SISSA, Trieste

### Measures of irreversibility in quantum phase space

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### Irreversibility and entropy production

System + Environment: exchange of energy (continuity equation):

$$\frac{d\langle H\rangle}{dt} = -\Phi_E$$

\* No such result holds for the entropy:

\* In equilibrium: 
$$\frac{dS}{dt} = -\frac{\Phi_E}{T} := \Phi$$
  $\left(\Delta S = \frac{\delta Q}{T}\right)$ 

- \* But out of equilibrium:  $\frac{dS}{dt} \ge -\Phi$
- \* The difference is called the *entropy production*  $\Pi$ 
  - \* It works as a quantifier of irreversibility  $\Pi \ge 0$  and  $\Pi = 0$  only in equilibrium

$$\frac{dS}{dt} = \Pi - \Phi$$

#### Ex: RL Circuit

Steady-state



$$\frac{dS}{dt} = 0 \qquad \Pi_{\rm ss} = \Phi_{\rm ss} = \frac{\mathcal{E}^2}{RT}$$

#### Ex: 2 inductively coupled RL circuits



 $\Pi_{\rm ss} = \frac{\mathcal{E}_1^2}{R_1 T_1} + \frac{\mathcal{E}_2^2}{R_2 T_2} + \frac{m^2 R_1 R_2}{(L_1 L_2 - m^2)(L_2 R_1 + L_1 R_2)} \frac{(T_1 - T_2)^2}{T_1 T_2}$ 

GTL, T. Tomé and M. J. de Oliveira, J. Phys A. 46 (2013) 395001

### Non-equilibrium open quantum systems

- \* How to quantify irreversibility in situations which do not fall under the usual thermodynamic paradigms?
- \* In quantum systems, one is no longer restricted to thermal baths.
- Any system-environment interaction may be described by a Kraus map:

$$o \to \sum_k M_k \rho M_k^{\dagger}, \qquad \sum_k M_k^{\dagger} M_k = 1$$

- By appropriately choosing the environment and the S+E interaction, we may generate any type of open system dynamics.
  - \* Reservoir engineering.

### Decoherence of quantum superpositions through coupling to engineered reservoirs

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Observation of non-Markovian micromechanical Brownian motion

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#### Selected for a Viewpoint in *Physics* PHYSICAL REVIEW X 7, 031044 (2017)

#### Squeezed Thermal Reservoirs as a Resource for a Nanomechanical Engine beyond the Carnot Limit

Jan Klaers,<sup>\*</sup> Stefan Faelt, Atac Imamoglu, and Emre Togan Institute for Quantum Electronics, ETH Zürich, CH-8093 Zürich, Switzerland (Received 25 April 2017; revised manuscript received 25 July 2017; published 13 September 2017)





#### from squeezed light on a mechanical oscillator

Jeremy B. Clark, Florent Lecocq, Raymond W. Simmonds, José Aumentado and John D. Teufel\*

### Entropy production in quantum systems

\* We must hold on to something. So we assume that irreversibility may be quantified in terms of an entropy production.

$$\frac{dS}{dt} = \Pi - \Phi$$

- \* The question is then how to separate into a flux and a production?
- \* For thermal baths, most approaches assume that we continue to have:

$$\Phi = \frac{\Phi_E}{T}$$

- Because it is assumed that the bath is in equilibrium, so equilibrium thermodynamics also holds from the bath side.
- \* But that's a very strong assumption!
  - \* And this expression is obviously nonsensical if the bath is not thermal.

