A magnonic logic gate in the: open Heisenberg chain

Gabriel T. Landi

Universidade Federal do ABC

In collaboration with Dragi Karevski from Université de Lorraine

Magnonic devices

nature COMMUNICATIONS

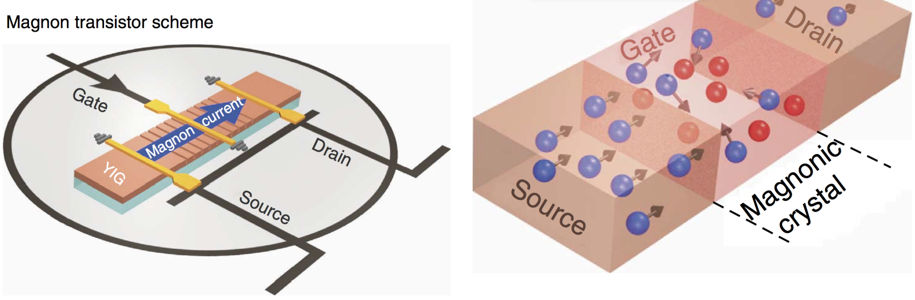
ARTICLE

Received 16 May 2013 | Accepted 15 Jul 2014 | Published 21 Aug 2014 DOI: 10.1038/ncomms5700 OPEN

Magnon transistor for all-magnon data processing

Andrii V. Chumak[1], Alexander A. Serga[1] & Burkard Hillebrands[1]

a



Open quantum systems

- The 1D Heisenberg chain is described by the Hamiltonian

$$H = \frac{1}{2} \sum_{i=1}^{N-1} \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_{i+1}$$

- Our goal is to describe this quantum system in contact with an external environment.
  - Describe the injection and absorption of excitations (magnons).

$$\frac{d\rho}{dt} = -i[H, \rho] + D_L(\rho) + D_R(\rho)$$

- We choose $D_L(\rho)$ to be a perfect injector of magnons, or a *magnon pump*.

$$D_L(\rho) = \gamma \left(2 \sigma_1^+ \rho \sigma_1^- - \{\sigma_1^- \sigma_1^+, \rho\}\right)$$

  - It injects γ magnons/second at site #1.
- Similarly, $D_R(\rho)$ is a perfect *magnon drain*.

$$D_R(\rho) = \gamma(2 \sigma_N^- \rho \sigma_N^+ - \{\sigma_N^+ \sigma_N^-, \rho\})$$

- This is a completely quantum-mechanical problem.
  - It may therefore present novel effects not observed in semi-classical calculations.
- Goal: to compute the *spin current* J.

Exact solution for the *steady-state*

- These types of many-body problems are usually very difficult to solve.
  - Analytically: maybe 3 or 4 spins
  - Numerically (without DMRG): maybe 10 spins.
  - Numerically with DMRG: maybe 100. Very difficult.
- This case is a nice exception.

PRL **110,** 047201 (2013) PHYSICAL REVIEW LETTERS week ending 25 JANUARY 2013

Exact Matrix Product Solution for the Boundary-Driven Lindblad *XXZ* Chain

D. Karevski,[1] V. Popkov,[2,3] and G. M. Schütz[4]
[1]*Institut Jean Lamour, Department P2M, Groupe de Physique Statistique, Université de Lorraine, CNRS, B.P. 70239, F-54506 Vandoeuvre les Nancy Cedex, France*
[2]*Dipartimento di Fisica, Università di Firenze, via Sansone 1, 50019 Sesto Fiorentino Firenze, Italy*
[3]*Max Planck Institute for Complex Systems, Nöthnitzer Straße 38, 01187 Dresden, Germany*
[4]*Institute of Complex Systems II, Forschungszentrum Jülich, 52428 Jülich, Germany*
(Received 29 November 2012; published 24 January 2013)

- An exact solution was found for *any chain size* in terms of matrix product states.
- With this solution J may be written as a product of matrices.
  - It may therefore be computed numerically for any chain size.

PHYSICAL REVIEW B **91**, 174422 (2015)

Open Heisenberg chain under boundary fields: A magnonic logic gate

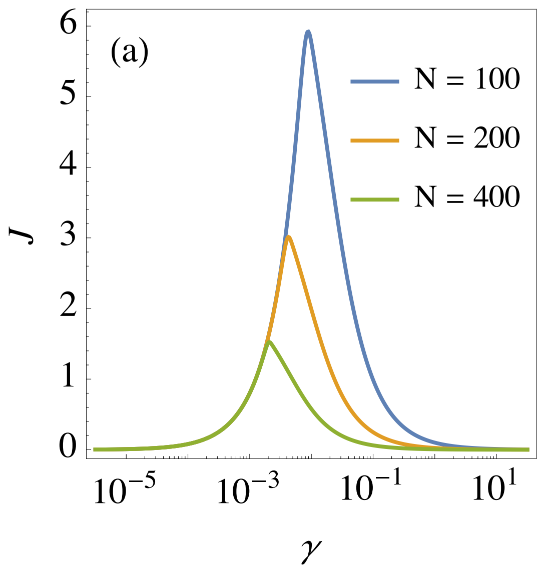
Gabriel T. Landi*
Departamento de Ciências Naturais e Humanas, Universidade Federal do ABC, 09210-580 Santo André, Brazil

