Entropic dynamics of a discontinuous dissipative phase transition

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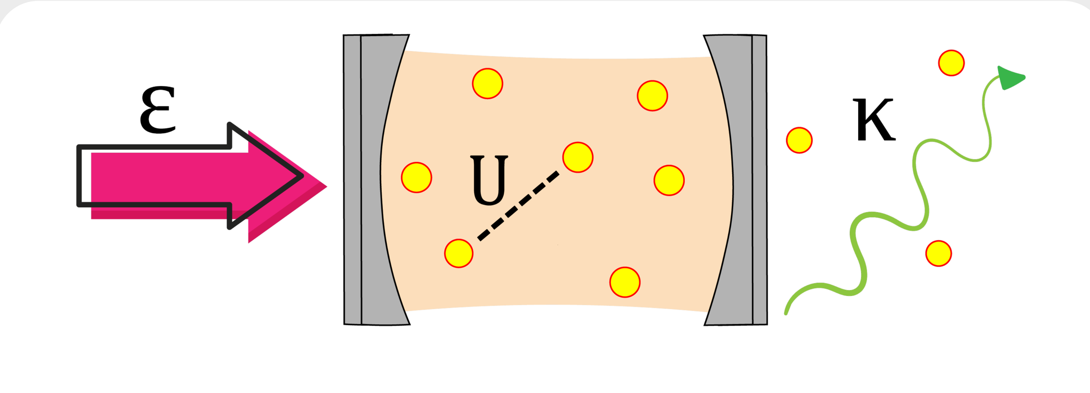
Optical bistability model

We are going to present our results for the optical bistability model, which is known to present a discontinuous phase transition in the thermodynamic limit.



Sketch of the considered system.

The hamiltonian that describes a single-mode field inside a cavity which contains a nonlinear dispersive medium, in the rotating frame is[1]

$$H = \Delta_{cp} a^\dagger a + i\mathcal{E}(a^\dagger - a) + \frac{U}{2} a^\dagger a^\dagger a a$$

- $a^\dagger$ (a) creats (annihilates) an excitation inside the cavity;
- $\Delta_{cp} = \omega_c - \omega_p$ is the detuning;
- $\mathcal{E}$ is the amplitude of the drive;
- U is the photon-photon interaction strength.

The one photon losses can be described within Born-Markov approximation, so the density matrix dynamics of the system can be described in terms of a Lindblad master equation:

$$\partial_t \rho = -i[H,\rho] + 2\kappa\left(a\rho a^\dagger - \frac{1}{2}\{a^\dagger a, \rho\}\right) \qquad (1)$$

- κ being the dissipation rate.

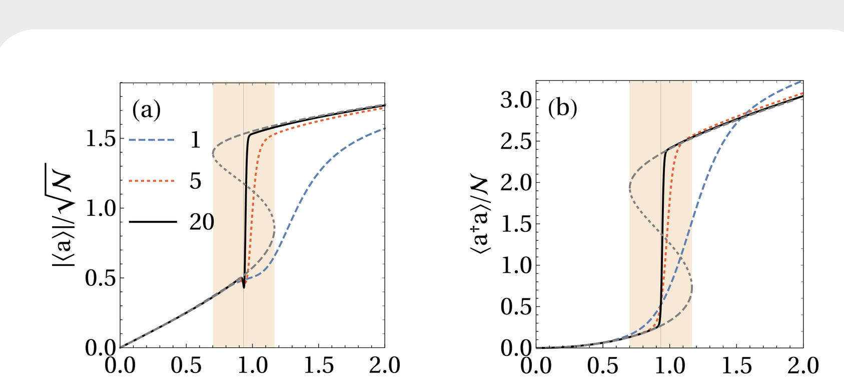
Due to the competition between the unitary dynamics and the dissipation a phase transition occurs for a given value ϵ_c.

