

PHYSICAL REVIEW LETTERS **120**, 210601 (2018)

Editors' Suggestion

Featured in Physics

Single-Atom Demonstration of the Quantum Landauer Principle

L. L. Yan,¹ T. P. Xiong,^{1,2} K. Rehan,^{1,2} F. Zhou,^{1,*} D. F. Liang,^{1,3} L. Chen,¹ J. Q. Zhang,¹
W. L. Yang,^{1,†} Z. H. Ma,⁴ and M. Feng^{1,3,5,6,‡}

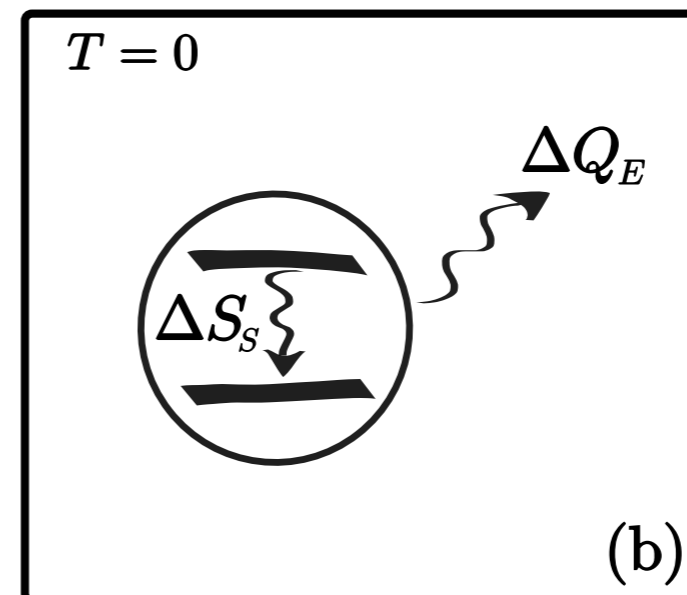
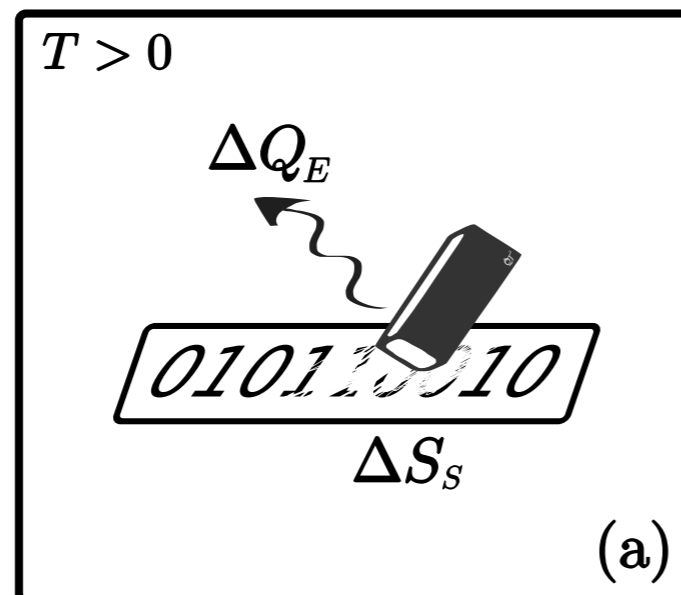
Trouble at $T \rightarrow 0$

Trouble at $T \rightarrow 0$

- Landauer's bound becomes trivial in the limit $T \rightarrow 0$:

$$\Delta Q_E \geq -T\Delta S_S \quad \rightarrow \quad \Delta Q_E \geq 0$$

- All it says is that the heat cost for erasure is non-negative. But otherwise, it is *independent* of the amount of erasure.
- Take, as an example, spontaneous emission:
 - To erase information about an atom, we must emit a photon.
 - Energy therefore *was* emitted. But the bound does not capture it.



