QUANTIFYING IRREVERSIBILITY IN QUANTUM SYSTEM

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Transport in strongly correlated quantum systems
July 23rd, 2018

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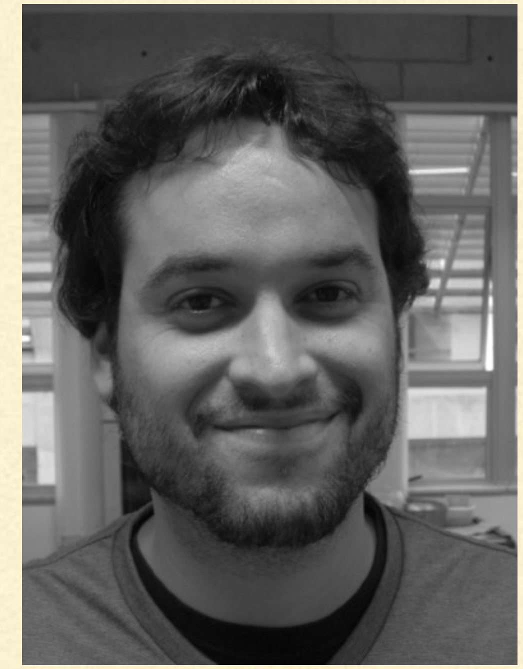

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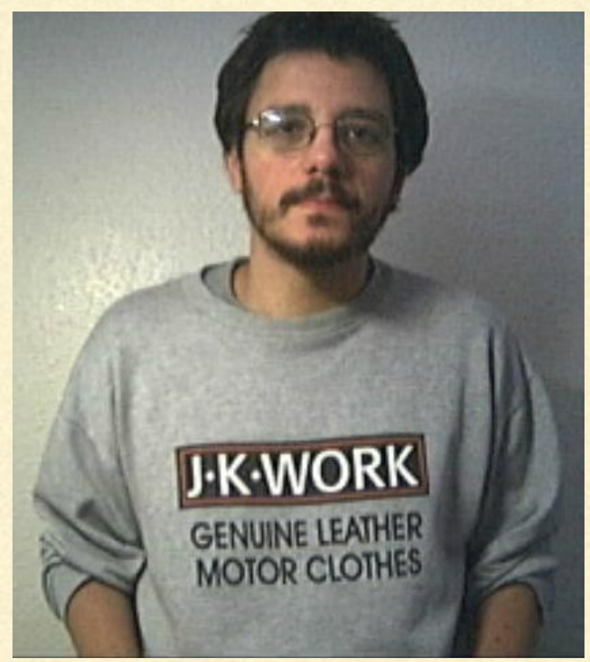

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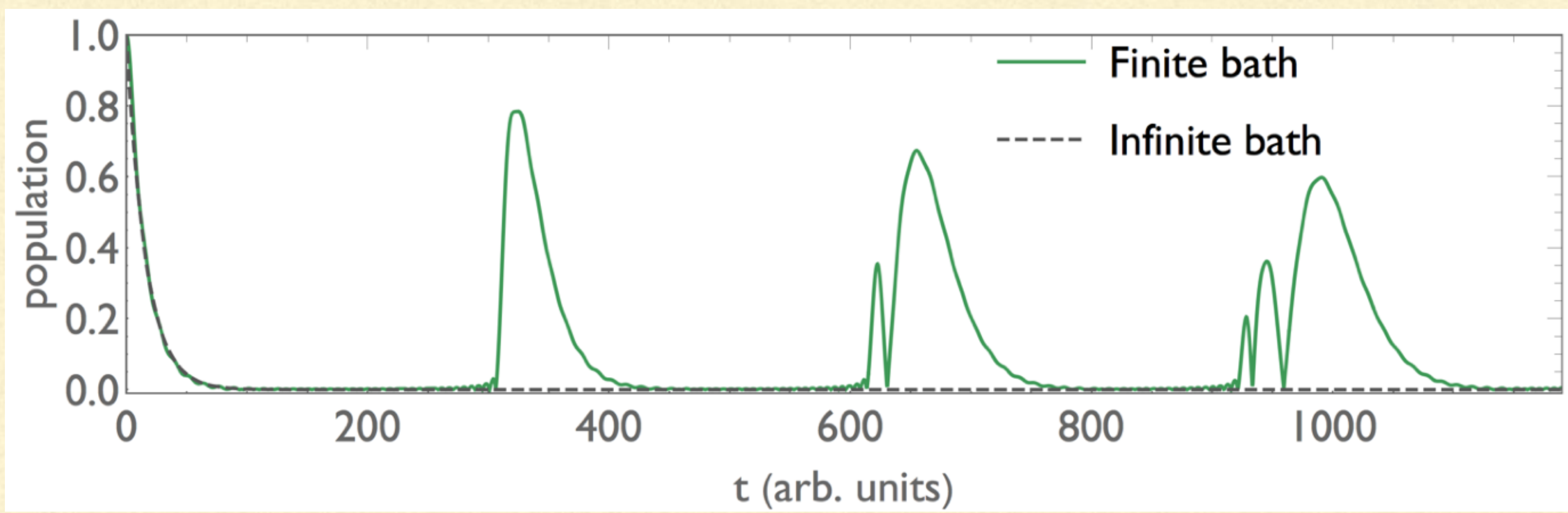
IRREVERSIBILITY

- Consider a system *S* connected to an environment *E*, undergoing some process.

- Information about S is diluted in the environment and some part (or all of it) may never return.



- Irreversibility ≔ the irretrievable loss of *any* resource.

- **Goal:** to quantify the degree of irreversibility.

ENTROPY PRODUCTION

- In thermodynamics the resources are heat and work, and irreversibility is quantified using the *entropy production*.

(Clausius inequality) $$\Delta S \geq \frac{\delta Q}{T} \qquad \longrightarrow \qquad \Sigma := \Delta S - \frac{\delta Q}{T} \geq 0$$

(entropy flux) (entropy production)

- We also express this in terms of rates:

$$\Pi = \frac{d\Sigma}{dt} \qquad \boxed{\frac{dS}{dt} = \Pi - \Phi} \qquad \Phi = -\frac{1}{T}\frac{dQ}{dt}$$