\* Master equations of Lindblad form and take the system to a Gibbs state, while satisfying detailed balance, are called *Davies maps*:

$$\frac{d\rho}{dt} = -i[H,\rho] + D(\rho), \qquad D(e^{-\beta H}) = 0$$

\* For Davies maps the entropy production becomes

. .

$$\Pi = -\frac{d}{dt}S(\rho||\rho_{\rm eq})$$

- \* where  $S(\rho || \rho^{eq}) = tr(\rho \ln \rho \rho \ln \rho^{eq})$  (Relative entropy)
- It is also interesting to note that the relative entropy may be written as

$$S(\rho||\rho_{eq}) = F(\rho) - F(\rho_{eq}), \qquad F(\rho) = U(\rho) - TS(\rho)$$

$$\Pi = -\frac{dF}{dt} \ge 0$$

Spohn, J. Math Phys. 1978

### Motivation and objectives

 Our main motivation was to look for alternative ways to formulate the entropy production problem that also hold for more general reservoirs.

### Alternative entropy measures

- The first thing we analyzed was the possibility of using other entropy measures instead of von Neumann's.
- \* We are not interested in standard thermodynamics, so we do not need to be able to define a *thermodynamic entropy*.
- \* Recently, it was shown that in thermal transitions involving arbitrary size quantum systems, the family of Rényi entropies play an essential role.

#### The second laws of quantum thermodynamics

Fernando Brandão<sup>a,1</sup>, Michał Horodecki<sup>b</sup>, Nelly Ng<sup>c</sup>, Jonathan Oppenheim<sup>c,d,2</sup>, and Stephanie Wehner<sup>c,e</sup>

PNAS | March 17, 2015 | vol. 112 | no. 11 | 3275–3279

#### The second laws of quantum thermodynamics

Fernando Brandão<sup>a,1</sup>, Michał Horodecki<sup>b</sup>, Nelly Ng<sup>c</sup>, Jonathan Oppenheim<sup>c,d,2</sup>, and Stephanie Wehner<sup>c,e</sup> PNAS | March 17, 2015 | vol. 112 | no. 11 | 3275–3279

Rényi entropy

$$S_{\alpha} = -\frac{1}{1-\alpha} \log \mathrm{tr} \rho^{\alpha}$$

von Neumann entropy

 $S_1 = -\mathrm{tr}(\rho \ln \rho)$ 

$$F_{\alpha}(\rho,\rho_{\beta}) \coloneqq kTD_{\alpha}(\rho \| \rho_{\beta}) - kT\log Z,$$

with the Rényi divergences  $D_{\alpha}(\rho \| \rho_{\beta})$  defined as

$$D_{\alpha}(\rho \| \rho_{\beta}) = \frac{\operatorname{sgn}(\alpha)}{\alpha - 1} \log \sum_{i} p_{i}^{\alpha} q_{i}^{1 - \alpha},$$

A transition is allowed when:

$$F_{\alpha}(\rho,\rho_{\beta}) \ge F_{\alpha}(\rho',\rho_{\beta})$$

Generalizes the second law. For macroscopic systems all Rényi entropies converge to von Neumann's.

# Probability currents

- In classical stochastic processes, entropy production can be neatly formulated in terms of *probability currents* which generate irreversibility in phase space.
- \* A Fokker-Planck equation, for instance, can be written as a continuity equation:

$$\frac{\partial P}{\partial t} = \sum_{i} \frac{\partial J_i(P)}{\partial x_i}$$

In this case, it is (reasonably) postulated that the entropy production should be an *even* function of the currents, whereas the entropy flux should be odd.

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U. Seifert, Rep. Prog. Phys. 75, 126001 (2012)
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# Toy model - open QHO

- Let us start with a quantum harmonic oscillator subject to a thermal bath.
- \* We will then generalize this to non-equilibrium environments later.

$$\frac{d\rho}{dt} = -i[H,\rho] + D(\rho) \qquad \qquad H = \omega(a^{\dagger}a + 1/2)$$

$$D(\rho) = \gamma(\bar{n}+1) \left[ a\rho a^{\dagger} - \frac{1}{2} \{ a^{\dagger}a, \rho \} \right] + \gamma \bar{n} \left[ a^{\dagger}\rho a - \frac{1}{2} \{ aa^{\dagger}, \rho \} \right] \qquad \bar{n} = \frac{1}{e^{\beta\omega} - 1}$$





