Seminário para a Universidade Federal de Minas Gerais

Quantum thermodynamics and irreversibility

Gabriel Teixeira Landi
FMT - IFUSP

IFUSP
Instituto de Física da USP

Summary

- Quantum Thermodynamics?
  - Motivation from Quantum Information Sciences.
- Recent progress and general trends in the field.
- Our contribution: quantifying irreversibility at the quantum level.

We live in the age of quantum technologies

- Since its conception, quantum mechanics has already provided us with remarkable technologies:
  - Lasers.
  - **Semiconductors**: solar panels, LEDs, computers, smartphones.
  - Nuclear magnetic resonance, electron microscopy, etc.
- These are now called Quantum Technologies 1.0 (UK Defence Science and Technology Laboratory)

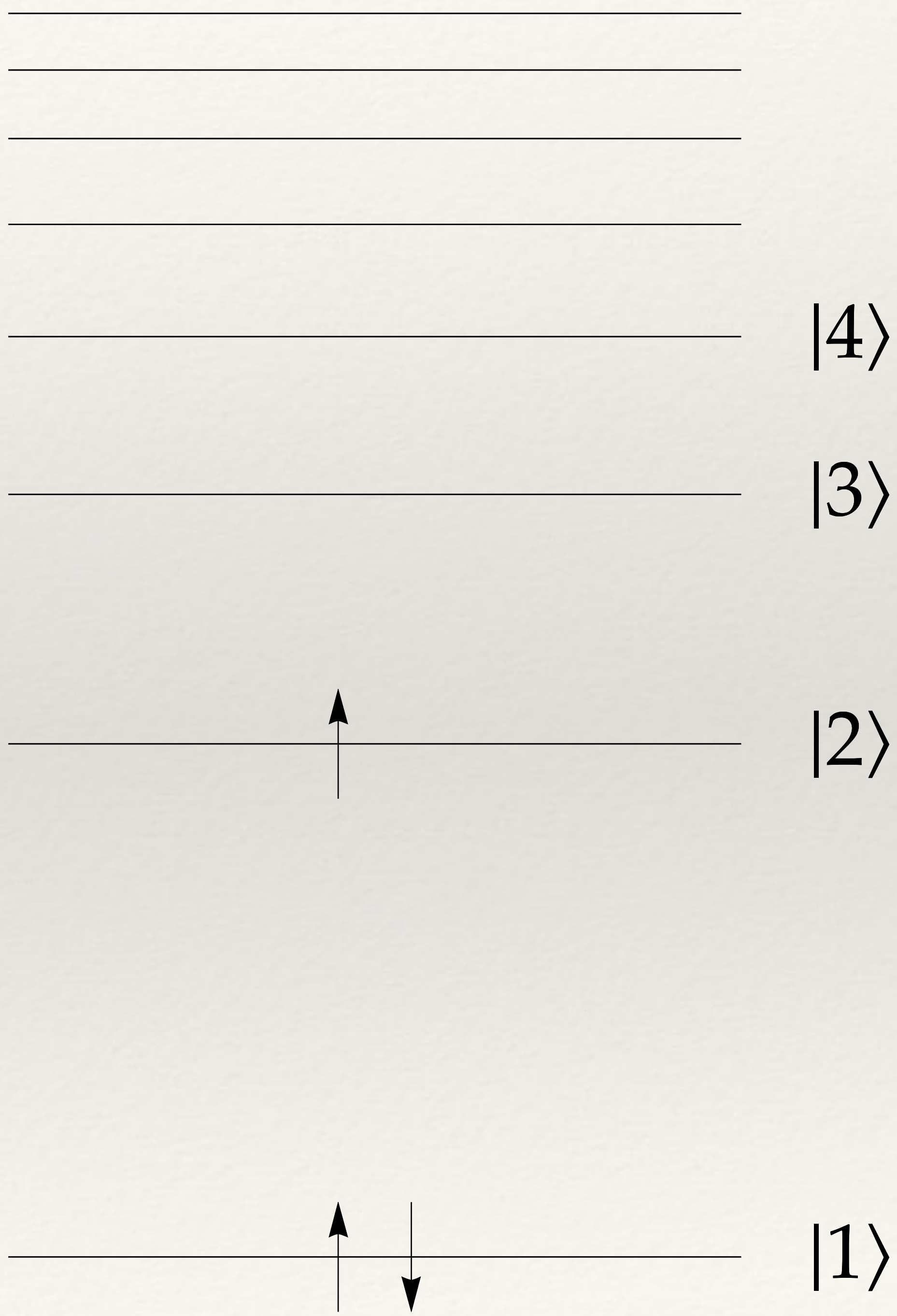
But quantum mechanics also predicts other properties, such as *coherence* and *entanglement*, which are not usually employed in these applications.

Coherence

- In QM we learn that a superposition of states is also a valid state:

$$|\psi\rangle = a|1\rangle + b|2\rangle$$

- But when we construct the periodic table, we don't care about this: we just "put" the electrons in each state.
- That's not very quantum:
  - Its quantum because the energy levels are discrete.
  - But other than that, its classical.