Dragi Karevski
Institut Jean Lamour, Department P2M, Groupe de Physique Statistique, Université de Lorraine, CNRS, Boîte Postale 70239, F-54506 Vandoeuvre les Nancy Cedex, France
(Received 29 January 2015; revised manuscript received 23 April 2015; published 20 May 2015)

- In this talk I want to focus on the *physics*.

Ballistic vs. sub-diffusive

- Spin current J vs. the pumping rate γ for different chain sizes.

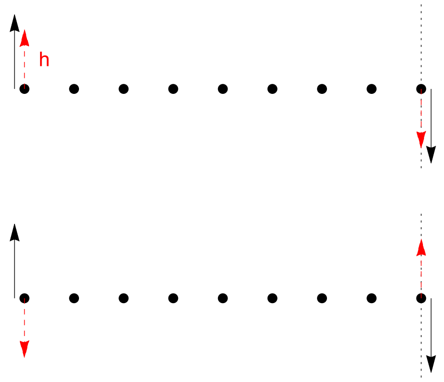


- Low γ → low magnon density → ballistic spin flux
  - Magnons propagate freely (they do not collide).
- High γ → sub-diffusive spin flux
  - Magnon scattering events hinder the flux.
- Transition occurs at $\gamma^* = 1/N$

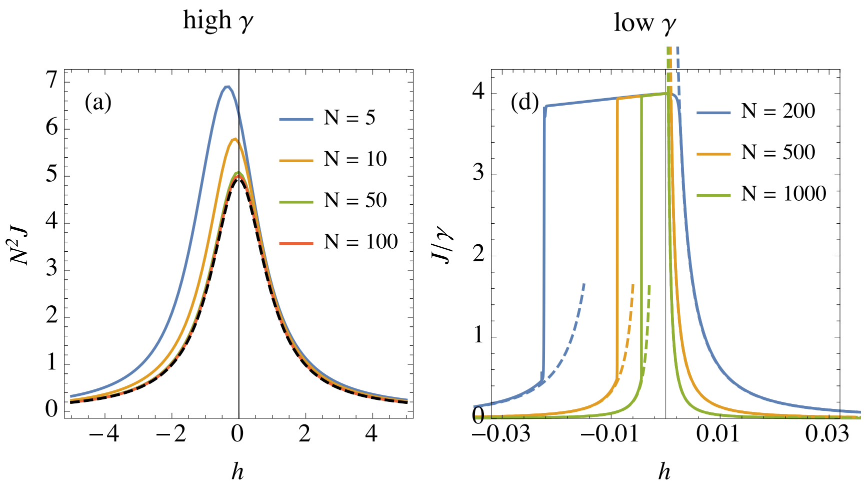
Boundary fields

- It was also possible to obtain a solution when the spins at the boundaries are subject to magnetic fields at opposite directions.

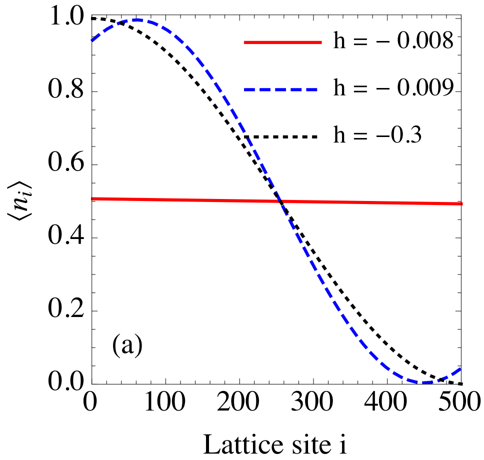
$$H = \frac{1}{2} \sum_{i=1}^{N-1} \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_{i+1} + h(\sigma_1^z - \sigma_N^z)$$



- In this case we obtain a quite interesting result:

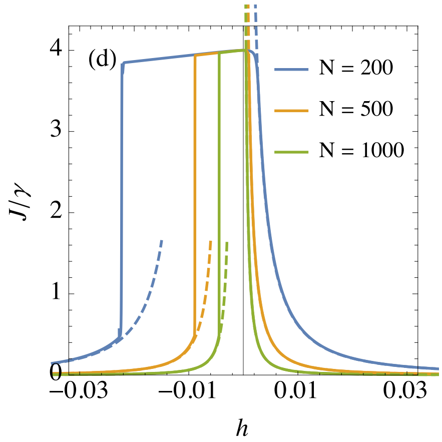


- At low γ, as we change the boundary fields, we observe an *abrupt* transition:
  - Ballistic inside the *plateau*.
  - Sub-diffusive outside.
- This can also be seen in the density of magnons along the chain:
  - Inside the *plateau* → flat density → no accumulation of magnons.
  - Outside → accumulation of magnons → strong magnon-magnon interaction.