# Move to phase space

\* We can describe the MEq in phase space through the Wigner function

$$W(\alpha, \alpha^*) = \frac{1}{\pi^2} \int d^2 \lambda e^{-\lambda \alpha^* + \lambda^* \alpha} \operatorname{tr} \left\{ \rho e^{\lambda a^\dagger - \lambda^* a} \right\}$$

\* Phase space is now the complex plane, with:

$$x = \sqrt{2} \operatorname{Re}(\alpha), \qquad p = \sqrt{2} \operatorname{Im}(\alpha)$$

\* Thermal equilibrium is a Gaussian

$$W_{\rm eq} = \frac{1}{\pi(\bar{n}+1/2)} \exp\left\{-\frac{|\alpha|^2}{\bar{n}+1/2}\right\}$$

 For T = 0 this gives the vacuum state, which still has a non-zero width: *quantum fluctuations*.



# Quantum Fokker-Planck equation

 In terms of the Wigner function, the Lindblad equation becomes a quantum Fokker-Planck equation:

$$\partial_t W = \mathcal{U}(W) + \partial_\alpha J(W) + \partial_\alpha^* J^*(W)$$

where U(W) is the unitary part and

$$J(W) = \frac{\gamma}{2} \left[ \alpha W + (\bar{n} + 1/2) \partial_{\alpha^*} W \right]$$

\* The QFP Eq. is also a *continuity equation* and J(W) is the irreversible component of the probability current.

 $J(W_{\rm eq}) = 0$ 

# Wigner entropy

\* The mist widely used Rényi entropy is the Rényi-2

 $S_2 = -\ln \mathrm{tr}\rho^2$ 

- \* It is therefore simply related to the purity of the state
- \* In *PRL* **109**, 190502 (2012) the authors showed that for Gaussian states, this actually coincides with the *Wigner entropy*

$$S = -\int d^2 \alpha W(\alpha, \alpha^*) \ln W(\alpha, \alpha^*)$$

 For Gaussian states, the Wigner function is also always positive, so S is always real.

### Production and flux

\* Starting with the Wigner entropy, we now postulate that the production must be even in J and the flux must be odd:

$$\frac{dS}{dt} = -\int d^2 \alpha \, (\partial_t W) \ln W$$
$$= -\int d^2 \alpha (\partial_\alpha J + \partial_{\alpha^*} J^*) \ln W$$
$$= \int d^2 \alpha \frac{1}{W} \left( J \partial_\alpha W + J^* \partial_{\alpha^*} W \right)$$

$$J(W) = \frac{\gamma}{2} \left[ \alpha W + (\bar{n} + 1/2)\partial_{\alpha^*} W \right] \qquad \Longrightarrow \quad \partial_{\alpha^*} W = \frac{1}{(\bar{n} + 1/2)} \left[ 2J/\gamma - \alpha W \right]$$

\* As a result we get

$$\Pi = \frac{4}{\gamma(\bar{n}+1/2)} \int d^2 \alpha \frac{|J(W)|^2}{W}$$
$$= -\frac{dS(W||W_{eq})}{dt}$$

$$\Phi = \frac{\gamma}{\bar{n} + 1/2} \left[ \langle a^{\dagger} a \rangle - \bar{n} \right] = \frac{\Phi_E}{\omega(\bar{n} + 1/2)}$$

\* At high temperatures  $\omega(\bar{n} + 1/2) \simeq T$  so we get

$$\Phi \simeq \frac{\Phi_E}{T}$$

\* Now both  $\Pi$  and  $\Phi$  remain finite at T = 0.