We introduce the dimensionless parameter N and perform the scaling[2]

$$U = u/N \qquad \mathcal{E} = \sqrt{N}\epsilon \qquad \alpha = \sqrt{N}\beta \qquad (2)$$

- The thermodynamic limit (TL) occurs when $N \to \infty$, where $\mathcal{E}^2 U$ = constant while the number of excitations $n = |\alpha|^2 \propto N$ grows up;
- In the TL quantum fluctuations became negligible and the system behaves in a semiclassical fashion, which justifies a mean field approximation (MFA),which gives us:

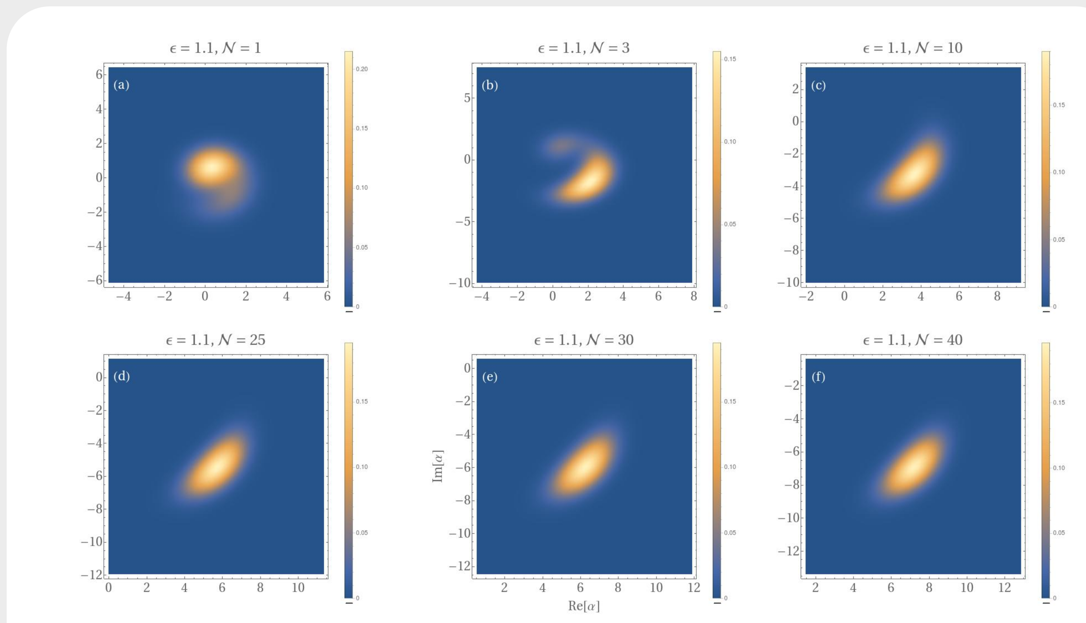
$$\partial_t \alpha = (\kappa - i\Delta_{cp} - iU|\alpha|^2)\alpha + \mathcal{E}$$



$\langle a\rangle/\sqrt{N}$ and $\langle a^\dagger a\rangle/N$, parameters were fixed at $\kappa = 1/2$, $\Delta = -2$ and $U = 1$.

Q-function

- In the quantum phase space the Q-function $Q(\alpha,\alpha^*) = \langle\alpha|\rho|\alpha\rangle/\pi$ tends to a gaussian.



Husimi Q-function contour plot: $Q = \langle\alpha|\rho|\alpha\rangle/\pi$ for $\epsilon = 1.1$ the values of N are (a) $N = 1$ (b) $N = 3$ (c) $N = 10$ (d) $N = 25$ (e) $N = 30$ (f) $N = 40$. Other parameters were fixed at $\kappa = 1/2$, $\Delta = -2$ and $U = 1$.

The Fokker-Planck equation and Wehrl entropy

- We can map eqn. 1 into a FP eqn.:[3,4]