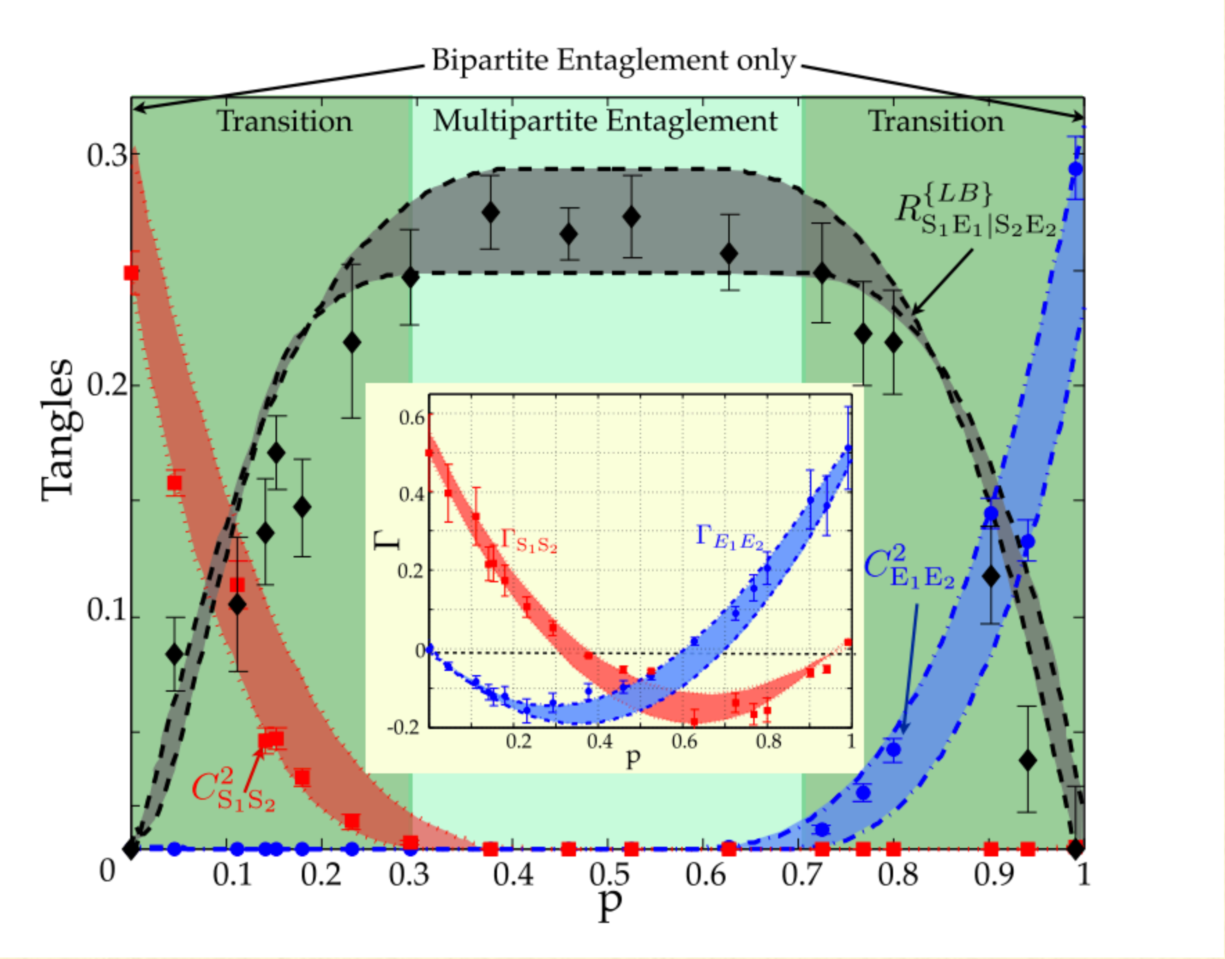
(entropy production *rate*) (entropy flux *rate*)

WHAT IS DIFFERENT IN QUANTUM SYSTEMS?

WHAT IS DIFFERENT IN QUANTUM SYSTEMS?

1. In quantum systems there are also other resources, such as entanglement and coherence.

   - They are also irretrievably lost due to the contact with the environment.



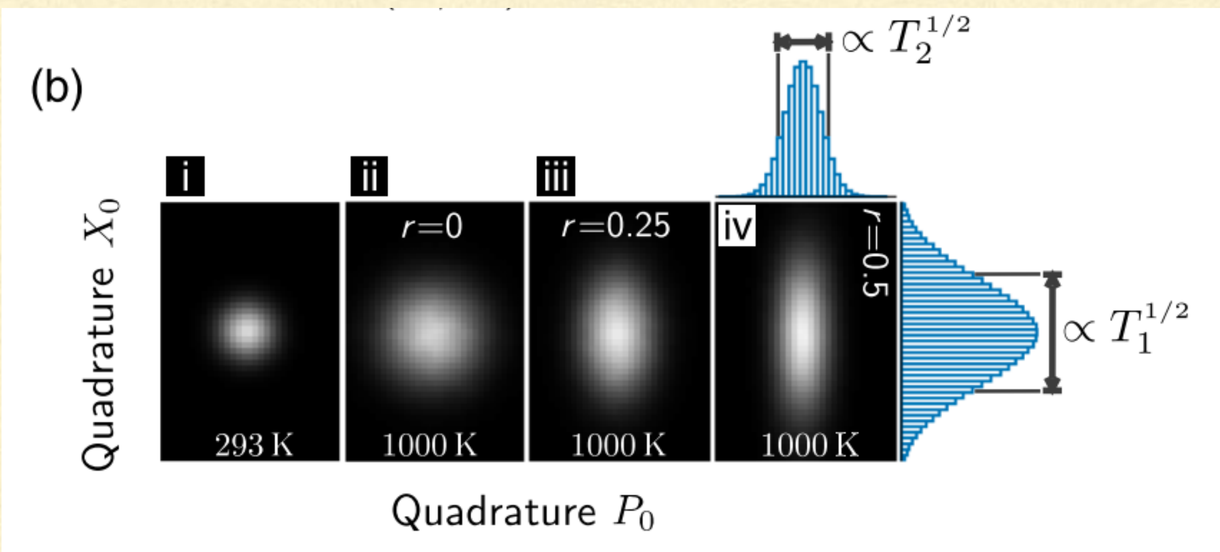
Aguilar, Valdés-Hernández, Davidovich, Walborn, Souto Ribeiro, *Phys. Rev. Lett*, **113**, 240501 (2014)

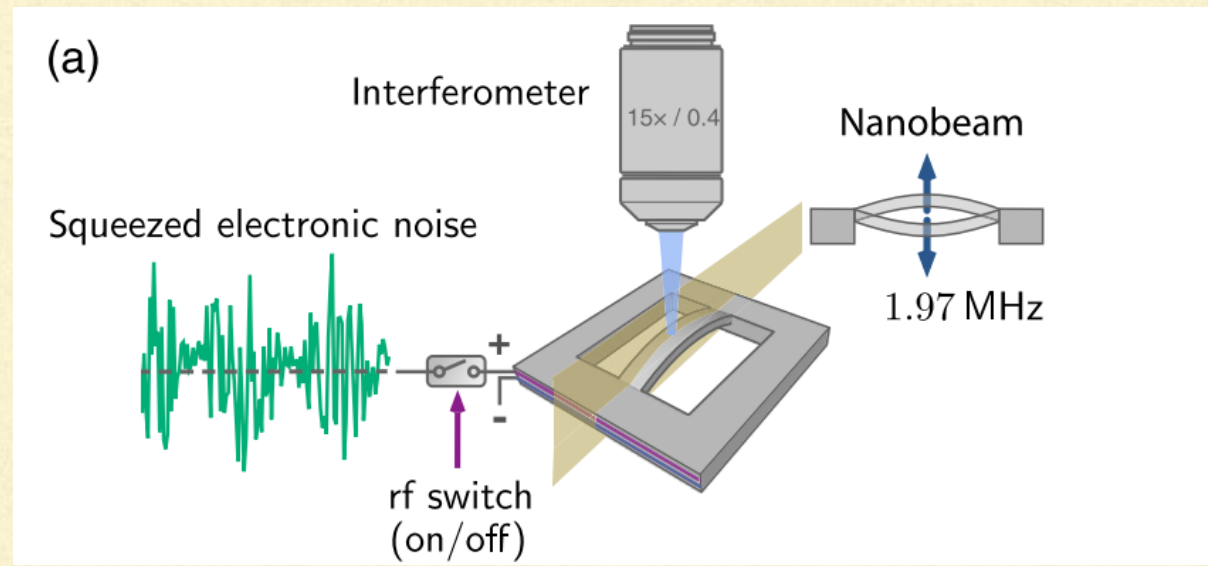
WHAT IS DIFFERENT IN QUANTUM SYSTEMS?

2. We are no longer restricted to equilibrium baths.

   - It is possible to work with *engineered environments*.