1.0
0.8
0.6
0.4
0.2
0.0
$\langle n_i \rangle$
h = − 0.008
h = − 0.009
h = −0.3
(a)
0
100
200
300
400
500
Lattice site i

Physical explanation

- The boundary fields act as scattering barriers which *confine* the magnons inside the chain.
- Low $\gamma \rightarrow$ low magnon density.
  - If h is low, the magnons propagate freely → ballistic flux.
  - If h is large, it confine the magnons → more scattering → sub-diffusive flux.
- The situation where we found an exact solution is peculiar, but the physical principle is quite general:
  - Use non-uniform magnetic fields to confine the magnons.
  - Tuning the field amplitude, you can tune the spin current.



- By tuning the field around the transition, you can get huge variations in the spin current.
  - This is a very efficient magnonic logic gate.
  - And this is a genuinely quantum mechanical effect.

Conclusions

- Open quantum systems may be used to describe magnonic circuits.
- The regime of the spin current depends on the density of magnons in the system.
- Magnetic fields can be used to confine magnons → induces scattering effects.
- The main results of this presentation are contained in

  G. T. Landi and D. Karevski, *Phys. Rev. B.* **91** 174422 (2015)
- For more details see:

  D. Karevski, V. Popkov, G. M. Schütz, *Phys. Rev. Lett.* **110** 047201 (2013)
  V. Popkov, D. Karevski, G. M. Schütz, *Phys. Rev. E.* **88** 062118 (2013)
  G. T. Landi, *et. al., Phys. Rev. E.* **90** 042142 (2014)

Thank you very much.

Matrix product solution

Quick answer

- The spin flux reads

$$J = \frac{2\gamma}{\gamma^2 + h^2} \frac{Z(N-1)}{Z(N)}$$

- Where Z(N) is the (0,0) element of a matrix B raised to the power N

$$Z(N) = \left(B^N\right)_{00}$$

$$B_{i,j} = 2\,|\,p - i\,|^2\,\delta_{i,j} + j^2\,\delta_{i,j-1} + |\,2\,p - j\,|^2\,\delta_{i,j+1}$$

$$p = \frac{i}{2\,(\gamma - i\,h)}$$

- Thus, to find J the procedure is:

1. Construct this N×N matrix B
2. Multiply it by itself N times (there are quick ways to do this)
3. Take the (0,0) entry.

Detailed solution

- Our goal is to find the solution of

$$i[H, \rho] = D_L(\rho) + D_R(\rho)$$

- First we decompose

$$\rho = \frac{S^\dagger S}{\mathrm{tr}(S^\dagger S)}$$

- We then write

$$S = \left\langle \phi \,\middle|\, \Omega^{\otimes N} \,\middle|\, \psi \right\rangle$$

- where Ω is an operator valued 2×2 matrix

$$\Omega = S_z\,\sigma_z + S_+\,\sigma_+ + S_-\,\sigma_-$$

- The operators S_a act on an auxiliary space.
  - By taking the inner product with $\langle\phi|$ and $|\psi\rangle$ we then recover S in the Hilbert space of the N spins.
- From the bulk structure of the Hamiltonian we find that the S_a must obey the SU(2) algebra
  - In the XXZ model this generalizes to the quantum U_q[SU(2)] algebra

$$[S_z, S_\pm] = \pm\, S_\pm$$

$$[S_+, S_-] = 2\, S_z$$

- We then choose a irreducible representation of this algebra as

$$S_z = \sum_{n=0}^{\infty} (p - n)\, |n\rangle\langle n|$$

- The boundary structure then fixes

$$p = \frac{i}{2\,(\gamma - i\,h)}$$

$$|\,\phi\rangle = |\,\psi\rangle = |\,0\rangle$$

- Which completes the formal solution.

Auxiliary functions

```
SetDirectory[NotebookDirectory[]];
<< "LinLib`";
<< "CustomTicks`";
load[filename_, size_] := Show[Import[filename], ImageSize → Scaled[size]];

SetOptions[Plot, Frame → True, Axes → False,
  BaseStyle → 20, ImageSize → 400, PlotStyle → {Black}];

SetOptions[InputNotebook[],
 DefaultNewCellStyle → "Item",
 ShowCellLabel → "False",
 CellGrouping → Manual,
 FontFamily → "Times",
 DefaultNewCellStyle → {"Text", FontFamily → "Times"},
 BaseStyle → {FontFamily → "Times"},
 MultiLetterItalics → False,
 SingleLetterItalics → Automatic
]
```