Weakly coherent collisional models

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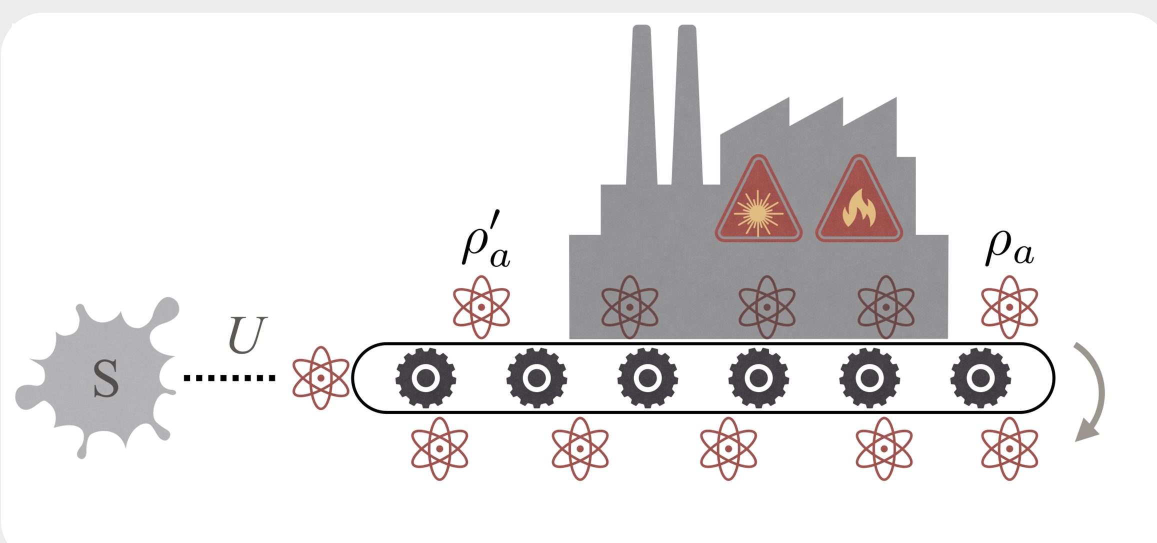
Collisional models



- Prepare system ρ_S and unit ρ_A uncorrelated.
- Interact: $U = \exp\{-iV\tau\}$.
- Throw unit away.
- Repeat.

Global Map:

$$\rho'_{SA} = U(\rho_S \otimes \rho_A)U^\dagger$$

For the system:

$$\rho'_S = \mathrm{Tr_A}\{\mathrm{U}(\rho_S \otimes \rho_A)\mathrm{U}^\dagger\}.$$

Perturbative coherence

Introducing weakly coherent states:

$$\rho_A = \rho^{th} + \sqrt{\tau}\lambda\chi.$$

Scaling for dissipative effect:

$$V \to V/\sqrt{\tau}$$

For small τ:

$$\rho'_S = \rho_S - i\tau[H_S + \lambda G, \rho_S] + \tau\mathcal{D}[\rho_S],$$

$$G = \mathrm{Tr_A}\{\mathrm{V}\chi\},$$

$$\mathcal{D} = -\frac{1}{2}\mathrm{Tr_A}\{[\mathrm{V}, [\mathrm{V}, \rho_S \otimes \rho^{\mathrm{th}}]]\},$$

- Thermal state induces dissipation
- Weak coherence induces non-commutative unitary evolution

Continuum limit

Taking

$$\dot\rho = \lim_{\tau\to 0}\frac{\rho'_S - \rho_S}{\tau},$$

then:

$$\dot\rho_S = -i[H_S + \lambda G, \rho_S] + \mathcal{D}[\rho_S]$$

- Exact master equation
- Lindblad dissipator

Parametrization

Parametrization of energy preserving interaction:

$$V = \sum_k g_k(L_k A_k^\dagger + L_k^\dagger A_k),$$

where the eigenoperators L_k and A_K are defined as

$$[H_S, L_k] = -\omega_k L_k,$$
$$[H_S, A_k] = -\omega_k A_k.$$

$$G = \sum_k g_k(L_k\langle A_k^\dagger\rangle_\chi + L_k^\dagger\langle A_k\rangle_\chi)$$

$$\mathcal{D}[\rho_S] = \sum_k |g_k|^2\left\{\langle A_k A_k^\dagger\rangle_{\mathrm{th}}\mathcal{D}[L_k] + \langle A_k^\dagger A_k\rangle_{\mathrm{th}}\mathcal{D}[L_k^\dagger]\right\}$$

Example : Qubits

$$V = g(\sigma_+^S\sigma_-^A + \sigma_-^S\sigma_+^A),$$

$$\rho_A = \begin{pmatrix} f & 0 \\ 0 & 1-f\end{pmatrix} + \begin{pmatrix} 0 & q \\ q & 0\end{pmatrix},$$