A tighter Landauer bound

- Now I would like to show how it is possible to derive a modified bound, which:
 - Is always tighter than the original.
 - Tends to it at high temperatures.
 - But yields non-trivial information when $T \rightarrow 0$.
- Landauer's bound stems from the positivity of:

$$\Sigma = I'(S : E) + D(\rho'_E || \rho_E) \geq 0$$

- Instead, we focus only on the positivity of:

$$I'(S : E) \geq 0$$

Derivation

- The initial bath state is thermal.
 - But its final state $\rho'_E = \text{tr}_S(\rho'_{SE})$ is not.
- Define a reference thermal state $\rho_E(T')$ which is thermal, but at a temperature T' such that

$$\text{tr}\{H_E \rho_E(T')\} = \text{tr}\{H_E \rho'_E\} := E_E(T')$$

- From the MaxEnt principle $S(\rho_E(T')) \geq S(\rho'_E)$ so that

$$\begin{aligned}\Delta S_S + \Delta S_E^{\text{th}} &\geq \Delta S_S + \Delta S_E \\ &= I'(S : E) \\ &\geq 0\end{aligned}$$

- Thus

$$\Delta S_S + \Delta S_E^{\text{th}} \geq 0$$

- This is the 1st result we will need.

- Next define the functions

$$\mathcal{Q}(T') = \Delta Q_E = \int_T^{T'} C_E(\tau) d\tau \quad \text{and} \quad \mathcal{S}(T') = \Delta S_E^{\text{th}} = \int_T^{T'} \frac{C_E(\tau)}{\tau} d\tau$$

- Here $C_E(T)$ is the equilibrium heat capacity of the bath.
- We may then write

$$\Delta S_E^{\text{th}} = \mathcal{S}(T') = \mathcal{S}(\mathcal{Q}^{-1}(\Delta Q_E))$$

- Plugging this in $\Delta S_S + \Delta S_E^{\text{th}} \geq 0$ we then finally get

$$\mathcal{S}(\mathcal{Q}^{-1}(\Delta Q_E)) \geq -\Delta S_S$$

- Finally, inverting \mathcal{S} and \mathcal{Q} , we get:

$$\Delta Q_E \geq \mathcal{Q}(\mathcal{S}^{-1}(-\Delta S_S))$$

About this modified bound

- Our new bound is:

$$\Delta Q_E \geq \mathcal{Q}(\mathcal{S}^{-1}(-\Delta S_S))$$

- It is identical in spirit to Landauer's original bound (same assumptions).
 - Landauer's is universal because it assumes only 1 thing: that the bath is thermal and has a temperature T .
- Compared to Landauer, we require only one additional piece of information: the bath equilibrium heat capacity $C_E(T)$.
 - However, our bound is always tighter.
 - Requires a bit more info, but is also always better.