Interaction with the environment

- Consider a 2-level system in an arbitrary state.
- For a pure state we have

$$|\psi\rangle = a|0\rangle + b|1\rangle \qquad \Longrightarrow \qquad \rho = |\psi\rangle\langle\psi| = \begin{pmatrix} |a|^2 & ab^* \\ a^*b & |b|^2 \end{pmatrix}$$

- More generally, we have

$$\rho = \begin{pmatrix} p_0 & q \\ q^* & p_1 \end{pmatrix}$$

- The interaction with the environment does two things:

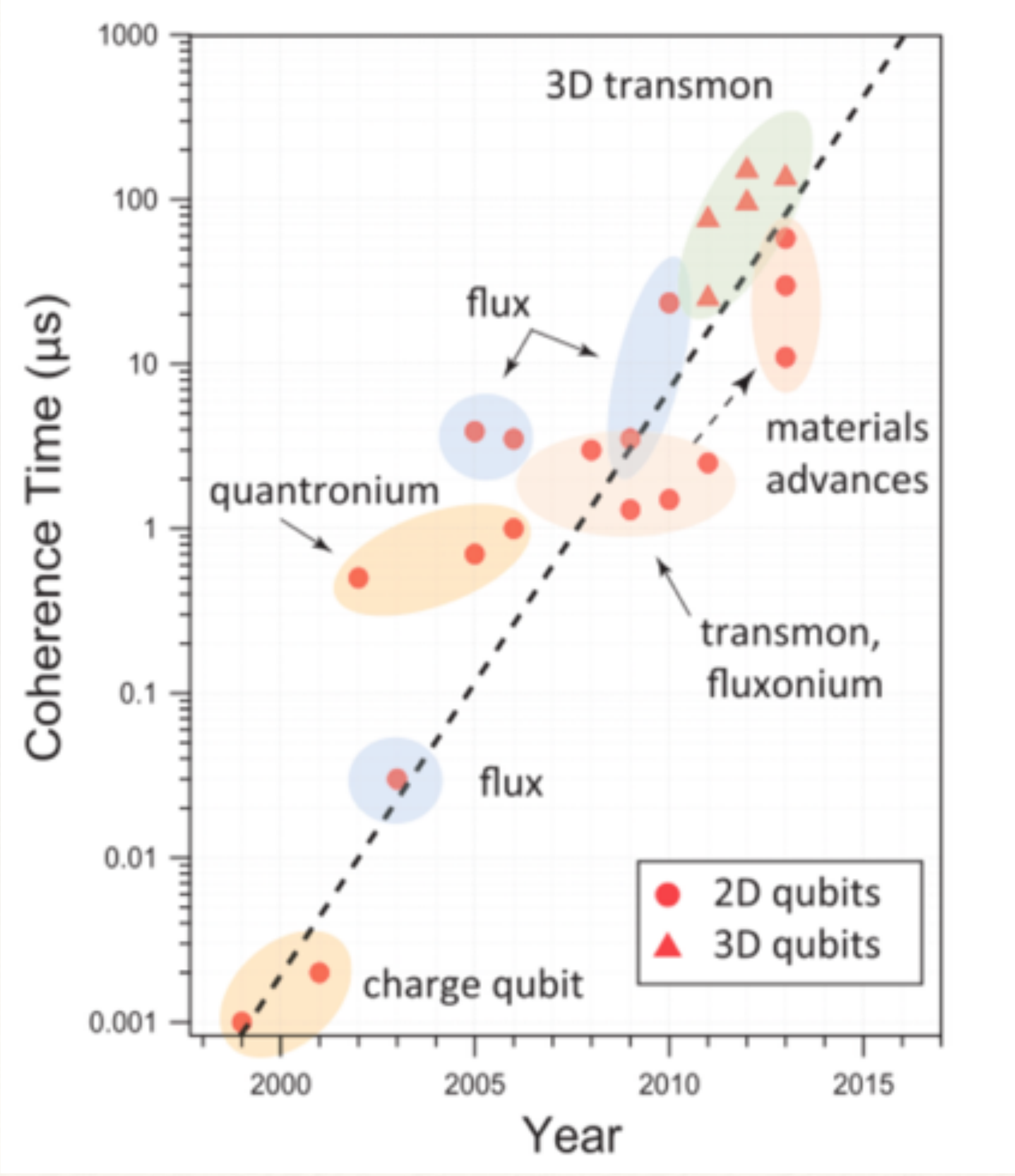
1. Changes the populations in a certain basis.
2. Destroys coherences.

$$\begin{pmatrix} p_0 & q \\ q^* & p_1 \end{pmatrix} \to \begin{pmatrix} p_0' & 0 \\ 0 & p_1' \end{pmatrix}$$

Isolate, experiment, understand…

- And so began a long quest to isolate, experiment and understand with these more exotic *quantum resources*:
  - Coherence, entanglement, squeezing, asymmetry, purity, discord, &c.
- We now have are many platforms where we can have impressive control over individual quantum systems:
  - Quantum optics, trapped ions, superconducting qubits, NMR, NV centers in diamond, Bose-Einstein condensates, ultra-cold atoms in optical lattices, &c.

Superconducting qubits



Quantum Technologies 2.0

- Together with these experimental advances, it also became clear that we could harness these quantum resources to produce new technologies:
  - Secure communications with *quantum cryptography*.
  - Exponentially faster algorithms with *quantum computers*.
  - Higher sensitivity with *quantum metrology*.
- Will any of these ever see the light of day?
  - Based on the history of physics, we will *definitely* see some applications.
- But even if no direct applications appear:
  - What we learned so far in this field is already helping in many other areas, such as e.g. *strongly correlated systems* (in this context correlation = entanglement).

