Rectification and out-of-time-order correlators

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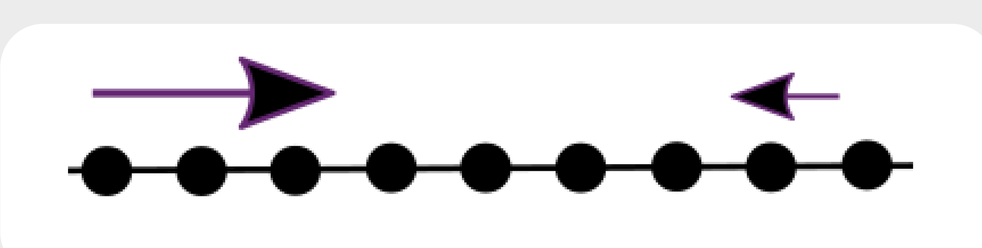
Introduction

This poster presents an ongoing investigation on the fundamental properties of rectification in the quantum realm. For that we developed some techniques grounded on OTOC formalism. We propose to achieve a quantifier associated with the underlying operator algebra.

Rectification

By analogy to electric and thermal diodes we look at an information rectifier as an one dimensional L-site quantum chain which carry a reasonable amount of information in one direction whilst barely does it in the other.



OTOC's

Out-of-time-order correlators(OTOC's) are used to quantify information scrambling in quantum systems.[1] We seek to associate an OTOC to each direction of propagation of information and compare them in order to quantify how information scrambles in each direction of a quantum chain.

The prototype of such idea is to track *local operators* and how they evolve in Heisenberg's picture. Let A_j be an operator of site j and time evolved operators having a t argument. We then define our OTOC as:

$$C_{ji} := \left\langle \mathcal{A}_{ji} \right\rangle \tag{1}$$

$$\mathcal{A}_{ji}(t) := \frac{[A_j(t), A_i]}{2i^*} \cdot \frac{[A_j(t), A_i]^\dagger}{2i} \tag{2}$$

Indeed we are interested in $\mathcal{A}_{ji}(t)$ so that we obtain a state-independent quantifier. One can thus associate to left and right information currents the quantities $\mathcal{A}_{1L}$ and $\mathcal{A}_{L1}$ so that a candidate of quantifier is:

$$Q = \mathcal{A}_{1L} - \mathcal{A}_{L1} \tag{3}$$

This quantity is interesting but has some flaws when one wants to track chain asymmetry. To fix that we allow ourselves to go beyond.

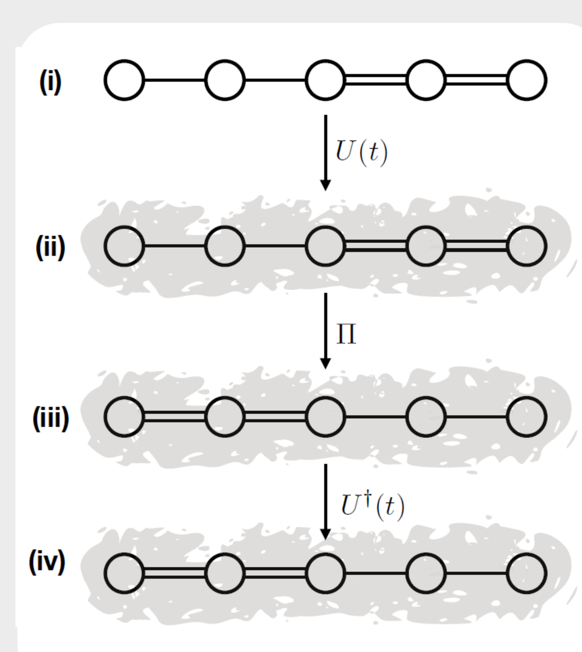
Quantifying Rectification

Beyond OTOC's

Despite our idea is largely based on the OTOC's framework it's essence lies in quantifying rectification independently of states and dissipator particularities.

Evolve, mirror and evolve back

Our general desire together with the sense that chain asymmetry is a cornerstone for rectification gives rise to another prototype: let an local operator evolve to t, apply a parity transformation to mirror the chain and evolve backwards in time through the reflected chain.



One now chooses either site one or L and compare the local operator to itself after applying such protocol. This is encompassed by the operator $\mathcal{P}$ which by construction yields zero for any parity invariant chain:

$$\mathcal{P} := [A_L, \Pi U \Pi U^\dagger A_L U \Pi U^\dagger \Pi] \tag{4}$$

Where Π maps a local operator at site j to $L+1-j$

Systems of interest

So far we are interested in applying $\mathcal{P}$ and Q to graded XXZ, XX, transverse Ising spin chains and bosonic/fermionic Tight Binding model. The site-dependent parameters are tuned to induce chain asymmetry.
In the case of the tight binding Hamiltonian:

$$\mathcal{H} = \sum_{i,j} w_{ij} a_i^\dagger a_j \tag{5}$$

Considering the matrix w defined by w_{ij} we used the Green's functions associated to forward evolution and mirrored backward evolution respectively:

$$W = e^{-itw} \tag{6}$$

$$G = e^{+itg} \tag{7}$$

$$g_{ij} = w_{L+1-i,L+1-j} \tag{8}$$

So that if we take our local operator to be $N_L = a_L^\dagger a_L$ we get:

$$\begin{aligned} \mathcal{P} &= [N_L, \Pi U \Pi U^\dagger N_L U \Pi U^\dagger \Pi] && (9) \\ &= \sum_j (W\cdot G)_{jj} (W\cdot G)^*_{LL} a_j^\dagger a_l - (W\cdot G)^*_{jj} (W\cdot G)_{jL} a_L^\dagger a_j && (10) \end{aligned}$$

C-Numbers

The final step is to associate a number to such operator. Norms are the way to go and the first candidate is of course the trace norm.

$$\|\mathcal{P}\|_1 := \mathrm{Tr}\left\{ \sqrt{\mathcal{P}^\dagger \mathcal{P}} \right\} \tag{11}$$

Yet trace norm can be inconvenient whilst doing symbolic computations and other possibilities are investigated together with upper bounds.
Bounds can be useful in cases like (10) for one can try to obtain the particular form of the hamiltonian parameters which saturate the bounds.

Prospects and ongoing work

- $\mathcal{P}$ is being applied to tight binding models, where analytical calculations are simpler to perform.
- Numerical and analytical computations of trace norms and bounds.
- More alternatives to quantifiers are being investigated.
- Applying $\mathcal{P}$ and Q to small chains in Mathematica.
- Benchmarking according to the results of[2] for small chains in Mathematica and for larger ones with DMRG techniques.

Open questions

- Is rectification possible in closed systems?
- In systems with dissipation does it depend on the particular form of the dissipation to occur?
- What is the precise relation between parity, time reversal and rectification?

Acknowledgements

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References

[1] J. R. G. Alonso, N. Y. Halpern, and J. Dressel, Out-of-time-ordered-correlator quasiprobabilities robustly witness scrambling (2018), `arXiv:1806.09637`.

[2] V. Balachandran, G. Benenti, E. Pereira, G. Casati, and D. Poletti, Perfect Diode in Quantum Spin Chains, Physical Review Letters **120**, 200603 (2018), `1707.08823`.