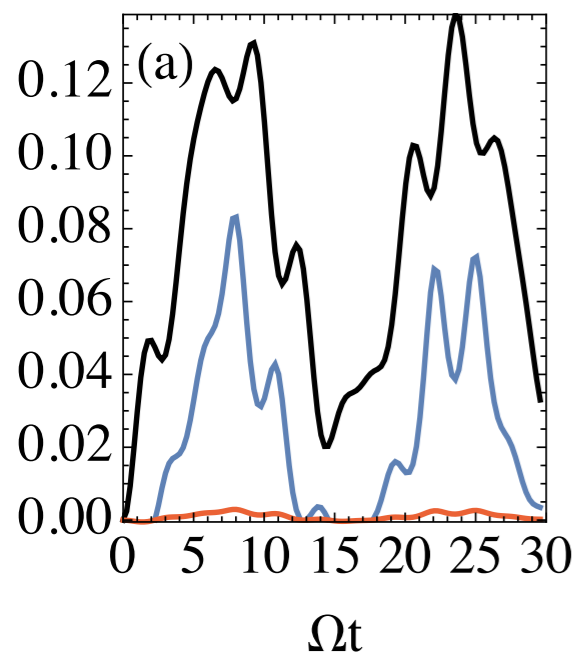
Applications

Cavity QED

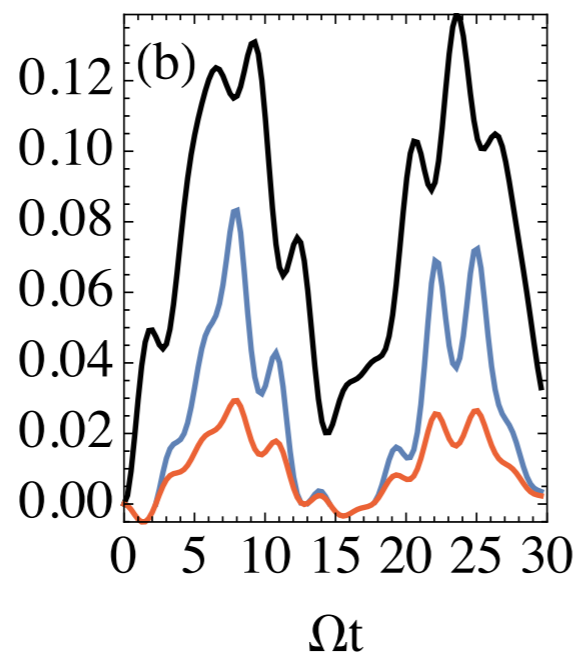
- Consider a 2-level atom interacting with an optical cavity field via the Rabi model:

$$H = \hbar\omega a^\dagger a + \frac{\hbar\omega}{2}\sigma_z + \hbar g(a + a^\dagger)\sigma_x$$

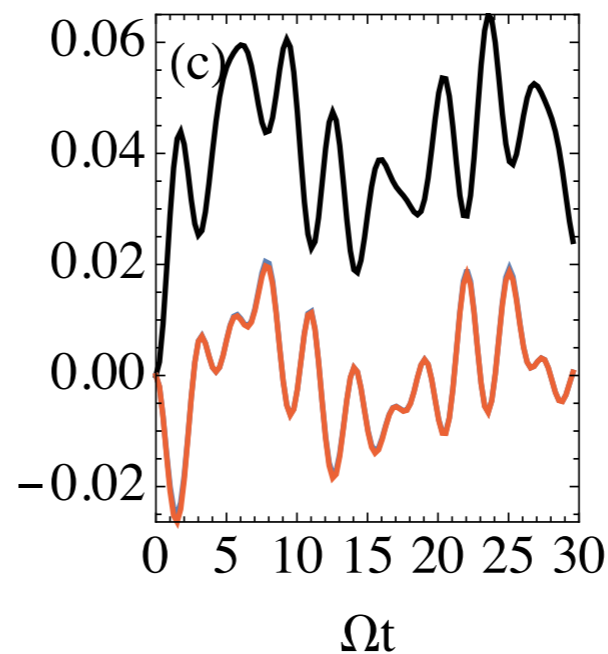
- The atom is the system and the cavity is the bath.



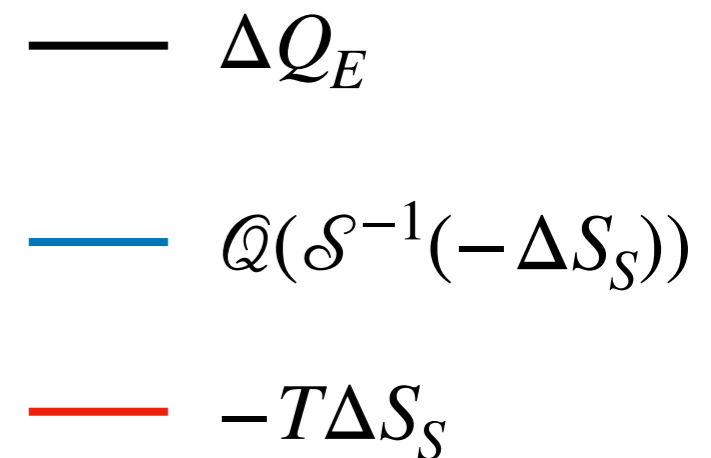
$$T/\omega = 0.01$$



$$T/\omega = 0.1$$



$$T/\omega = 0.4$$



Waveguide QED

- Next consider the emission of the atom into a 1D waveguide of length L .
- The waveguide is characterized by a continuum of modes with dispersion relation $\omega_k = ck$. Hence

$$E_E(T) = \sum_k \frac{\hbar\omega_k}{e^{\beta\omega_k} - 1} = \frac{\pi L}{12\hbar c} T^2$$

- Similarly, the entropy can be found to be

$$S_E(T) = \frac{\pi L}{6\hbar c} T$$

- Thus

$$\Delta Q_E \geq -T\Delta S_S + \frac{3\hbar c}{\pi L} \Delta S_S^2$$

- The first term is the original bound; it vanishes when $T \rightarrow 0$
 - But the 2nd remains.