J. P Santos, GTL and M. Paternostro, *Physical Review Letters* **118** 220601 (2017)

### Stochastic trajectories and fluctuation theorems

- \* We can also arrive at the same result using a completely different method.
  - \* We analyze the stochastic trajectories in the complex plane.
- \* The quantum Fokker-Planck equation is equivalent to a Langevin equation in the complex plane:

$$\frac{dA}{dt} = -i\omega A - \frac{\gamma}{2}A + \sqrt{\gamma(\bar{n} + 1/2)}\xi(t)$$
  
$$\langle \xi(t)\xi(t')\rangle = 0, \qquad \langle \xi(t)\xi^*(t')\rangle = \delta(t - t')$$

 We can now define the entropy produced in a trajectory as a functional of the path probabilities for the forward and reversed trajectories:

$$\Sigma[\alpha(t)] = \ln \frac{\mathcal{P}[\alpha(t)]}{\mathcal{P}_R[\alpha^*(\tau - t)]}$$

\* This quantity satisfies a fluctuation theorem

$$\langle e^{-\Sigma} \rangle = 1$$

\* We show that we can obtain exactly the same formula for the entropy production rate if we define it as

$$\Pi = \frac{\langle d\Sigma[A(t)] \rangle}{dt}$$

# Non-equilibrium baths

# Examples of non-equilibrium baths

- \* The standard entropy production framework is not applicable to any MEq which is not a Davies map.
- \* We may therefore construct very silly examples. For instance:

$$H = \omega(a^{\dagger}a + b^{\dagger}b) + \lambda(a^{\dagger}b + b^{\dagger}a)$$

 $\mathcal{D} = \gamma_a (\bar{n}_a + 1) D[a] + \gamma_a \bar{n}_a D[a^{\dagger}]$ 

 $+\gamma_b(\bar{n}_b+1)D[b]+\gamma_b\bar{n}_bD[b^{\dagger}]$ 

 If you naïvely apply the standard formulation, you may get "violations of the second law".

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#### from squeezed light on a mechanical oscillator

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# Squeezed bath

\* A more interesting non-equilibrium bath is the squeezed bath:

$$S_{z} = e^{\frac{1}{2}(za^{\dagger}a^{\dagger} - aa)}, \qquad z = re^{i\theta}$$
$$\rho = S_{z}^{\dagger} \left(\frac{e^{-\beta\omega a^{\dagger}a}}{Z}\right) S_{z}$$
$$\sim \exp\left\{-\eta a^{\dagger}a - \frac{\zeta}{2}a^{\dagger}a^{\dagger} - \frac{\zeta^{*}}{2}aa\right\}$$



$$\mathcal{D}_{z}(\rho) = \gamma(N+1) \left[ a\rho a^{\dagger} - \frac{1}{2} \{ a^{\dagger}a, \rho \} \right]$$
$$+ \gamma N \left[ a^{\dagger}\rho a - \frac{1}{2} \{ aa^{\dagger}, \rho \} \right]$$
$$- \gamma M \left[ a^{\dagger}\rho a^{\dagger} - \frac{1}{2} \{ a^{\dagger}a^{\dagger}, \rho \} \right]$$
$$- \gamma M^{*} \left[ a\rho a - \frac{1}{2} \{ aa, \rho \} \right]$$

$$N + 1/2 = (\bar{n} + 1/2) \cosh 2r$$
  
 $M = -(\bar{n} + 1/2)e^{i\theta} \sinh(2r)$ 

- \* The procedure for a squeezed bath is now identical as the other one.
- \* First we identify the relevant current from the QFP equation:

$$J_z(W) = \frac{\gamma}{2} \left[ \alpha W + (N+1/2)\partial_{\alpha^*} W + M_t \partial_{\alpha} W \right]$$

\* Then we carry out the separation:

 $\Phi$ 

$$\Pi = \frac{4}{\gamma(\bar{n}+1/2)} \int \frac{d^2\alpha}{W} \left| J_z(W) \cosh r + J_z^*(W) e^{i\theta} \sinh r \right|^2$$
$$= \frac{\gamma}{\bar{n}+1/2} \left\{ \langle a^{\dagger}a \rangle \cosh 2r - \bar{n} + \sinh^2 r - \frac{\sinh 2r}{2} \left[ e^{i\theta} \langle a^{\dagger}a^{\dagger} \rangle + e^{-i\theta} \langle aa \rangle \right] \right\}$$