Quantum Thermodynamics

- It is now straightforward to define what is the goal of "Quantum Thermodynamics":
  - *To understand the role of quantum resources in thermodynamic quantities such as heat and work.*
- Topics of current interest include:
  - The role of measurements in thermodynamic processes.
  - Thermal transformations under the presence of quantum fluctuations.
  - How coherence, entanglement and squeezing affect the operation of heat engines.
  - *Irreversibility at the quantum level.*

Review of the recent literature

Quantum measurement

PHYSICAL REVIEW E **75**, 050102(R) (2007)

Fluctuation theorems: Work is not an observable

Peter Talkner, Eric Lutz, and Peter Hänggi

PRL **118**, 070601 (2017) PHYSICAL REVIEW LETTERS week ending 17 FEBRUARY 2017

No-Go Theorem for the Characterization of Work Fluctuations in Coherent Quantum Systems

Martí Perarnau-Llobet,[1,*] Elisa Bäumer,[1,2,†] Karen V. Hovhannisyan,[1,‡] Marcus Huber,[3,4,§] and Antonio Acin[1,5,¶]

The role of quantum measurement in stochastic thermodynamics

Cyril Elouard[1], David A. Herrera-Martí[1], Maxime Clusel[2] and Alexia Auffèves[1]

npj Quantum Information (2017)3:9

Thermal operations

Published 27 Jun 2014

Work extraction and thermodynamics for individual quantum systems

Paul Skrzypczyk[1], Anthony J. Short[2] & Sandu Popescu[2]

NATURE COMMUNICATIONS | 5:4185 | DOI: 10.1038/ncomms5185 |

Published 26 Jun 2013

Fundamental limitations for quantum and nanoscale thermodynamics

Michal Horodecki[1,*] & Jonathan Oppenheim[2,3,*]

NATURE COMMUNICATIONS | 4:2059 | DOI: 10.1038/ncomms3059 |

The second laws of quantum thermodynamics

Fernando Brandão[a,1], Michał Horodecki[b], Nelly Ng[c], Jonathan Oppenheim[c,d,2], and Stephanie Wehner[c,e]

PNAS | **March 17, 2015** | vol. 112 | no. 11 | **3275–3279**

The second laws of quantum thermodynamics

Fernando Brandão[a,1], Michał Horodecki[b], Nelly Ng[c], Jonathan Oppenheim[c,d,2], and Stephanie Wehner[c,e]

PNAS | March 17, 2015 | vol. 112 | no. 11 | 3275–3279

Rényi entropy $$S_\alpha = -\frac{1}{1-\alpha} \log \mathrm{tr} \rho^\alpha$$

von Neumann entropy $$S_1 = -\mathrm{tr}(\rho \ln \rho)$$

$$F_\alpha(\rho, \rho_\beta) := kTD_\alpha(\rho \| \rho_\beta) - kT \log Z,$$

with the Rényi divergences $D_\alpha(\rho\|\rho_\beta)$ defined as

$$D_\alpha(\rho\|\rho_\beta) = \frac{\mathrm{sgn}(\alpha)}{\alpha - 1} \log \sum_i p_i^\alpha q_i^{1-\alpha},$$

A transition is allowed when: $$F_\alpha(\rho, \rho_\beta) \geq F_\alpha(\rho', \rho_\beta)$$

Generalizes the second law. For macroscopic systems all Rényi entropies converge to von Neumann.

More general heat engines

APS physics | Physics

Viewpoint: Squeezed Environment Boosts Engine Performance

James Millen, Vienna Center for Quantum Science and Technology, University of Vienna, 1090 Vienna, Austria