Heat capacity examples

- We can also provide explicit forms for the bound by assuming different scalings for the bath's heat capacity.
- We focus on $T = 0$.
- Phonons: $C_E = aT^3$

$$\therefore \Delta Q_E \geq \frac{3^{4/3}}{4} \frac{(-\Delta S_S)^{4/3}}{a^{1/3}}$$

- Gapped system (e.g. BCS superconductor): $C_E = be^{-\delta/T}$

$$\therefore \Delta Q_E \geq \delta \frac{(-\Delta S_S)}{\ln(-b/\Delta S_S)}$$

Summary

- Landauer's bound $\Delta Q_E \geq -T\Delta S_S$ provides a fundamental link between thermodynamics and information.
- The bound is powerful because it is universal.
 - Assumes minimal information about the system and process.
- But it trivializes when $T \rightarrow 0$.
- In this talk I showed how one can derive a modified bound which
 - Is always tighter than the original.
 - Tends to it at high temperatures.
 - Yields non-trivial information when $T \rightarrow 0$.
- The bound has exactly the same spirit as the original and assumes only 1 additional ingredient: knowledge of the bath's heat capacity.

Landauer's Principle at Zero Temperature

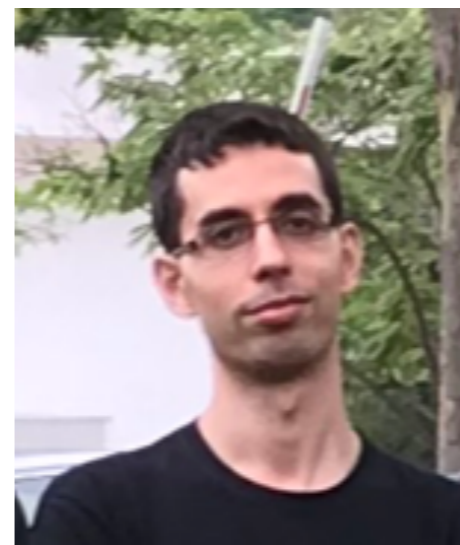
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²*Instituto de Física da Universidade de São Paulo, 05314-970 São Paulo, Brazil*

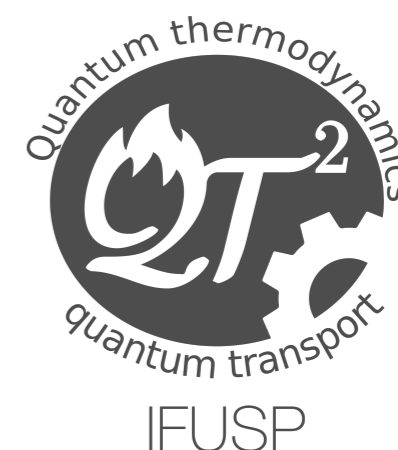


Jader P. Santos
USP



André M. Timpanaro
UFABC

Thank you ❄️🔄🔥🙄



Extra slides

Flow of heat

- The 2nd law reads

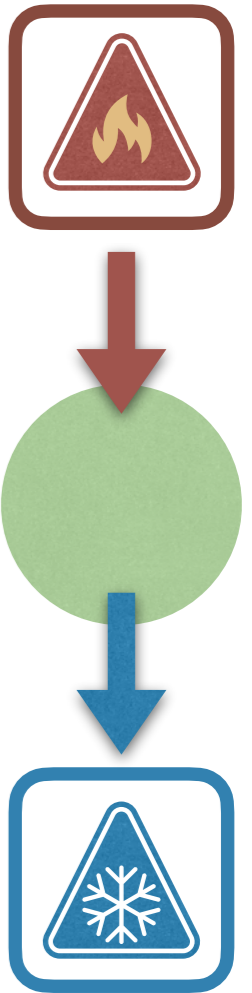
$$\dot{\Sigma} = -\frac{\dot{Q}_h}{T_h} - \frac{\dot{Q}_c}{T_c} \geq 0$$