$$\partial_t Q(\alpha,\alpha^*) = \mathcal{U}(Q) + \mathcal{D}(Q)$$

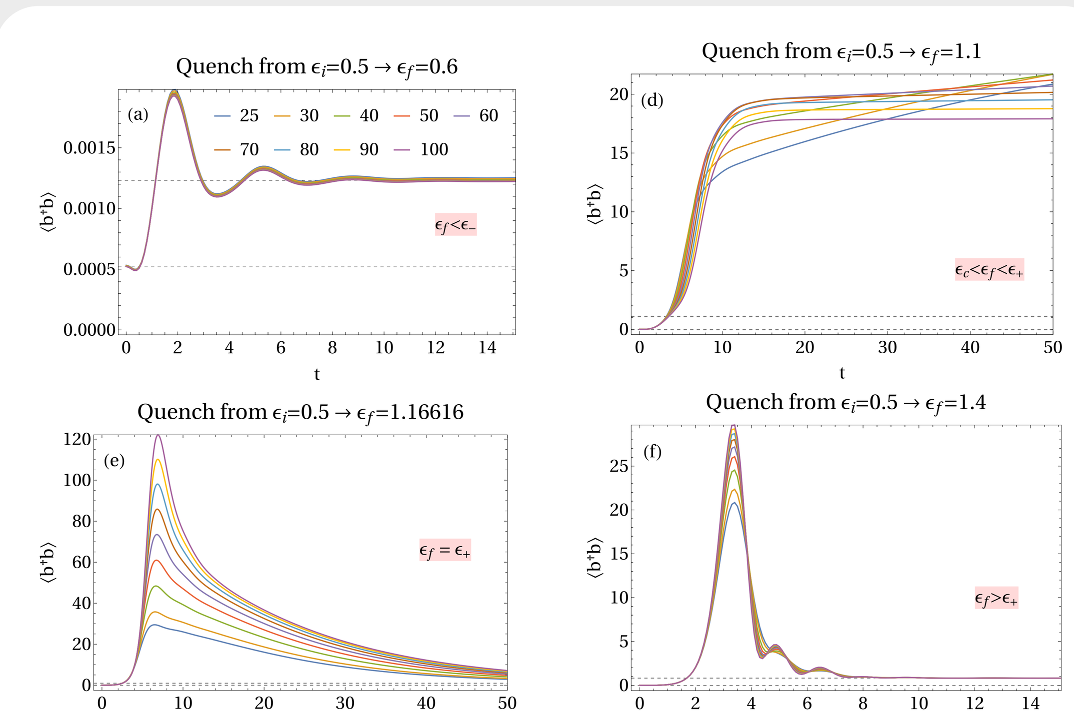
- And defining the entropy:

$$S_Q = -\int d^2\alpha\, Q(\alpha,\alpha^*)\ln(Q(\alpha,\alpha^*))$$

$$\frac{dS_Q}{dt} = -\int d^2\alpha\, \mathcal{U}(Q)\ln(Q) - \int d^2\alpha\, \mathcal{D}(Q)\ln(Q)$$
$$= \Psi + (\Pi - \Phi)$$
$$\Psi = \Psi_{\Delta_{cp}} + \Psi_{\mathcal{E}} + \Psi_U \to \Psi = \Psi_U$$