September 13, 2017 • *Physics* 10, 99

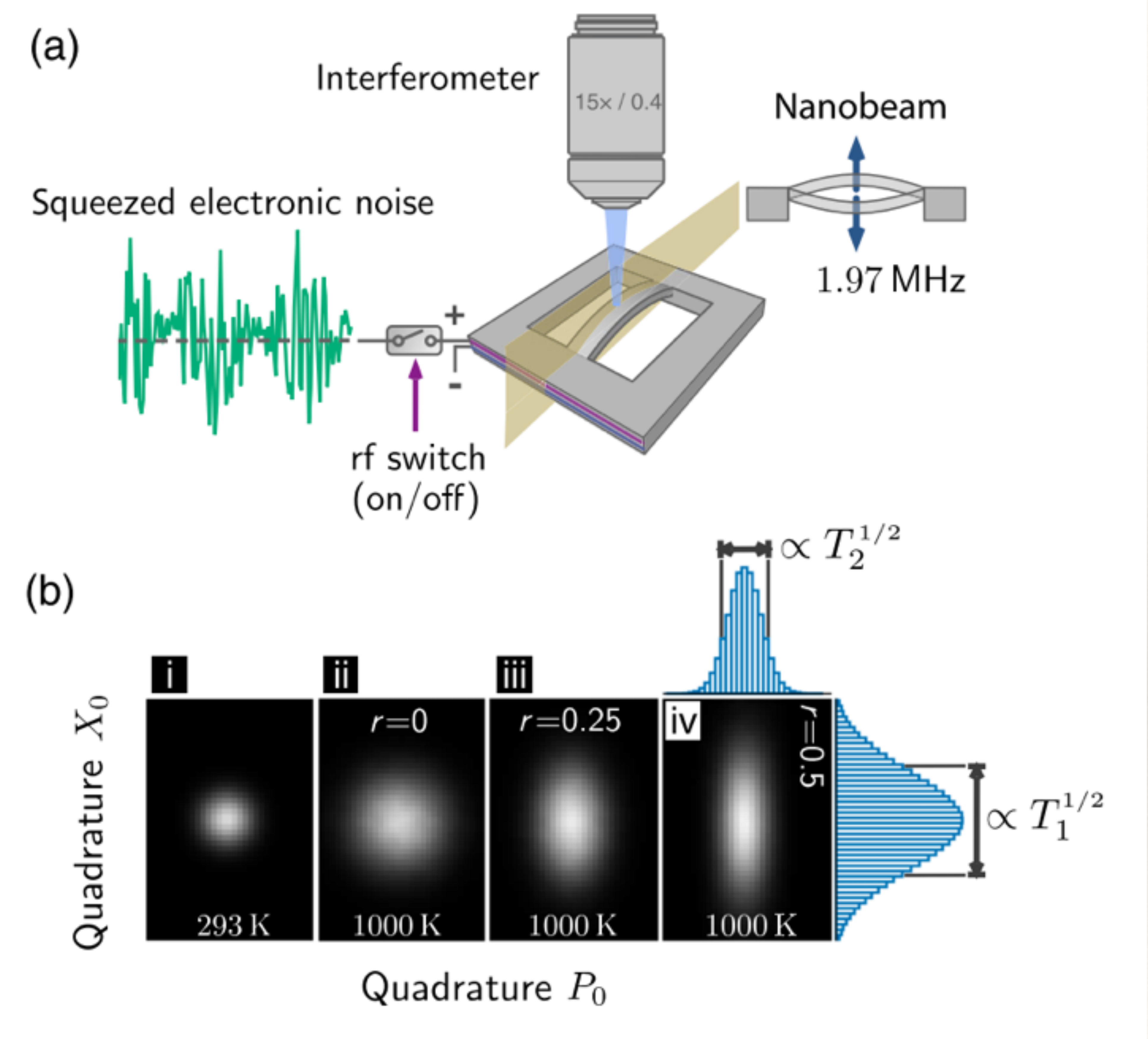
Selected for a Viewpoint in *Physics*
PHYSICAL REVIEW X **7**, 031044 (2017)

Squeezed Thermal Reservoirs as a Resource for a Nanomechanical Engine beyond the Carnot Limit

Jan Klaers,* Stefan Faelt, Atac Imamoglu, and Emre Togan

Institute for Quantum Electronics, ETH Zürich, CH-8093 Zürich, Switzerland

(Received 25 April 2017; revised manuscript received 25 July 2017; published 13 September 2017)