- Example: *squeezed thermal bath:*





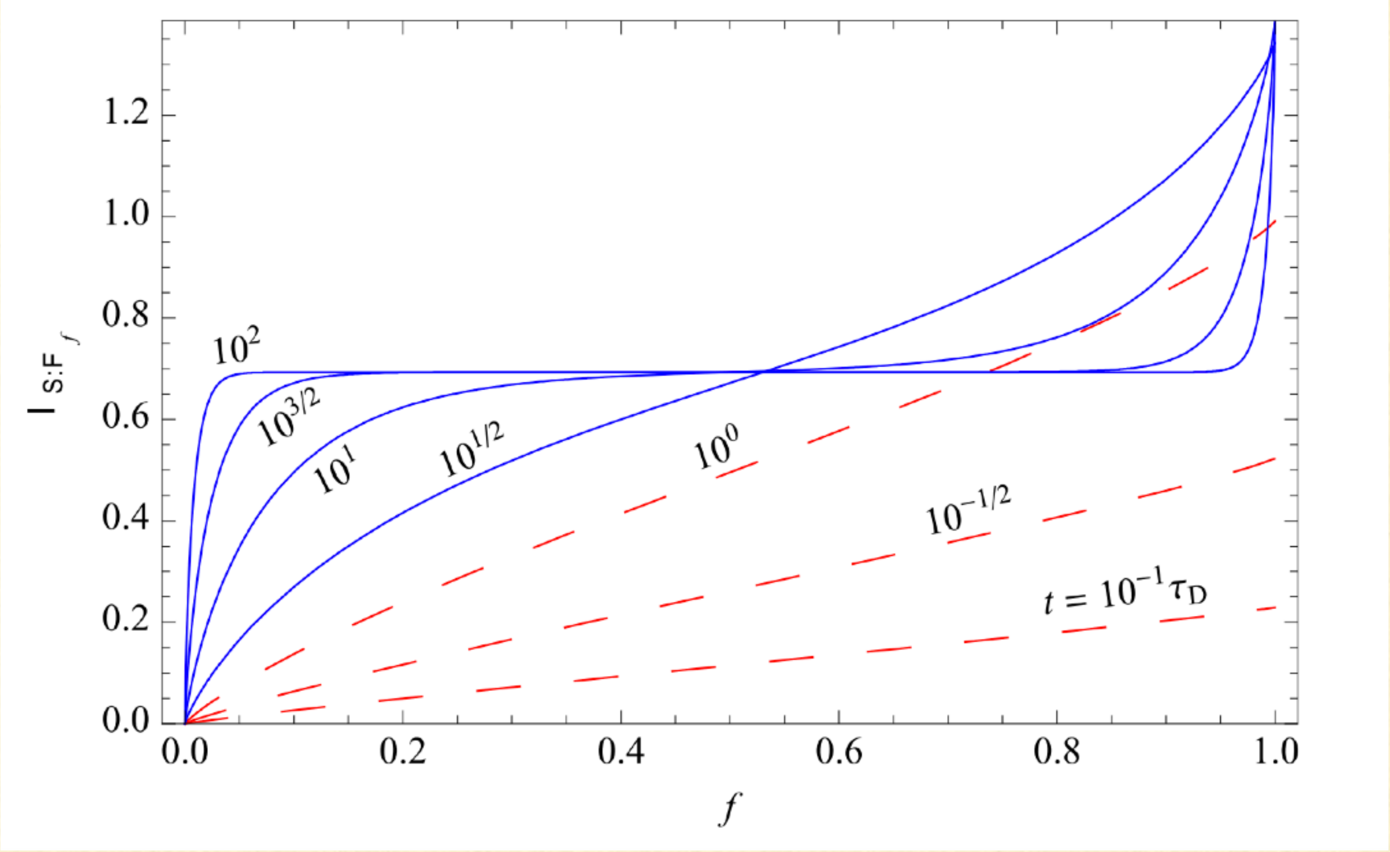
Klaers, Faelt, Imamoglu, Togan, *Phys. Rev. X*, **7**, 031044 (2017)

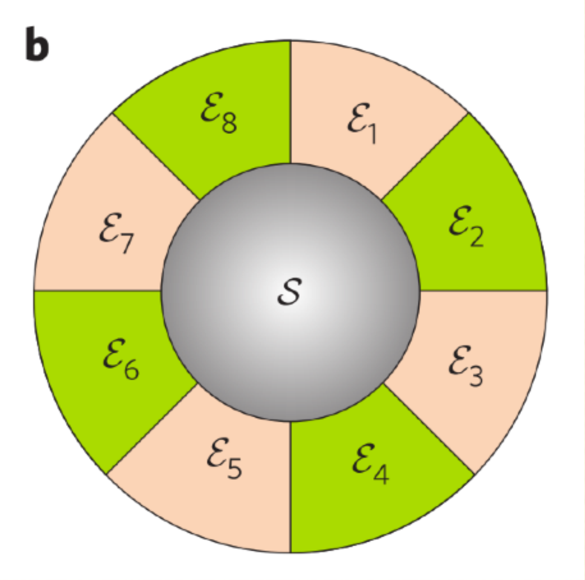
Move beyond the standard paradigms of thermodynamics.

WHAT IS DIFFERENT IN QUANTUM SYSTEMS?

3. Information becomes an essential concept:

- *"The fragility of states makes quantum systems very difficult to isolate. Transfer of information (which has no effect on classical states) has marked consequences in the quantum realm. So, whereas fundamental problems of classical physics were always solved in isolation (it sufficed to prevent energy loss), this is not so in quantum physics (leaks of information are much harder to plug)."*

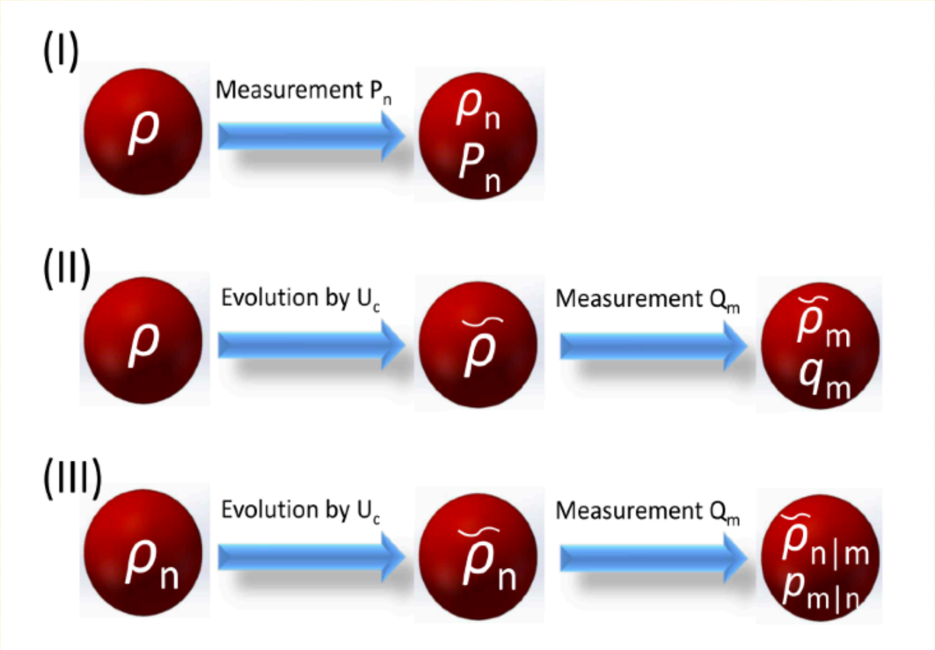




W. J. Zurek,
Nature Physics, **5**, 181 (2009)

WHAT IS DIFFERENT IN QUANTUM SYSTEMS?