# Onsager theory for squeezing

- \* A squeezed state is a GGE, much like the Grand canonical ensemble.
- Thus, we may think about the joint transport induced by gradients of temperature and squeezing
- \* In fact, let

$$J_E = \frac{d\langle a^{\dagger}a \rangle}{dt} = \gamma(N - \langle a^{\dagger}a \rangle)$$
$$J_C = \frac{d\langle aa \rangle}{dt} = \gamma(M - \langle aa \rangle)$$

 Now assume the state of the system and bath are infinitesimally close. We may expand to get:

$$J_E = \mathcal{T}_{1,1}\delta\bar{n} + \mathcal{T}_{1,2}\delta r$$
$$J_C = \mathcal{T}_{2,1}\delta\bar{n} + \mathcal{T}_{2,2}\delta r$$



- \* We may also construct a full thermodynamic analysis as follows.
- \* Define the conjugated thermodynamic variables:

$$f_E = \frac{\partial S}{\partial \langle a^{\dagger} a \rangle}, \qquad f_C = \frac{\partial S}{\partial \langle a a \rangle}$$

 Then the ugly formulas for the entropy production and flux may be written as

$$\Phi = f_C J_E + f_C J_C + f_C^* J_C^*$$
$$\Pi = (\bar{f}_E - f_E) J_E + (\bar{f}_C - f_C) J_C + (\bar{f}_C^* - f_C^*) J_C^*$$
$$= \mathcal{F}_E J_E + \mathcal{F}_C J_C + \mathcal{F}_C^* J_C^*$$

\* where  $\mathcal{F}_i$  are the generalized *affinities*, or *thermodynamic forces*, associated with the flow of energy and compression.

Bathview: Entropy production from the perspective of the environment

- \* Entropy production is an emergent property.
- \* It is also an informational property:
  - Occurs due to the inexorable loss of information when dealing with many particles.

- \* It would be nice to be able to understand entropy production from the perspective of the bath and the *system-bath correlations*.
- \* But that is impossible:
  - \* Different baths produce the same dynamics.
  - The reduced-system descriptions involve a bunch of approximations so we loose track of where the irreversibility comes from.

# A dilation based approach

- \* We will try to approach this problem in a different way.
- \* Consider the simplest and one of the most widely used open-system evolutions:

$$\frac{d\rho_S}{dt} = \gamma \left[ a\rho_S a^{\dagger} - \frac{1}{2} \{ a^{\dagger} a, \rho_S \} \right]$$

- We now ask, what are all the possible <u>dilations</u> which generate this evolution <u>exactly.</u>
  - \* That is, what is the most general unitary U between a system and bath which, from the perspective of the system, makes it evolve according to this equation.

$$\rho_S(t) = \operatorname{tr}_E \left\{ U(t) \left[ \rho_S(0) \otimes \rho_E(0) \right] U^{\dagger}(t) \right\}$$

- \* We assume that the bath is bosonic, specified by an arbitrary number of bosonic operators and that it starts in a global vacuum.
- \* It then turns out that the most general evolution is generated by a Hamiltonian of the form:

$$H = \omega a^{\dagger} a + \sum_{k} \Omega_{k} b_{k}^{\dagger} b_{k} + \sum_{k} \lambda_{k} (a^{\dagger} b_{k} + b_{k}^{\dagger} a)$$

 Of course, the coefficients must be chosen in a specific way to reproduce the exact dynamics, but the shape of the Hamiltonian is this one.