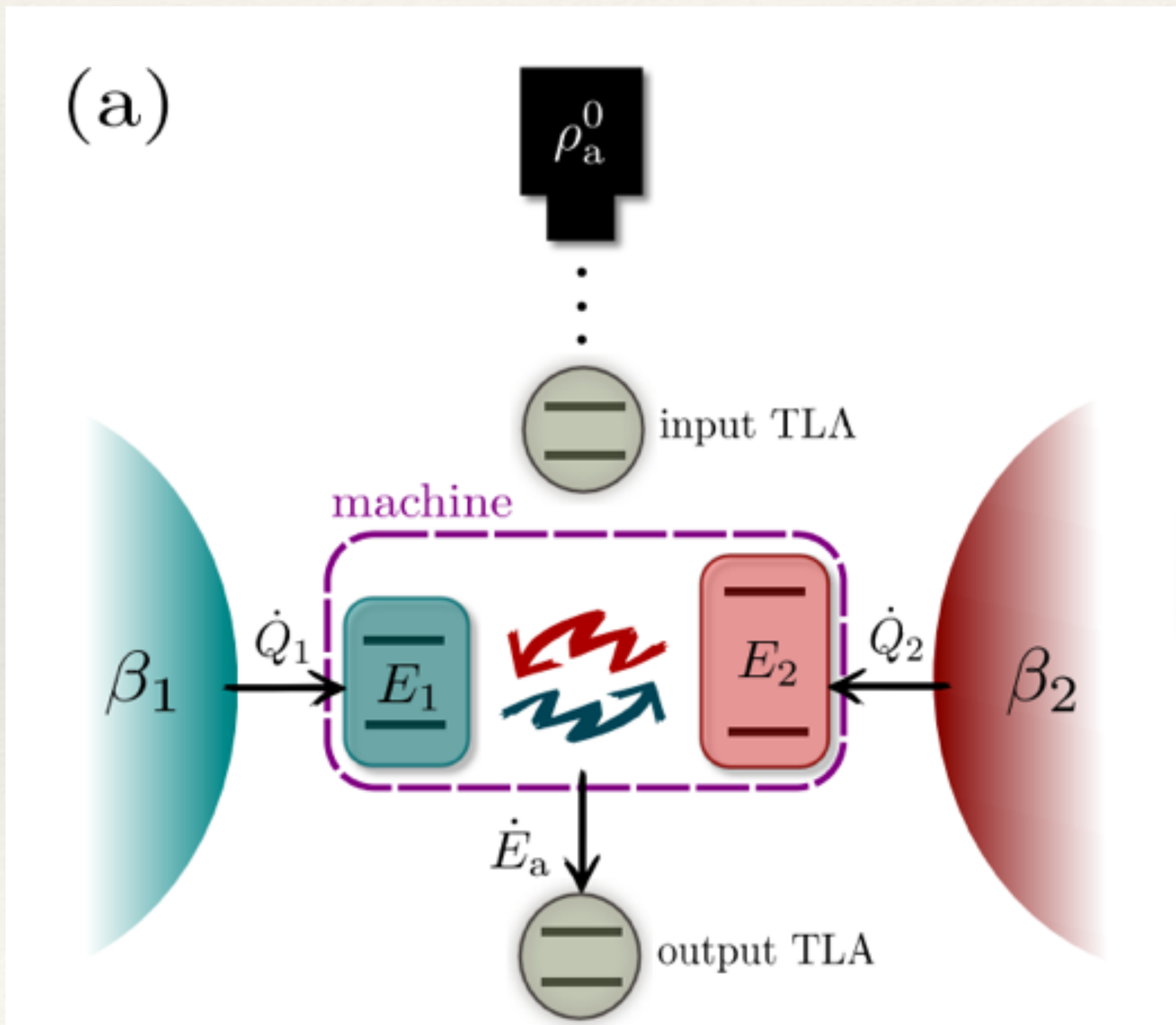


(a)
Interferometer
15× / 0.4
Nanobeam
Squeezed electronic noise
1.97 MHz
+
-
rf switch
(on/off)
(b)
$\propto T_2^{1/2}$
i
ii
iii
iv
r=0
r=0.25
r=0.5
$\propto T_1^{1/2}$
Quadrature X_0
293 K
1000 K
1000 K
1000 K
Quadrature P_0

Coherence producing engine

Autonomous thermal machine for amplification and control of energetic coherence

Gonzalo Manzano,[1,2] Ralph Silva,[3] and Juan M.R. Parrondo[1]



Measures of Irreversibility

Entropy production

- The energy of a system satisfies a continuity equation:

$$\frac{d\langle H\rangle}{dt} = -\Phi_E$$

- For the entropy that is not true:

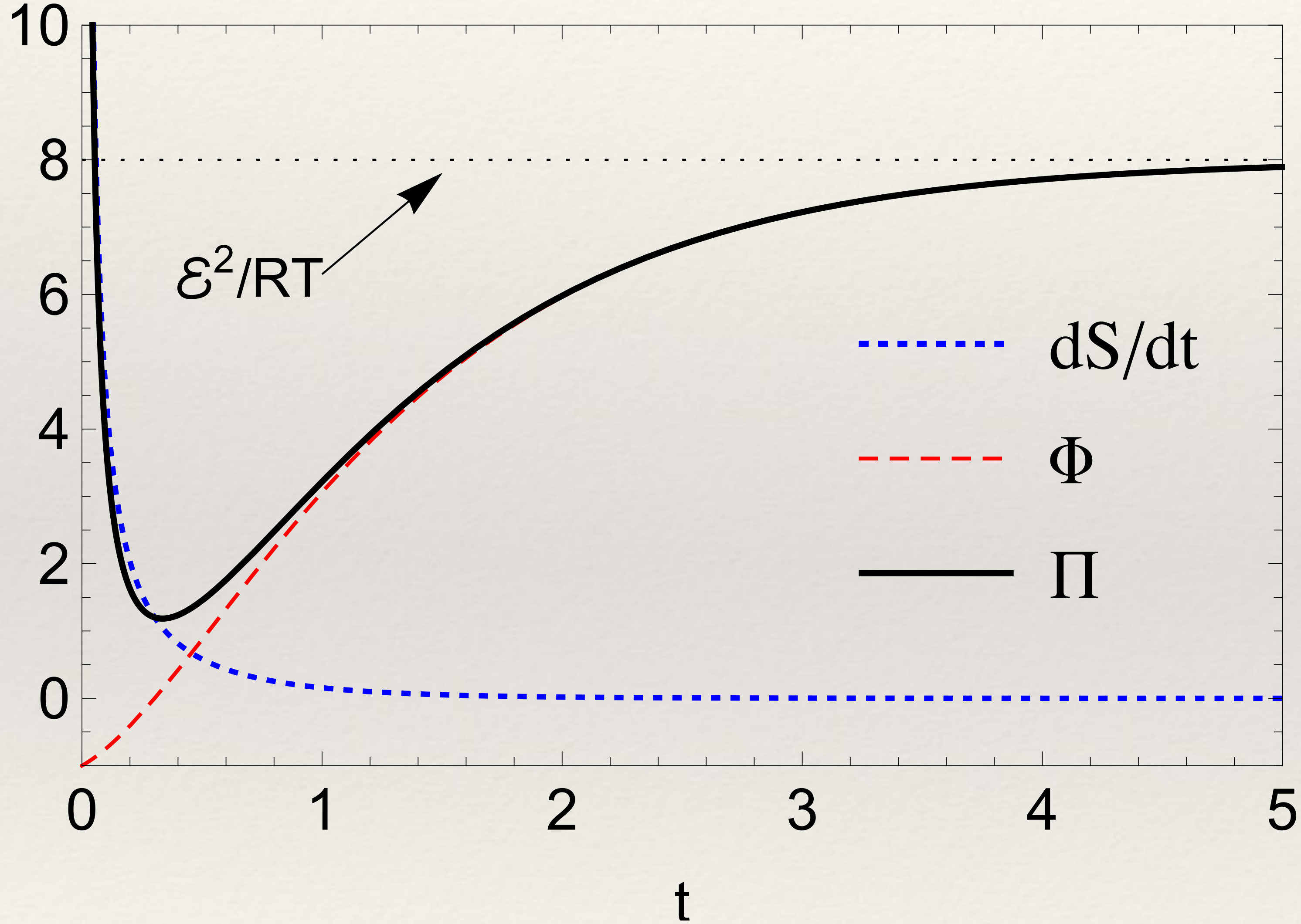
$$\frac{dS}{dt} = \Pi - \Phi$$

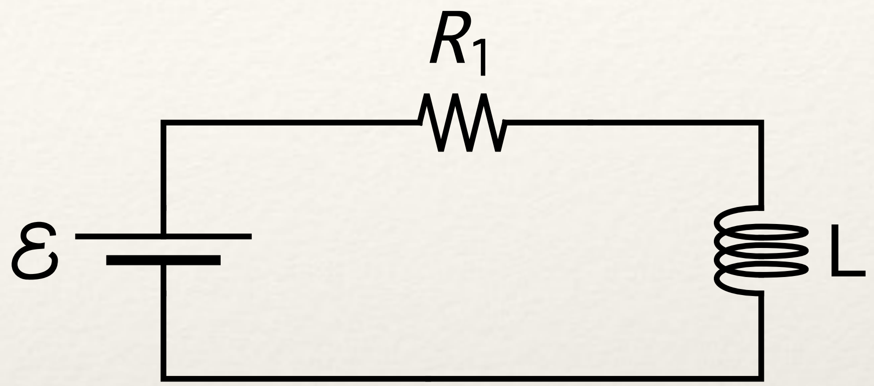
- Π represents the entropy production rate due to the irreversible dynamics:

$$\Pi \geq 0 \quad \text{and} \quad \Pi = 0 \;\; \text{only in equilibrium}$$

Dinâmica estocástica e irreversibilidade, T. Tomé e M. J. de Oliveira

Example: RL circuit



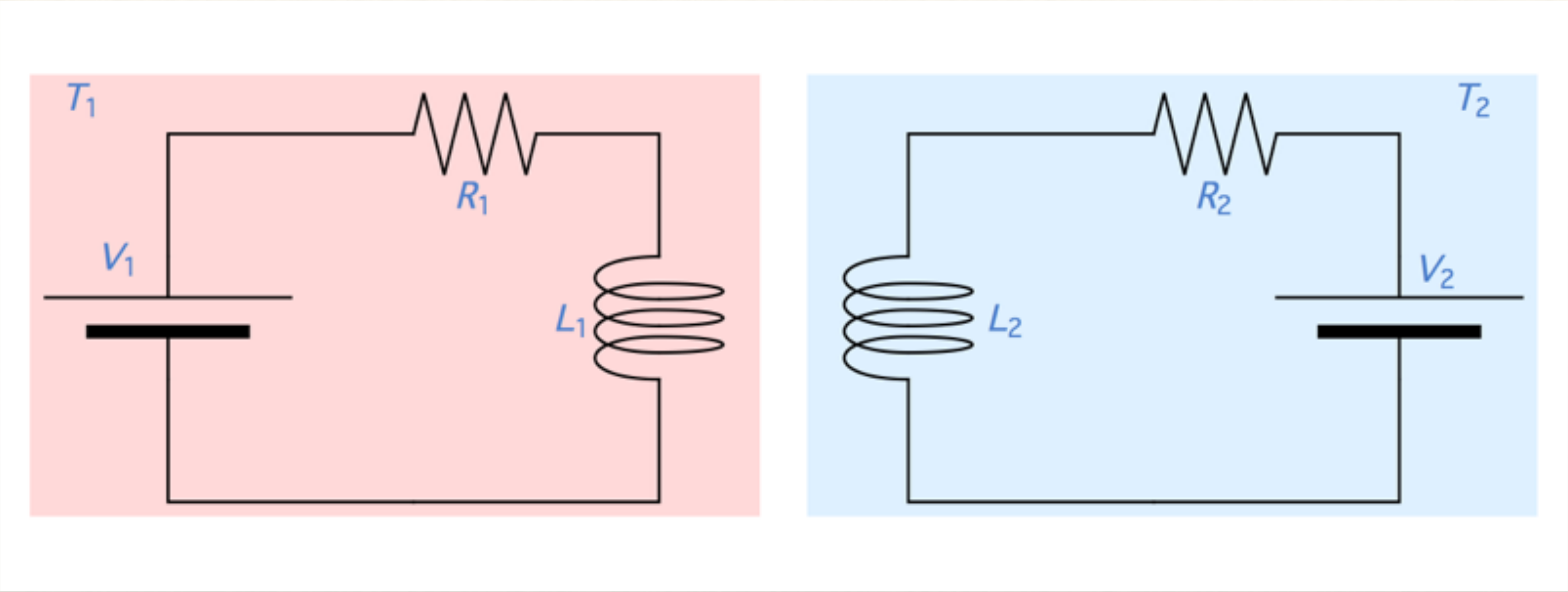


Steady-state

$$\frac{dS}{dt} = 0$$

$$\Pi_{\rm ss} = \Phi_{\rm ss} = \frac{\mathcal{E}^2}{RT}$$

Example: two inductively coupled RL circuits



$$\Pi_{\mathrm{ss}} = \frac{\mathcal{E}_1^2}{R_1 T_1} + \frac{\mathcal{E}_2^2}{R_2 T_2} + \frac{m^2 R_1 R_2}{(L_1 L_2 - m^2)(L_2 R_1 + L_1 R_2)} \frac{(T_1 - T_2)^2}{T_1 T_2}$$