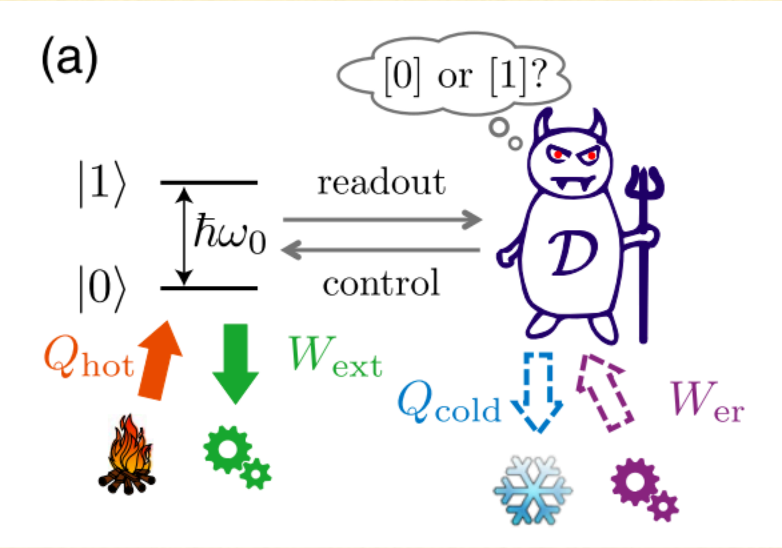
4. Measurement plays a central role:



Xiong, *et. al.*, *Phys. Rev. Lett.* **120**, 010601 (2018)

- Processes depend on deltas.
- Back-action (state collapse) affects how we extract thermodynamic information.

- Measurements can be directly implemented in thermodynamic engines.
- Maxwell's demons and information engines.

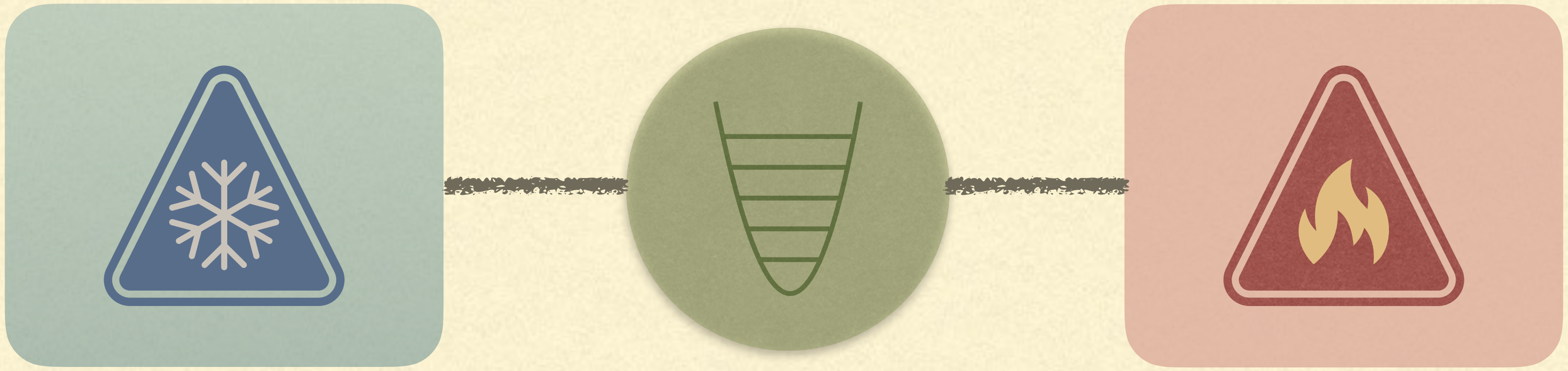


Elouard, Herrera-Martí, Huard, Auffèves, *Phys. Rev. Lett*, **118**, 260603 (2017)

SUMMARY

1. Quantum vs. Classical master equations: role of quantum coherence.

2. Entropy production in quantum non-equilibrium steady-states.



CLASSICAL VS. QUANTUM MASTER EQUATIONS

Jader P. Santos, Lucas C. Céleri, Gabriel T. Landi and Mauro Paternostro
The role of quantum coherence in non-equilibrium entropy production
arXiv 1707.08946 (submitted to *Nature Quantum Information*)

Jader P. Santos, Lucas C. Céleri, Frederico Brito, Gabriel T. Landi and Mauro Paternostro
Spin-phase-space-entropy production
arXiv 1806.04463 (PRA)

Jader P. Santos, Alberto L. de Paula, Raphael Drumond, Gabriel T. Landi and Mauro Paternostro
Irreversibility at zero temperature from the perspective of the environment.
arXiv 1804.02970 (*PRA Rapid Communications*).

- Consider a system with discrete energy levels and let p_n denote de probability of being found in state n.

- In a classical approach, the dynamics of the system in contact with a bath would be described by a Pauli master equation:

$$\frac{dp_n}{dt} = \sum_m \Big\{ W(n|m)p_m - W(m|n)p_n \Big\}$$

- Let us assume the steady-state is thermal equilibrium $\qquad p_n^{\text{eq}} = \dfrac{e^{-\beta E_n}}{Z}$

- Using the Shannon entropy, Schnakenberg proposed the following expression for the entropy production [Rev. Mod. Phys., **48**, 571 (1976)].

$$\Pi = -\frac{dS(\mathbf{p}(t)||\mathbf{p}^{\text{eq}})}{dt} \qquad S(\mathbf{p}||\mathbf{p}^{\text{eq}}) = \sum_n p_n \ln p_n / p_n^{\text{eq}}$$

(relative entropy)

Π due to system adapting to new population imposed by the bath.

QUANTUM MASTER EQUATION

- Now consider a quantum master equation: $$\frac{d\rho}{dt} = -i[H,\rho] + D(\rho)$$
- This equation will describe the evolution of both populations *and* coherences.
- e.g.: $$D(\rho) = \gamma(1-f)\left[\sigma_-\rho\sigma_+ - \frac{1}{2}\{\sigma_+\sigma_-,\rho\}\right] + \gamma f\left[\sigma_+\rho\sigma_- - \frac{1}{2}\{\sigma_-\sigma_+,\rho\}\right]$$

$$f = \frac{1}{e^{\beta\Omega}+1}$$

$$\rho = \begin{pmatrix} p_0 & q \\ q^* & p_1 \end{pmatrix}$$

$$\frac{dp_0}{dt} = \gamma f p_1 - \gamma(1-f)p_0$$

$$\frac{dp_1}{dt} = \gamma(1-f)p_0 - \gamma f p_1$$

(Pauli master equation)

$$\boxed{\frac{dq}{dt} = -\frac{\gamma}{2}q}$$

ENTROPY PRODUCTION

- Here we consider **Thermal Operations** (or Davies maps), which have simple thermal properties.
  - Thermalize correctly.
  - Populations evolve according to classical M Eq.
- The entropy flux does not depend on the coherences: $\Phi = -\frac{1}{T}\frac{dQ}{dt}$
- But the entropy production, on the other hand, becomes

$$\boxed{\Pi = -\frac{dS(\rho||\rho_{\mathrm{eq}})}{dt}}$$

$$S(\rho||\rho_{\mathrm{eq}}) = \mathrm{tr}\Big\{\rho(\ln\rho - \ln\rho_{\mathrm{eq}})\Big\}$$

ENTROPY PRODUCTION FROM GLOBAL DYNAMICS

- We can instead think about entropy production in terms of the *global* unitary dynamics of S+E. Then one may show that

$$\Pi = -\frac{d\mathcal{I}_{SE}}{dt} - \frac{dS(\rho_E(t)||\rho_E^{\mathrm{th}})}{dt}$$

- Thus, entropy production stems from:

1. *Mutual information* built up between S and E that is lost.

2. The state of the environment being pushed away from equilibrium.

1707.08946 and 1804.02970
see also: M. Esposito, K. Lindenberg, and C. Van Den Broeck, NJP **12**, 013013 (2010).