- But if there is no work involved,

$$\dot{Q}_c = -\dot{Q}_h$$

$$\therefore \dot{\Sigma} = \left(\frac{1}{T_c} - \frac{1}{T_h} \right) \dot{Q}_h \geq 0$$

- *Heat flows from hot to cold.*



Clausius' statement of the 2nd law

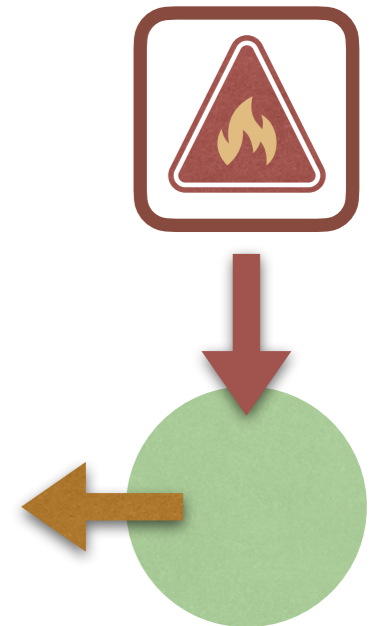
“Heat can never pass from a colder to a warmer body without some other change, connected therewith, occurring at the same time.”

Work from a single bath

- Finally, suppose there is only one bath present:

$$\dot{W} = -\dot{Q}_h$$

$$\dot{\Sigma} = -\frac{\dot{Q}_h}{T_h} = \frac{\dot{W}}{T_h} \geq 0$$



- Positive work (in my definition) means an external agent is *doing* work on the system.