GTL, T. Tomé and M. J. de Oliveira, *J. Phys A.* **46** (2013) 395001

Traditional formulation

- The traditional theory of entropy production, for both quantum and classical systems, is based on the following formulas:

$$\frac{dS}{dt} = \Pi - \Phi$$

Entropy flux

$$\Phi = \frac{\Phi_E}{T}$$

$$\left(dS = \frac{dE}{T} \right)$$

Entropy production

$$\Pi = -\frac{d}{dt} S(\rho || \rho_{\text{eq}})$$

$$S(\rho || \rho^{\text{eq}}) = \text{tr}(\rho \ln \rho - \rho \ln \rho^{\text{eq}})$$

(Relative entropy)

J. Schnakenberg, *Rev. Mod. Phys.* **48**, 571 (1976).
H. Spohn, *J. Math. Phys.*, **19**, 1227 (1978)
T. Tomé and M. J. de Oliveira, *Phys. Rev. Lett*, **108**, 020601 (2012)

Entropy production and loss of coherence

- The environment selects a preferred basis for the system.

$$\rho = \begin{pmatrix} p_0 & q \\ q^* & p_1 \end{pmatrix}$$

- When the system interacts with an environment, two things happen simultaneously:
  - The *populations* adjust to the levels imposed by the bath: $p_n = \langle n|\rho|n\rangle$
  - The system looses coherence.
- We may write the relative entropy as

$$S(\rho||\rho_{\text{eq}}) = S(p||p_{\text{eq}}) + C(\rho)$$

$$S(p||p_{\text{eq}}) = \sum_n p_n \ln p_n/p_n^{\text{eq}}$$

$$C(\rho) = S(p) - S(\rho)$$

$$\therefore \qquad \Pi = \Pi_d + \Pi_{\text{coh}}$$

Entropy is produced due to the "classical" transitions between energy levels and also due to the loss of coherence

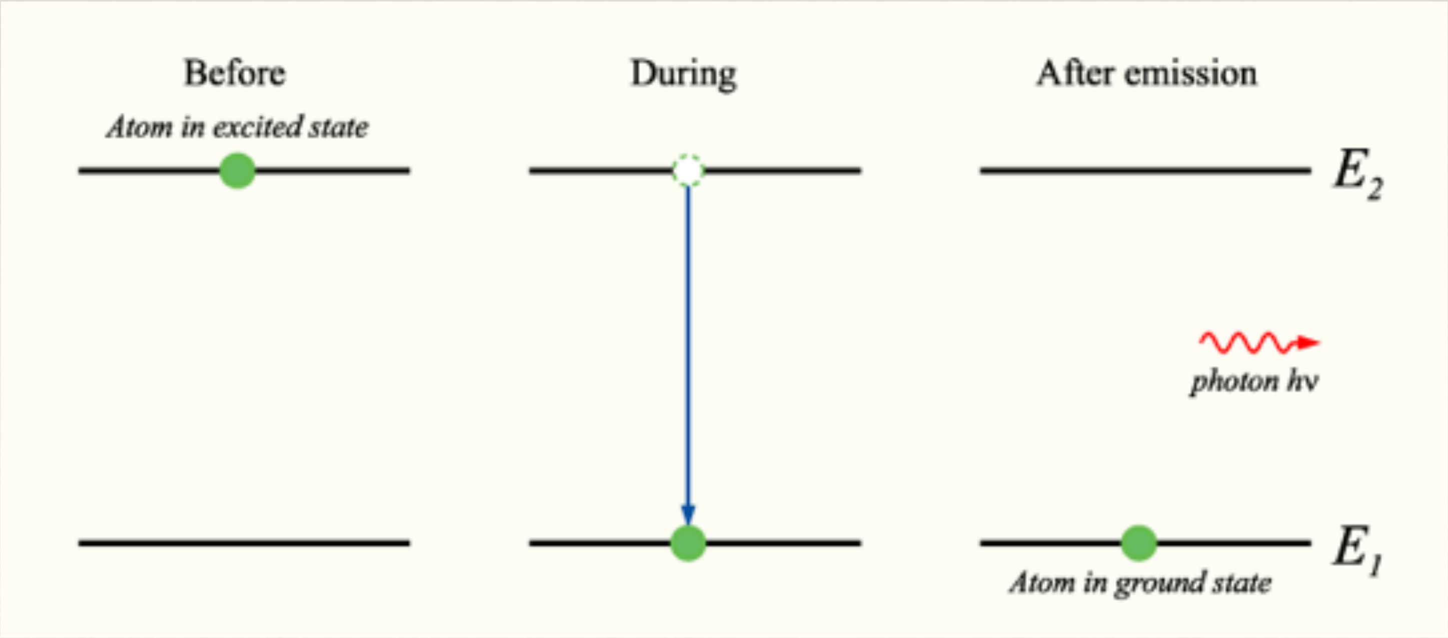
Problems with the standard formulation

$$\frac{dS}{dt} = \Pi - \Phi \qquad \Pi = -\frac{d}{dt} S(\rho || \rho_{\mathrm{eq}}) \qquad \Phi = \frac{\Phi_E}{T}$$

- Difficult to extend to systems connected to multiple reservoirs.
- Cannot be extended to non-equilibrium reservoirs:
  - Squeezed baths, dephasing baths, engineered baths, &c.
- Breaks down at T = 0.