CONTRIBUTION FROM QUANTUM COHERENCES

- But now we can separate: $S(\rho||\rho_{\rm eq}) = S(\mathbf{p}||\mathbf{p}^{\rm eq}) + \mathcal{C}(\rho)$

  $\mathcal{C}(\rho) = S(\Delta_H(\rho)) - S(\rho)$ (Entropy of coherence)

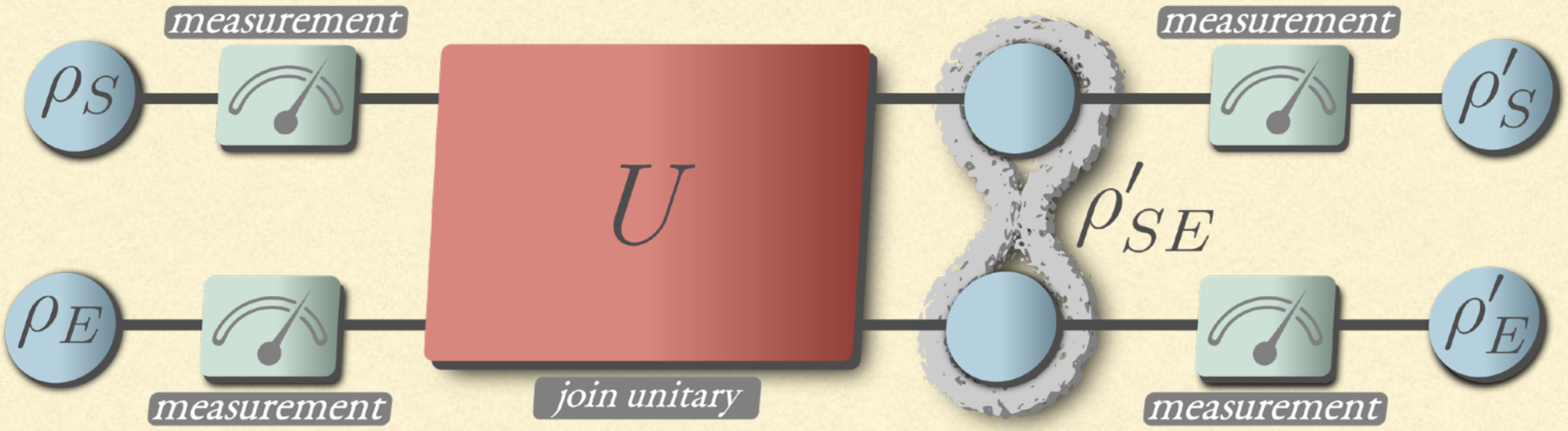
- As a result, we find that the entropy production can be divided in two parts:

$$\boxed{\Pi = -\frac{dS(\mathbf{p}(t)||\mathbf{p}^{\rm eq})}{dt} - \frac{\mathcal{C}(\rho)}{dt}}$$

- One part is the classical: entropy production due to population change.
- But the other is genuinely quantum mechanical:
  - Entropy production due to loss of coherence.

QUANTUM TRAJECTORIES

- Entropy production is not an observable.
  - But in certain cases it can be related to observables (e.g. currents in Onsager's theory).
- Otherwise, to access the entropy in the lab, we need to perform 2 quantum measurements.



- Initially the environment is thermal and the system is in an arbitrary state:

$$\rho_E(0) = \sum_\mu q_\mu^{\mathrm{th}} |\mu\rangle\langle\mu|, \qquad \rho_S(0) = \sum_\alpha p_\alpha |\psi_\alpha\rangle\langle\psi_\alpha|$$

- In general the system is *not* diagonal in the energy eigenbasis:

$$H_S|n\rangle = E_n|n\rangle$$

- **Step 1**: At $t = 0$ we then measure both S and E in the basis $|\psi_\alpha\rangle \otimes |\mu\rangle$
- Obtain outcomes with probability $p_\alpha q_\mu^{\mathrm{th}}$
- **Step 2**: evolve with a unitary *U* to obtain a final state ρ'_{SE}
- Now define $\rho'_S = \mathrm{tr}_E \rho'_{SE} := \sum_\beta p'_\beta |\psi'_\beta\rangle\langle\psi'_\beta|$
- **Step 3**: measure again S and E in the basis $|\psi'_\beta\rangle \otimes |\nu\rangle$

- Quantum trajectory: $\mathcal{X} = \{\alpha, \mu, \beta, \nu\}$

$$\mathcal{P}[\mathcal{X}] = p(\beta, \nu | \alpha, \mu) p_\alpha q_\mu^{\text{th}} = |\langle \psi'_\beta, \nu | U | \psi_\alpha, \mu \rangle|^2 p_\alpha q_\mu^{\text{th}}$$

- Now we define the stochastic entropy production

$$\boxed{\sigma[\mathcal{X}] = -\ln \left(\frac{p'_\beta q_\nu^{\text{th}}}{p_\alpha q_\mu^{\text{th}}} \right)}$$

- Its average gives the entropy production we had before: $\langle \sigma[\mathcal{X}] \rangle = \Sigma$

- And it satisfies a fluctuation theorem: $\langle e^{-\sigma[\mathcal{X}]} \rangle = 1$

CONTRIBUTION FROM QUANTUM COHERENCES

- But now we can ask, on this *stochastic level*, what is the meaning of separating the entropy production in two parts?

$$\boxed{\Pi = -\frac{dS(\mathbf{p}(t)||p^{\text{eq}})}{dt} - \frac{d\mathcal{C}(\rho)}{dt}}$$

- Define an *augmented* quantum trajectory: $\tilde{\mathcal{X}} = \{\alpha, n, \mu, \beta, m, \nu\}$

$$\mathcal{P}[\tilde{\mathcal{X}}] = \mathcal{P}[\mathcal{X}] p_{n|\alpha} p'_{m|\beta}$$

- where we defined the conditional probabilities

$$p_{n|\alpha} = |\langle n|\psi_\alpha\rangle|^2$$

$$p'_{m|\beta} = |\langle m|\psi'_\beta\rangle|^2$$

- We then find that $\boxed{\sigma[\tilde{\mathcal{X}}] = \sigma_{\text{classical}}[\tilde{\mathcal{X}}] + \xi[\tilde{\mathcal{X}}]}$

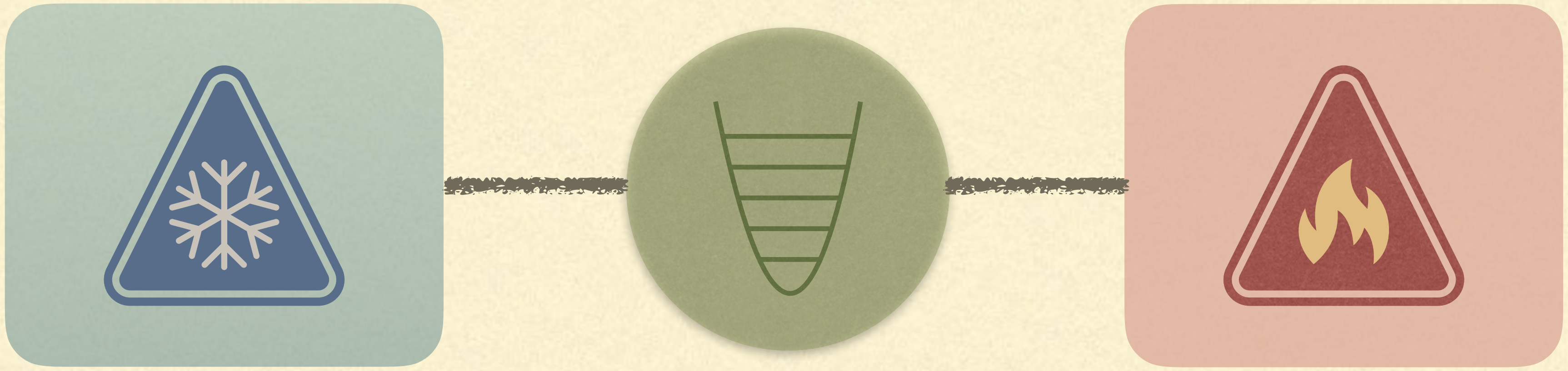
- where

$$\sigma_{\text{classical}}[\tilde{\mathcal{X}}] = -\ln\left(\frac{p'_m q_\nu^{\text{th}}}{p_n q_\mu^{\text{th}}}\right)$$

$$\boxed{\xi[\tilde{\mathcal{X}}] = -\ln\left(\frac{p_n}{p_\alpha}\right) - \ln\left(\frac{p'_m}{p'_\beta}\right)}$$

- The coherence contribution is precisely the *information gain:*
  - That is, the amount of information that the bases $|n\rangle$ and $|\psi_\alpha\rangle$ share with each other.
- This is therefore related to the fundamental incompatibility of different basis sets.