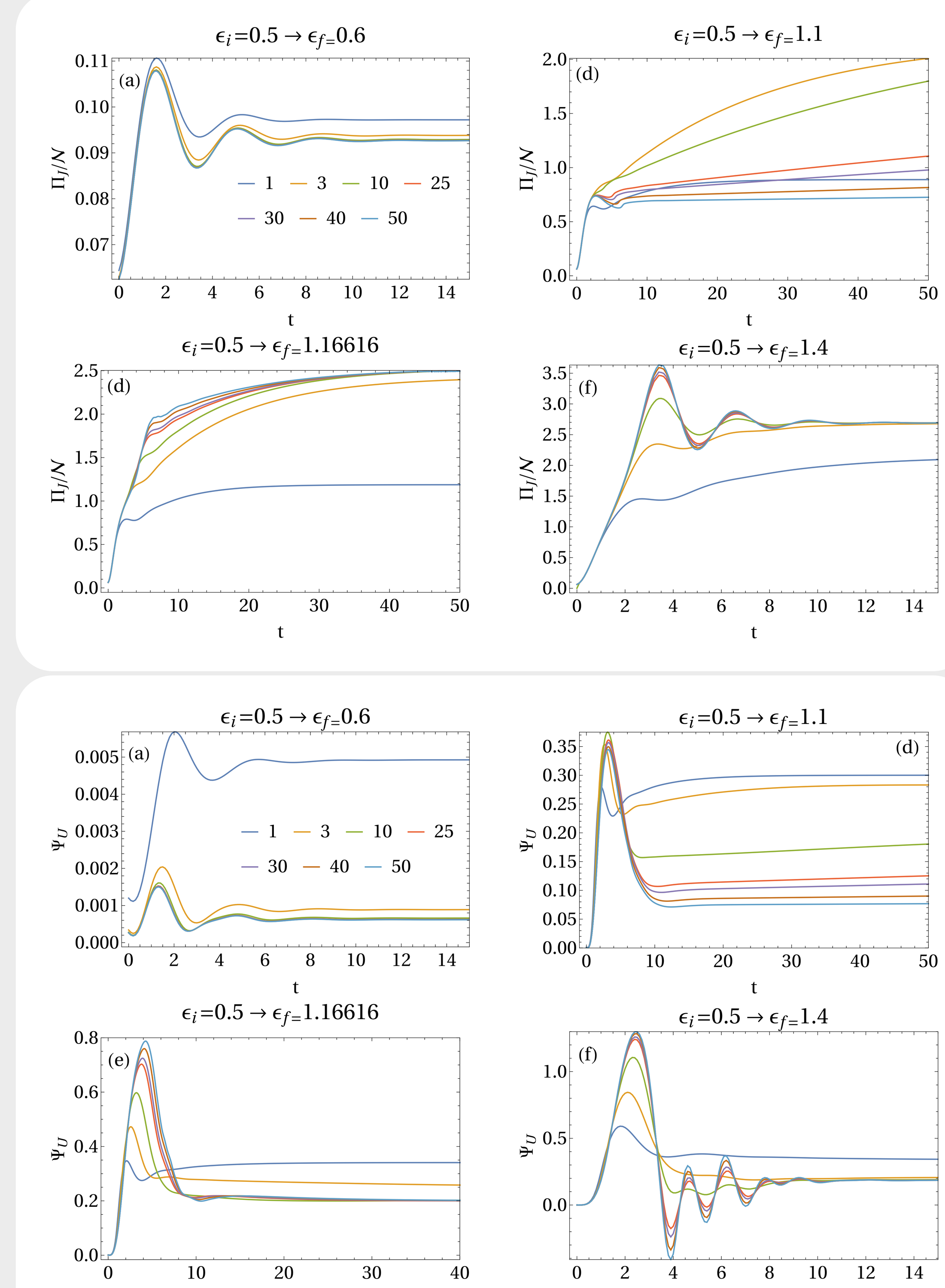
$$\Pi = \frac{2}{\kappa}\int d^2\alpha \frac{|J(Q)|^2}{Q}$$
$$J(Q) = \kappa(\alpha Q + \partial_{\alpha^*} Q)$$
$$\Phi = 2\kappa\langle a^\dagger a\rangle = 2\kappa(|\alpha|^2 + \langle b^\dagger b\rangle) = \Phi_M + \Phi_Q$$
$$\Psi = \frac{iU}{2}\langle \partial_{\alpha^*}((\alpha^*)^2\partial_{\alpha^*}(\ln(Q)) - h.c.\rangle$$

- Ψ and Φ_Q does not depend on N;



Quantum entropy flux $\Phi_Q = 2\kappa\langle b^\dagger b\rangle_t$ for a quench dynamics scenario as a function of time t: The curves are for nine different values of N, as shown in image (a). The dashed black horizontal lines corresponds to NESS value of $\langle b^\dagger b\rangle$ for each pair of (ϵ_i, ϵ_f), computed from the exact solution of Ref.[1] The initial ϵ_i is fixed at $\epsilon_i = 0.5$. (a) $\epsilon_f = 0.6$ (d) $\epsilon_f = 1.1$ (e) $\epsilon_f = \epsilon_p$ (f) $\epsilon_f = 1.4$. Other parameters were fixed at $\kappa = 1/2$, $\Delta = -2$ and $U = 1$.

Entropic dynamics



Π_I/N **and** Ψ_U **entropic dynamics:** The curves are for seven different values of N, as shown in images (a). The initial ϵ_i is fixed $\epsilon_i = 0.5$. (a) $\epsilon_f = 0.6$ (d) $\epsilon_f = 1.1$ (e) $\epsilon_f = \epsilon_p$ (f) $\epsilon_f = 1.4$. Other parameters were fixed at $\kappa = 1/2$, $\Delta = -2$ and $U = 1$.

Conclusions

- We have developed a theorical framework based on the Husimi Q-function to study the entropic dynamics of a dissipative phase transition, which have 3 contributions;
- We have simulated the dynamics of the system in a quench scenario and we can note that even quantities that does not depend on N, in principle, shows a scalling behaviour when crossing the ϵ_c in the trasient dynamics.

Acknowledgements

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References

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