then

$$G = gq\sigma_x,$$

and

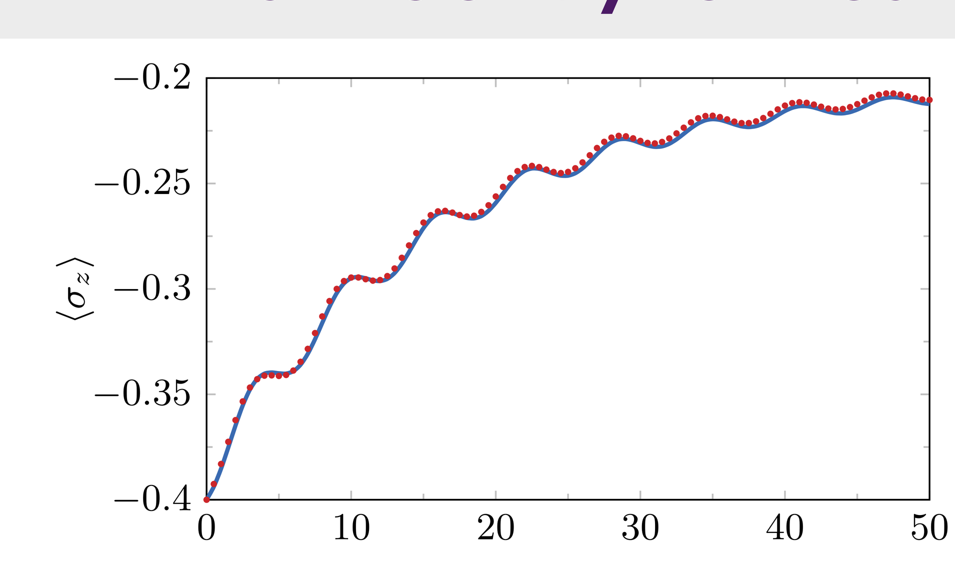
$$D(\rho_S) = \gamma^-\left[\sigma_-^S\rho_S\sigma_+^S - \frac{1}{2}\{\sigma_+^S\sigma_-^S, \rho_S\}\right] + \gamma^+\left[\sigma_+^S\rho_S\sigma_-^S - \frac{1}{2}\{\sigma_-^S\sigma_+^S, \rho_S\}\right],$$

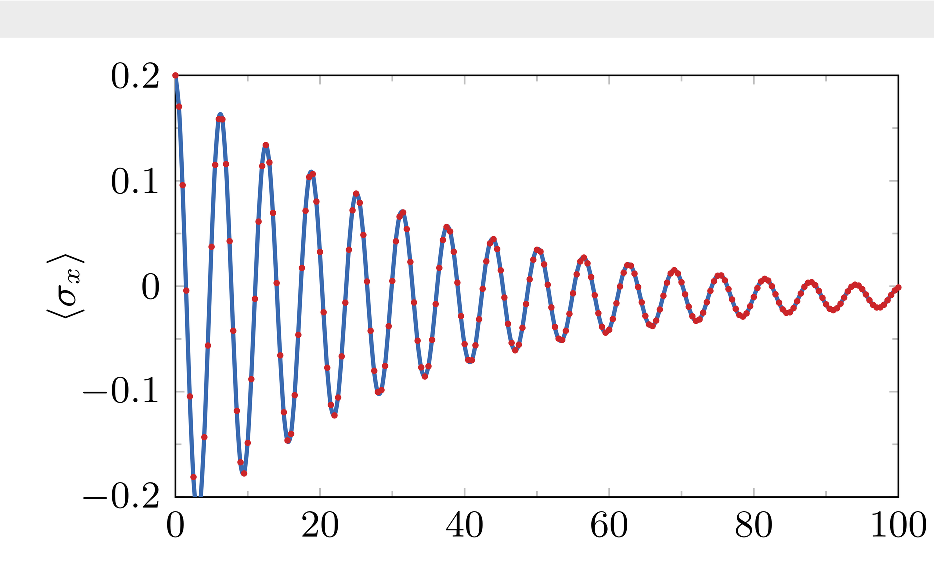
where

$$\gamma^- = g^2(1-f), \qquad \gamma^+ = g^2 f. \qquad (1)$$

For $\sqrt{\tau}\lambda = 0.7$, $q = 0.1$, $\omega = 1$, $g = 0.25$ and $f = 0.4$:

EME vs Exact Dynamics





Energy Balance

$$\dot H_A \equiv \dot Q$$
$$\dot H_S = i\lambda\,\mathrm{tr}\{[H_S, G]\rho_S\} + \mathrm{tr}\{\mathcal{D}[\rho]H_S\}.$$

$$\dot{\mathcal{W}}_c \equiv i\lambda\,\mathrm{tr}\{[H_S, G]\rho_S\}$$
$$\dot{\mathcal{Q}}_{inc} \equiv \tau\,\mathrm{tr}\{\mathcal{D}[\rho]H_S\}.$$

$$\dot H_S = -\dot Q = \dot{\mathcal{W}}_c + \dot{\mathcal{Q}}_{inc}.$$

- Transformation process: Disordered to ordered energy.
- Quantum coherence as a resource.
- No work! Interaction preserves energy.[1]

Entropy production

Defined as:[2]

$$\Sigma = \mathcal{I}(S:A)' + K(\rho_A'\|\rho_A) \geq 0,$$

where $\mathcal{I}$ and K are the mutual information and Kullback-Leibler divergence respectively.

With the map above:

$$\dot{\mathcal{W}}_c \geq -T\,\dot{C}_A$$

- C_A is the coherence entropy, is a measure for coherence.
- Coherence consumption bounds ordered energy produced

Moreover:

$$\dot\Sigma = \beta(\dot{\mathcal{W}}_c - \dot F_S) = \dot S_S - \beta\dot{\mathcal{Q}}_{inc}.$$

- Modified second law

Next Steps

- Study thermal engines mixing classical and quantum resources
- Include external work

References

[1] G. De Chiara, G. Landi, A. Hewgill, B. Reid, A. Ferraro, A. J. Roncaglia, and M. Antezza, Reconciliation of quantum local master equations with thermodynamics, New Journal of Physics **20**, 113024 (2018), 1808.10450.

[2] P. Strasberg, G. Schaller, T. Brandes, and M. Esposito, Quantum and Information Thermodynamics: A Unifying Framework based on Repeated Interactions, arXiv **021003**, 1 (2016), 1610.01829.

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