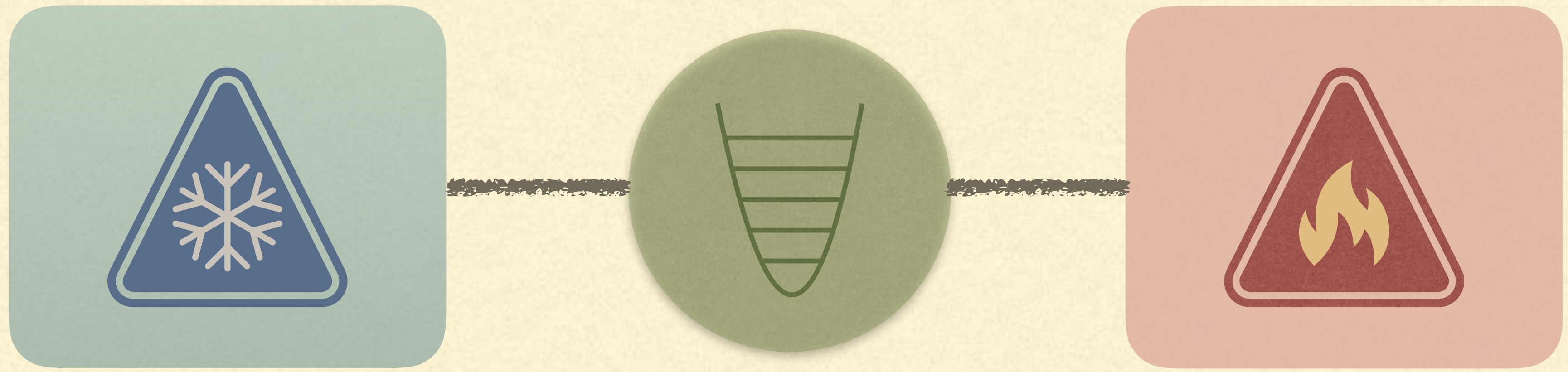
ENTROPY PRODUCTION IN QUANTUM NON-EQUILIBRIUM STEADY-STATES



Jader P. Santos, Gabriel T. Landi and Mauro Paternostro
The Wigner entropy production rate
PRL, **118**, 220601 (2017)

M. Brunelli, L. Fusco, R. Landig, W. Wieczorek, J. Hoelscher-Obermaier, G. T. Landi, F Semião, A. Ferraro, N. Kiesel, T. Donner, G. De Chiara, and M. Paternostro
Measurement of irreversible entropy production in mesoscopic quantum systems out of equilibrium.
arXiv 1602.06958 (submitted to *PRL*)

NESS



- We consider now the case of a system connected to multiple reservoirs.

$$\frac{dS}{dt} = \Pi - \sum_n \Phi_n, \qquad \Phi_n = -\frac{1}{T_n}\frac{dQ_n}{dt}$$

- This system will eventually reach a non-equilibrium steady-state, characterized by a current of heat from hot to cold.

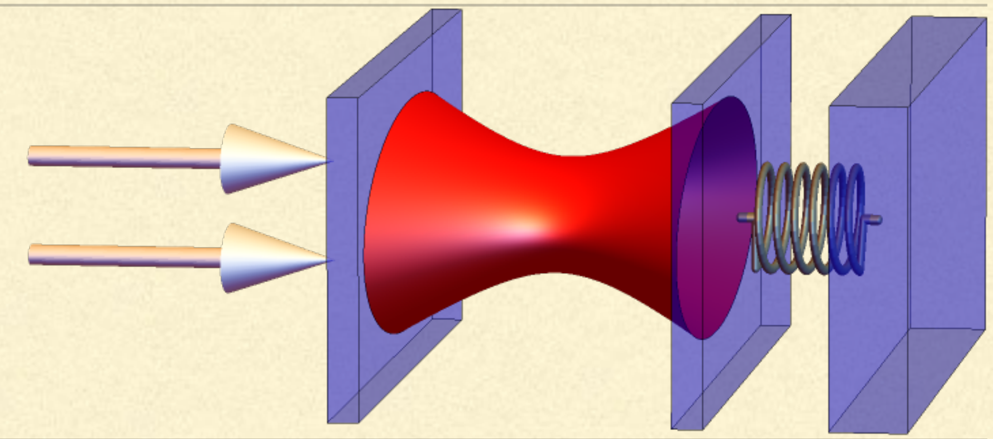
- In the NESS we get

$$\boxed{\Pi = \sum_n \Phi_n \geq 0}$$

- Meaning all entropy produced in the system flows towards the environments.

MODELS OF A QUANTUM NESS IN DRIVEN-DISSIPATIVE SYSTEMS

OPTOMECHANICS

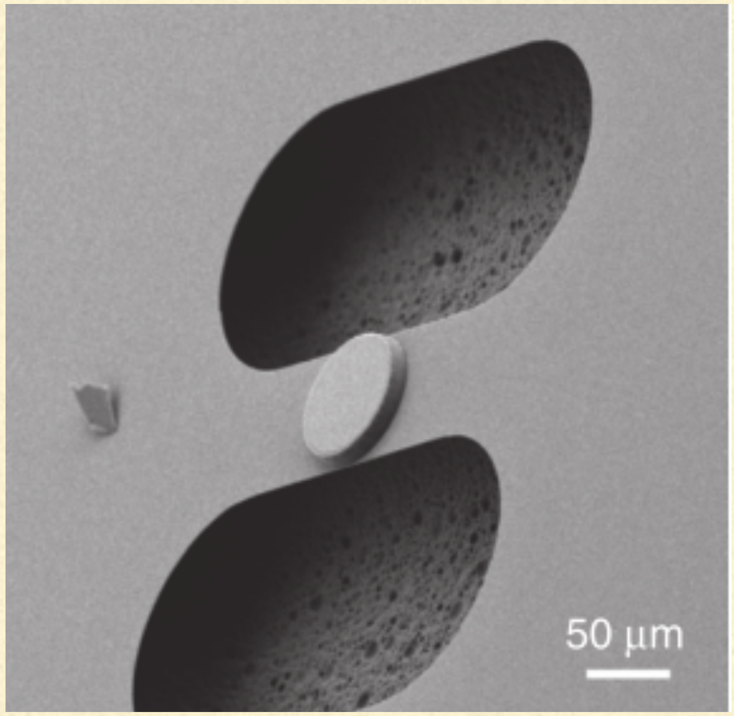


- A thin membrane is allowed to vibrate in contact with radiation trapped in a cavity.

$$H = \omega_c a^\dagger a + \left(\frac{p^2}{2m} + \frac{1}{2} m \omega_m^2 x^2 \right)$$

$$- g a^\dagger a x + \epsilon (a^\dagger e^{-i\omega_p t} + a e^{i\omega_p t})$$

$$\frac{d\rho}{dt} = -i[H, \rho] + D_c(\rho) + D_m(\rho)$$



Aspelmeyer group
Viena

Groeblacher, *et. al.*, Nature Communications, **6**, 7606 (2015)

$$\frac{d\rho}{dt} = -i[H,\rho] + D_c(\rho) + D_m(\rho)$$

- The system tends to a NESS because there are two dissipation channels.