- \* <u>Reasons:</u>
  - 1.The reduced dynamics is Gaussian preserving, so for an exact dilation this must also be true.
  - \* For real baths, the preservation of Gaussianity is a consequence of weak-coupling. Here it must be imposed exactly.
  - With the exception of very accidental cases (of which we have so far not found any) Gaussian dynamics can only be generated by Gaussian Hamiltonians.
  - 2.The global vacuum  $|0\rangle_S \otimes |0\rangle_E$  must be an exact fixed point so there can be no terms of the form  $a^{\dagger}b_k^{\dagger}$  or  $ab_k$
  - 3. Interactions among modes can also be eliminated by a simple diagonalization and amount only to a renormalization of  $\Omega_k$  and  $\lambda_k$

$$\frac{d\rho_S}{dt} = \gamma \left[ a\rho_S a^{\dagger} - \frac{1}{2} \{ a^{\dagger} a, \rho_S \} \right]$$
$$H = \omega a^{\dagger} a + \sum_k \Omega_k b_k^{\dagger} b_k + \sum_k \lambda_k (a^{\dagger} b_k + b_k^{\dagger} a)$$

### Exact solution

\* This model can be solved exactly for any number of oscillators. All properties can be reduced to two auxiliary functions satisfying

$$\frac{dg}{dt} = -i\sum_{k} \gamma_k e^{(\omega - \Omega_k)t} f_k(t), \qquad g(0) = 1$$
$$\frac{df_k}{dt} = -i\gamma_k e^{-(\omega - \Omega_k)t} g(t), \qquad f_k(0) = 0$$

\* The Markovian behavior we are interested in is recovered by appropriately choosing the spectral density

$$J(\Omega) = 2\pi \sum_{k} \lambda_k^2 \delta(\Omega - \Omega_k)$$

\* In this limit we have

$$g(t) = e^{-\kappa t}, \qquad \kappa = 2\gamma$$



Figure 1: Example of the behavior of the function g(t), Eq. (32) for different numbers of bath oscillators, respectively K = 10, 30, 80 and 120. The functions were computed assuming  $\omega = 1$ ,  $\gamma_k = 1/K$  and  $\Omega_k = 1/2 + (k-1)/(K-1)$ (i.e., a linear interpolation from 1/2 to 3/2). The red-dashed curve correspond to the Markovian solution (37). For this particular choice of frequencies, the function g is real.



Figure 2: Example of the behavior of the function g(t) for a Lorentzian spectral density, Eq. (40), for different values of  $\gamma/\kappa$ , respectively 0.2, 1, 2 and 10. The red-dashed curve corresponds to Eq. (37).

### Mutual information

\* The interesting aspect of this model is that it gives us full access to all system-environment correlations.



### Example: evolution of a coherent state

 Consider the evolution of a harmonic oscillator starting from a coherent state:

 $\rho(0) = |\mu\rangle\langle\mu|$ 

The evolution remains as a (pure) coherent state:

$$\rho(t) = |\mu_t\rangle \langle \mu_t|$$
$$\mu_t = \mu e^{-(i\omega + \gamma/2)t}$$



\* The system remains in a pure state throughout, so that the system and bath remain uncorrelated at all times, even though they exchange energy.

# Entropy production

- Finally, we can use the Wigner phase space formalism to describe the entropy production and the entropy flux from the perspective of the bath.
- By tracing out the environment, we can obtain the following Fokker-Planck equation for the system:

$$\partial_t W_S = \partial_\alpha J_S(W) + \partial_{\alpha^*} J_S^*(W)$$

$$J_S(W) = \Gamma(t) \left( \alpha + \frac{\partial_{\alpha^*}}{2} \right) W, \qquad \Gamma(t) = -\frac{\dot{g}}{g}$$

\* This is the exact same equation as before, except that now we have a time-dependent rate. In the Markovian limit  $\Gamma(t) \rightarrow \kappa$ 

\* Thus, the entropy production from respect to the system will be

$$\Pi = -\frac{d}{dt}S(W_S||W_S^\infty) = \frac{4}{\Gamma(t)}\int d^2\alpha \frac{|J_S(W_S)|^2}{W_S},$$

- \* It will be positive as long as the dynamics is Markovian.
- \* We can also derive the following conservation law:

$$\frac{dS(W_{SE}||W_S^{\infty}W_E(0))}{dt} = 0$$

- \* So the distance to the global vacuum is preserved.
- \* From this we may finally separate the entropy production as

$$\Pi = \frac{d\mathcal{I}_{SE}}{dt} + \frac{dS(W_E||W_E(0))}{dt}$$

\* Thus, one contribution is a local entropy production within the bath and the other is the rate of change of the system-bath correlations.