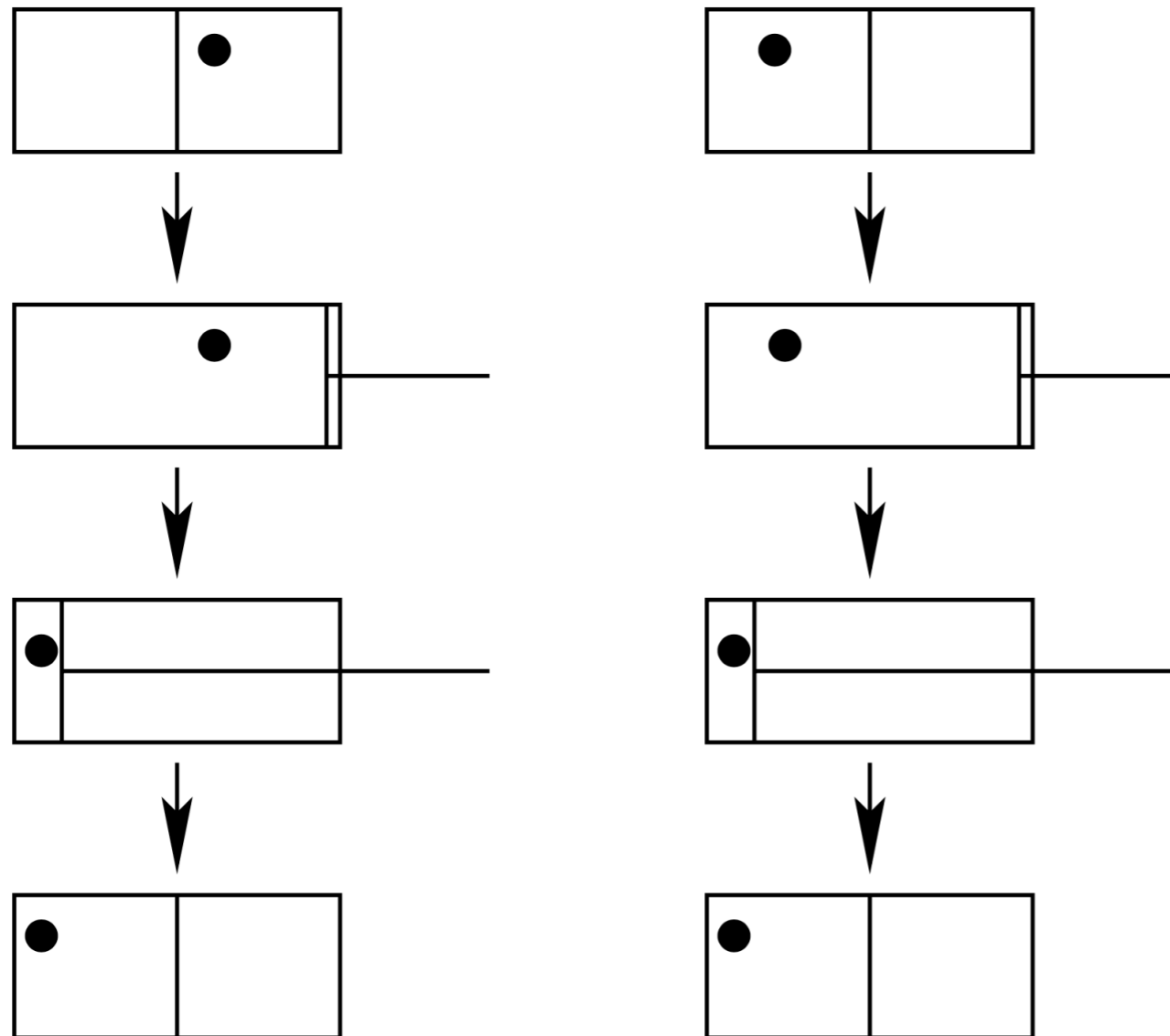
Kelvin-Planck statement of the 2nd law

“It is impossible to devise a cyclically operating device, the sole effect of which is to absorb energy in the form of heat from a single thermal reservoir and to deliver an equivalent amount of work.”

Information content

- Suppose you have an object in your hand, such as a coin or a deck of cards.
 - What is the *information content* of this object?
- Call a friend:
 - The two of you share some background information to make information possible (like establishing what a “coin” is).
 - But your friend does not know the state of the object (e.g. if the coin is Head or Tails).
- Information content = size of the set of instructions that your friend requires to be able to reconstruct the state of the object.

- We can understand Landauer's principle using basic thermodynamics.
- The particle is an ideal gas so $pV = T$



- The work in an isothermal compression is

$$W = - \int_{V_1}^{V_2} p dV = T \ln V_1 / V_2$$

- Minimum work is when one compresses up to the middle of the partition, so $V_1 = 2V_2$