- The mechanical oscillator has the usual damping:

$$D_m(\rho) = \gamma(n_m+1)\left[b\rho b^\dagger - \frac{1}{2}\{b^\dagger b,\rho\}\right] + \gamma n_m\left[b^\dagger\rho b - \frac{1}{2}\{bb^\dagger,\rho\}\right]$$

- where γ is the coupling rate to the environment and $n_m = \dfrac{1}{e^{\omega_m/T}-1}$

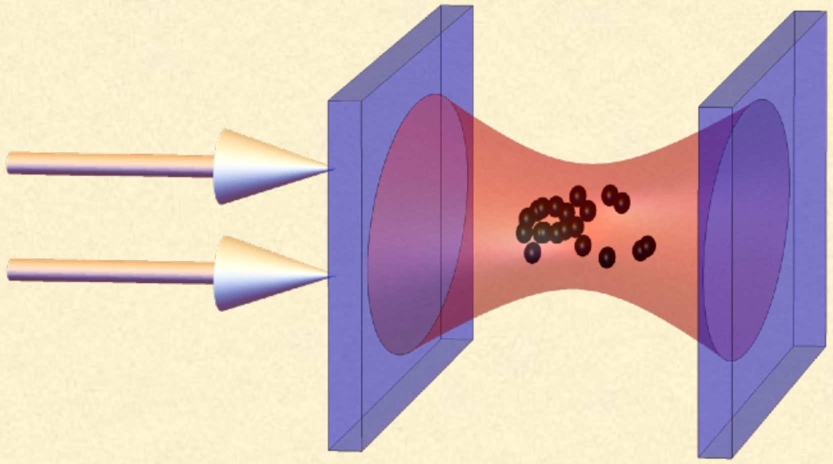
- On the other hand, the cavity can also loose photons (this is how they measure the cavity), which is described by

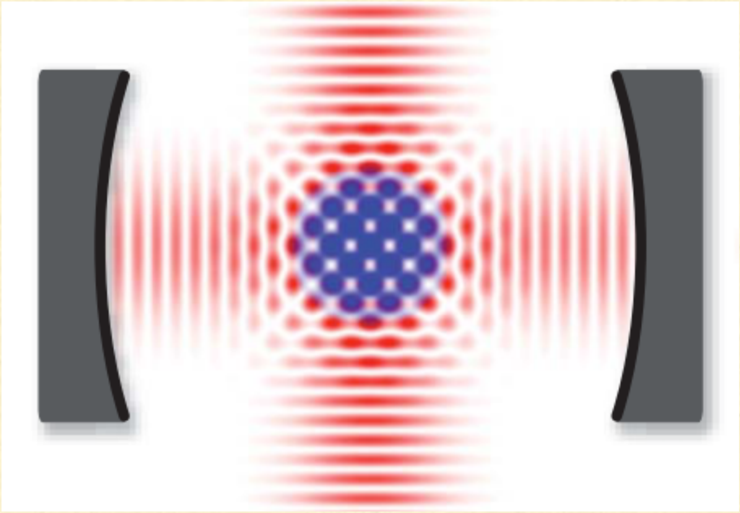
$$D_c(\rho) = 2\kappa\left[a\rho a^\dagger - \frac{1}{2}\{a^\dagger a,\rho\}\right]$$

DRIVEN-DISSIPATIVE BEC

Esslinger group
ETH

- Another interesting quantum NESS is that of a BEC interacting with a cavity field.





b_0 and b_1 are bosonic operators of the ground-state and first excited state of the BEC

$$H = \omega_c a^\dagger a + \frac{\omega_0}{2}(b_1^\dagger b_1 - b_0^\dagger b_0) + \frac{2\lambda}{\sqrt{N}}(a + a^\dagger)(b_0^\dagger b_1 + b_1^\dagger b_0)$$

Baumann, *et. al.*, Nature, **464**, 1301 (2010)

TROUBLE @T = 0

$$D_c(\rho) = 2\kappa\left[a\rho a^\dagger - \frac{1}{2}\{a^\dagger a, \rho\}\right]$$

- Both models clearly correspond tend to a quantum NESS.
- However, in both cases one of the reservoirs is the photon loss bath.
    - But this bath behaves exactly as a thermal bath at zero temperature.
- And the usual description of entropy production breaks down at T = 0.
    - Both production and flux diverge.

$$\Pi = -\frac{dS(\rho||\rho_{\mathrm{eq}})}{dt} \qquad \Phi = -\frac{1}{T}\frac{dQ}{dt}$$

- Is this divergence physical?
  - I don't think so.
- This divergence would be physical if we were talking about a thermal bath.
  - But photon loss is not a thermal bath.
    - It is an engineered bath.
- But here we shall not worry too much about this. Let's me pragmatic.
  - The process is clearly irreversible…
    - … and we want to quantify this irreversibility.

RÉNYI-2 AND WIGNER ENTROPY

- Recently there has been many discussions about using Rényi entropies as alternatives for constructing thermodynamics.
- We propose to use the Rényi-2 entropy. For Gaussian bosonic states, it actually coincides with the Wigner entropy:

$$S_2(\rho) = -\ln \mathrm{tr}\rho^2 = -\int W \ln W$$

W = Wigner function

Santos, GTL, Paternostro, *PRL*, **118,** 220601 (2017)

Adesso, Girolami, Serafini, *PRL*, **109**, 190502 (2012)

Brandão, Horodecki, Ng, Oppenheim, Wehner PNAS **112** 3275 (2015)

QUANTUM FOKKER-PLANCK EQUATION

- The master equation can be converted into a Quantum Fokker-Planck equation for the Wigner function.

- For instance: $$\frac{d\rho}{dt} = \gamma(n+1)\left[a\rho a^\dagger - \frac{1}{2}\{a^\dagger a, \rho\}\right] + \gamma n\left[a^\dagger \rho a - \frac{1}{2}\{a a^\dagger, \rho\}\right]$$

$$\frac{\partial W}{\partial t} = \partial_\alpha J(W) + \partial_{\alpha^*} J^*(W)$$

Probability current

$$J(W) = \frac{\gamma}{2}\left[\alpha W + (n+1/2)\partial_{\alpha^*} W\right]$$

- *$J(W_{eq}) = 0$* so we may "define" equilibrium as the state in which there are no currents.

WIGNER ENTROPY PRODUCTION

- Based on methods from classical stochastic processes, we show that the Wigner entropy production rate and the Wigner entropy flux rate are:

$$\Pi = \frac{4}{\gamma(n+1/2)} \int d^2\alpha \, \frac{|J(W)|^2}{W} = -\frac{dS(W||W_{\text{eq}})}{dt}$$

$$\Phi = \frac{\gamma}{n+1/2}\Big[\langle a^\dagger a\rangle - n\Big] = -\frac{1}{\omega(n+1/2)}\frac{dQ}{dt}$$

- At high temperatures $\omega(n+1/2) \simeq T$ which leads to $\Phi \simeq -\frac{1}{T}\frac{dQ}{dt}$

- But now both remain *finite* at *T = 0 (n = 0).*

Santos, GTL, Paternostro, *PRL*, **118,** 220601 (2017)

BACK TO THE NESS

- Now let's go back to the two models we discussed before.
- Both models can be Gaussianized for large drive and converted into an effective system of two harmonic oscillators

$$H = \omega_a a^\dagger a + \omega_b b^\dagger b + g(a + a^\dagger)(b + b^\dagger)$$