## Non-Markovianity

- Non-Markovianity can be characterized in a neat way by monitoring the entanglement between a system and an ancila, which do not interact, but which began in an entangled state.
- As a proof of principle, we consider the case where the system and ancilla start in a two-mode squeezed state which, locally, is identical to a thermal state. We then find that:

$$\begin{split} \frac{\mathrm{d}\mathcal{I}_{AS}}{\mathrm{d}t} &= -\frac{2N}{N+1} \bigg\{ |g|^2 [N(1-|g|^2)+1/2] \bigg\} \Pi \\ &= -\frac{2N}{N+1} \bigg\{ (1-|g|^2) [N|g|^2+1/2] \bigg\} \frac{\mathrm{d}S(W_E||W_E(0))}{\mathrm{d}t} \\ &= \frac{N}{N+1} (1-2|g|^2) \frac{\mathrm{d}\mathcal{I}_{SE}}{\mathrm{d}t}. \end{split}$$

Ángel Rivas, Susana F. Huelga, and Martin B. Plenio Phys. Rev. Lett. 105, 050403 (2010)

### Conclusions

- \* The theory of entropy production for non-equilibrium environments is well established in classical physics.
- \* But not in the quantum case.
- Here we show that, for the case of bosonic and spin systems, an alternative formulation can be obtained in terms of phase-space entropic measures.
- This has the advantage that it allows us to recognize currents within the system which are locally responsible for the emergent irreversible behavior.

Thank you.

Extension to interacting theories and non-bosonic systems

# Non-gaussian bosonic systems

- \* The phase-space framework can be extended to more general situations which do not necessarily involve Gaussian states or even bosons.
- \* For non-Gaussian bosonic systems one may use the Husimi-Q function instead:

$$Q(\alpha, \alpha^*) = \frac{1}{\pi} \langle \alpha | \rho | \alpha \rangle$$

- \* Its corresponding phase space entropy is known as Wehrl entropy.
- \* Since *Q* is always non-negative, this theory also holds for arbitrary non-Gaussian states.
- \* The extension to multiple bosonic modes is also straightforward.
- \* The downside is that the Q function is a coarse graining and therefore the unitary part may change the entropy.

## Spins and qubits

Spin coherent states:  $|\Omega\rangle = e^{-\phi J_z} e^{-\theta J_y} |J, J\rangle$ 

Husimi-Q function:  $Q(\Omega) = \langle \Omega | \rho | \Omega \rangle$ 

Wehrl entropy: 
$$\Sigma = -\int d\Omega Q(\Omega) \ln Q(\Omega)$$

 The Quantum Fokker-Planck equation is now written in terms of orbital angular momentum operators.

$$-i[J_z,\rho] \quad \to \quad \mathcal{J}_z(\mathcal{Q}) = -i\frac{\partial}{\partial\phi}\mathcal{Q}$$

\* We have studied two types of baths with this framework:

$$D(\rho) = -\frac{\lambda}{2} [J_z, [J_z, \rho]]$$
  
$$D(\rho) = \gamma(\bar{n}+1) \left[ J_-\rho J_+ - \frac{1}{2} \{J_+J_-, \rho\} \right] + \gamma \bar{n} \left[ J_+\rho J_- - \frac{1}{2} \{J_-J_+, \rho\} \right]$$

J. P. Santos, L. C. Céleri, GTL, M. Paternostro, arXiv:1707.08946

M. Brunelli, et. al. arXiv:1602.06958

### Experiments