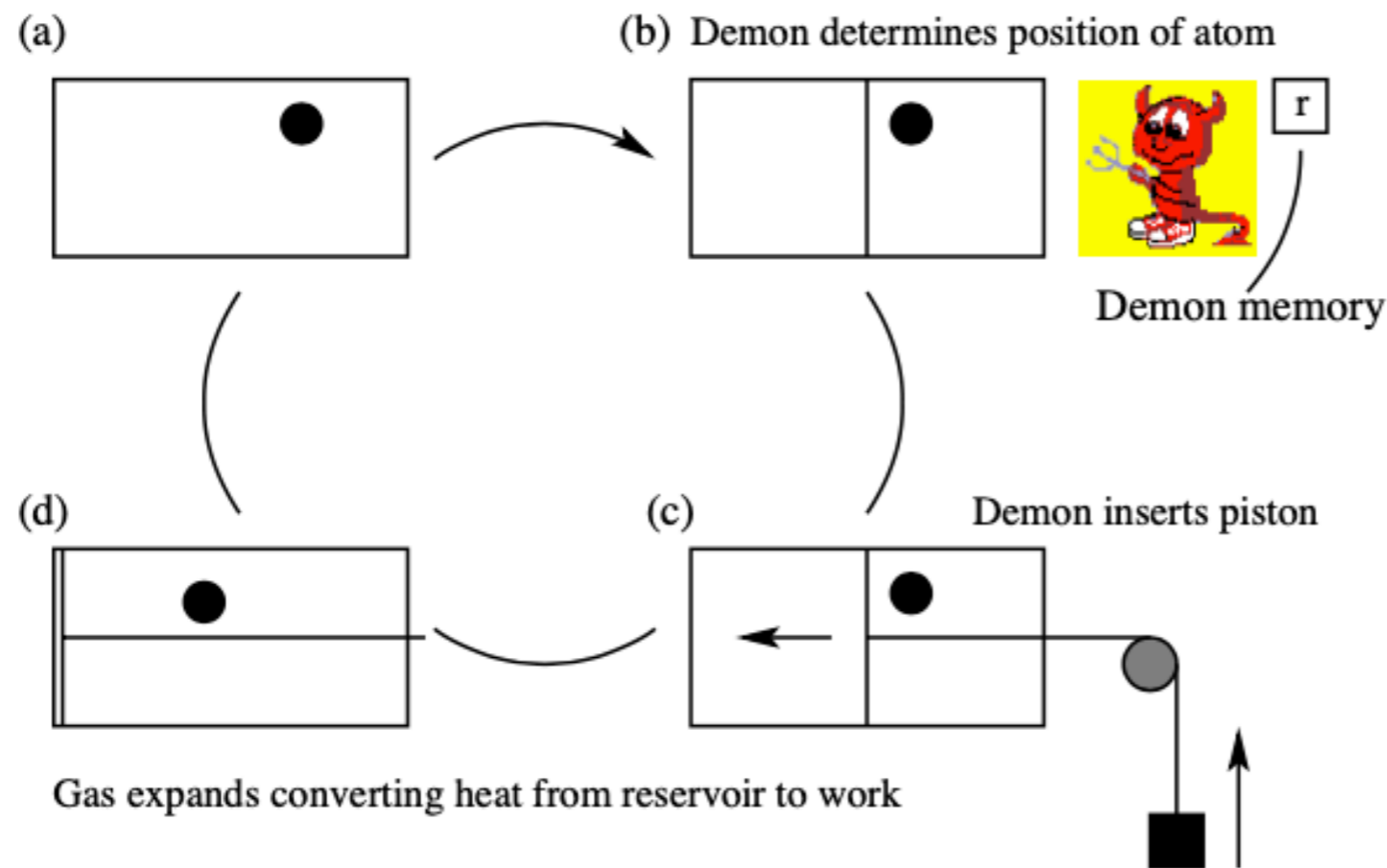
$$W_{\min} = T \ln 2$$

- The energy of the particle $U(T)$ remains constant, so this work must be converted into heat:

$$\Delta Q_E = W_{\min} = T \ln 2$$

Maxwell demon

Seem to violate the 2nd law.



- Bennet used Landauer's principle to solve this paradox:
- There is no energy cost for the demon to acquire information.
- But there is an energy cost to erase it!

Kelvin-Planck statement of the 2nd law

“It is impossible to devise a cyclically operating device, the sole effect of which is to absorb energy in the form of heat from a single thermal reservoir and to deliver an equivalent amount of work.”

Thermal case

- Let us check that we indeed recover the standard expressions when the bath is thermal.
- Since the dynamics is unitary

$$S(\rho'_{SE}) = S(\rho_S \otimes \rho_E) = S(\rho_S) + S(\rho_E)$$

- Whence

$$\begin{aligned} I'(S : E) &= S(\rho'_S) + S(\rho'_E) - S(\rho'_{SE}) \\ &= S(\rho'_S) + S(\rho'_E) - S(\rho_S) - S(\rho_E) \\ &= \Delta S_S + \Delta S_E \end{aligned}$$

- The entropy production will then be

$$\Sigma = \Delta S_S + S(\rho'_E) - S(\rho_E) + D(\rho'_E || \rho_E)$$

- Opening up the last terms:

$$\begin{aligned}
 S(\rho'_E) - S(\rho_E) + D(\rho'_E || \rho_E) &= -\text{tr}(\rho'_E \ln \rho'_E) + \text{tr}(\rho_E \ln \rho_E) \\
 &\quad + \text{tr}(\rho'_E \ln \rho'_E - \rho'_E \ln \rho_E) \\
 &= \text{tr}\{(\rho_E - \rho'_E) \ln \rho_E\}
 \end{aligned}$$

- This result is still general. Now we assume that the bath is in a thermal state

$$\rho_E = \frac{e^{-\beta H_E}}{Z_E}$$

- This is then seen to be the heat entering the bath:

$$\text{tr}\{(\rho_E - \rho'_E) \ln \rho_E\} = \beta \Delta Q_E = \beta(\langle H_E \rangle' - \langle H_E \rangle)$$

- Whence:

$$\Sigma = \Delta S_S + \beta \Delta Q_E$$