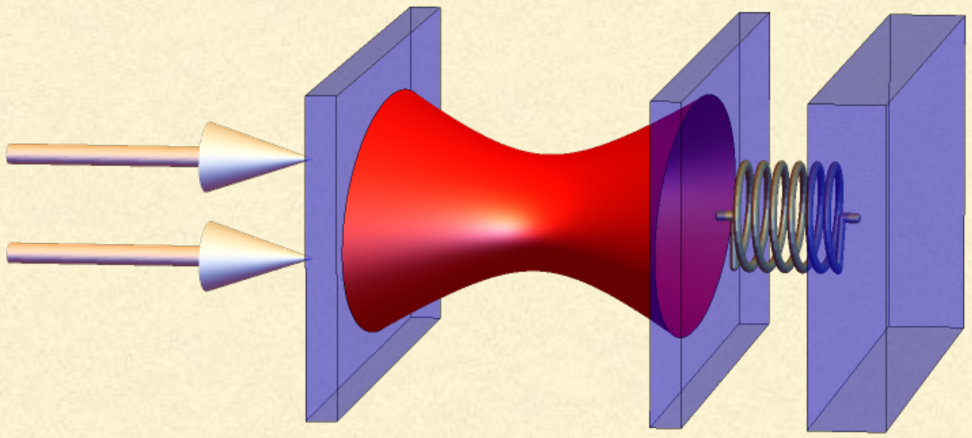
- The Wigner entropy production then becomes

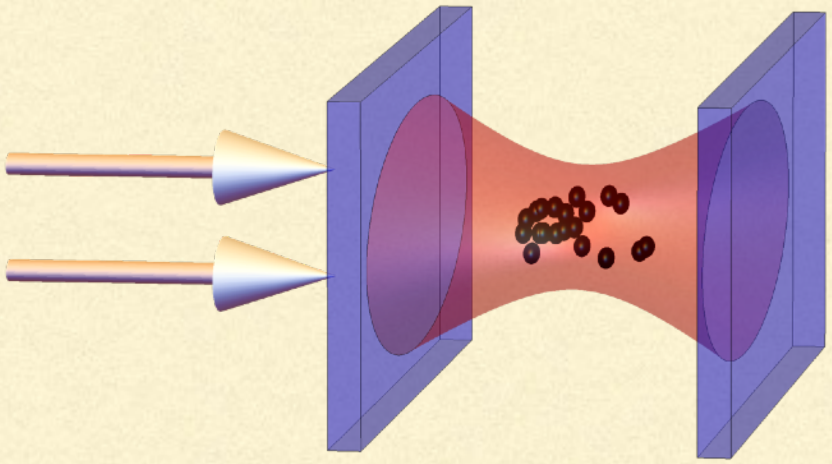
$$\Pi = 2\kappa \langle a^\dagger a \rangle + \frac{\gamma}{n + 1/2}(\langle b^\dagger b \rangle - n)$$

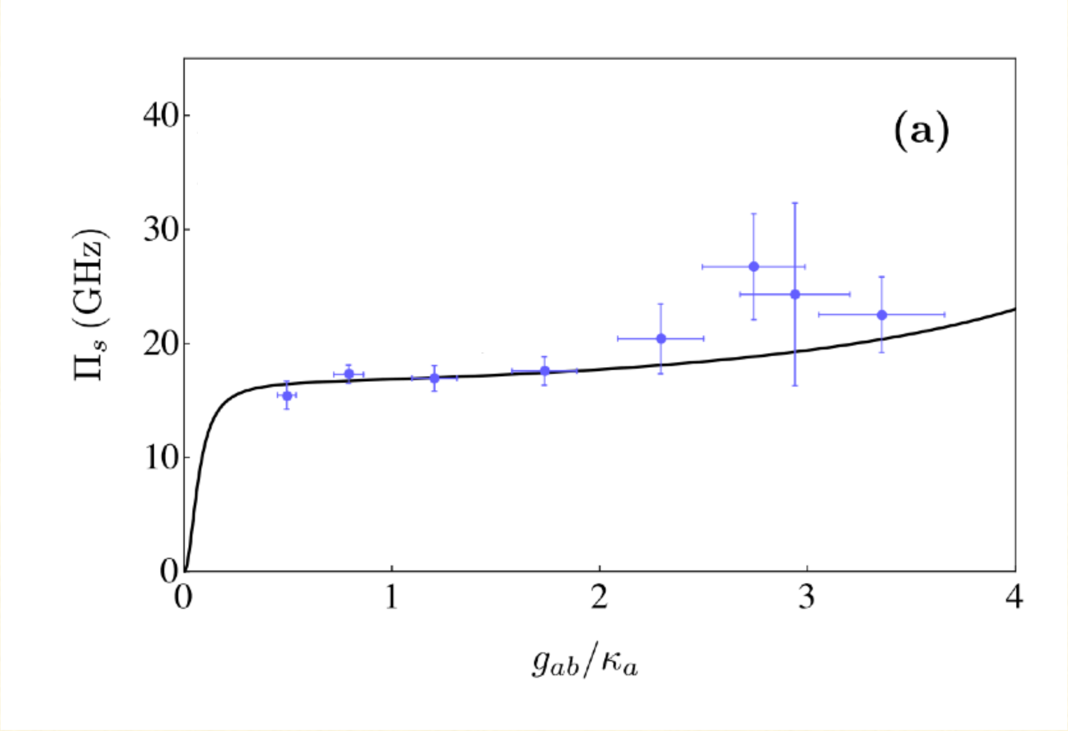
- It depends only on easily accessible quantities, in both experimental setups.

arXiv 1602.06958

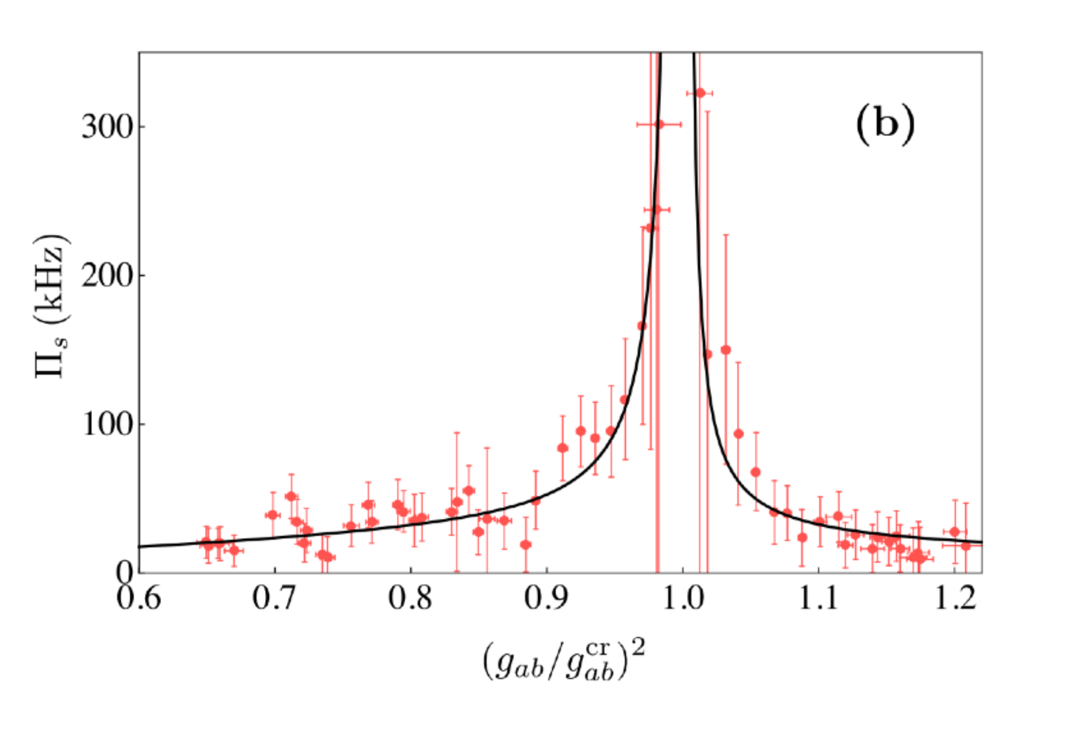
RESULTS







optomechanics



BEC

Before I finish…

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