Spontaneous emission is at T = 0

- Every system is nature is connected to a bath:
  - *Vacuum fluctuations act as a zero-temperature bath.*
  - Explains why atoms emit photons and relax to the ground-state.
- The theory of open quantum systems accounts for this type of process quite naturally.
- Everything is well behaved.
- But Π and Φ diverge when $T \to 0$.



Example: evolution of a coherent state

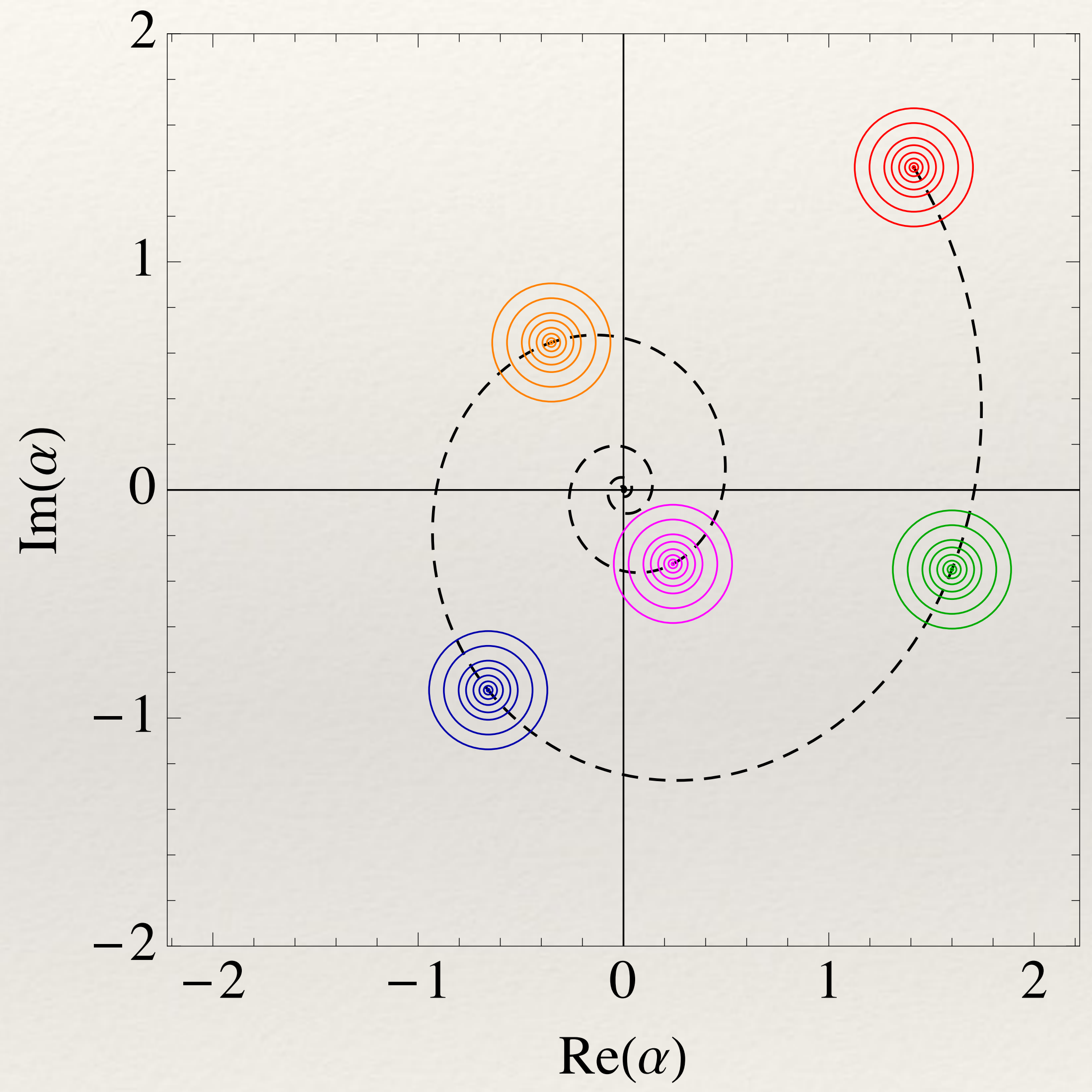
- Consider the evolution of a harmonic oscillator starting from a coherent state:

$$\rho(0) = |\mu\rangle\langle\mu|$$

- The evolution remains as a (pure) coherent state:

$$\rho(t) = |\mu_t\rangle\langle\mu_t|$$

$$\mu_t = \mu e^{-(i\omega+\gamma/2)t}$$



- The entropy is zero throughout, but Π and Φ would both be infinite.
- This is clearly an inconsistency of the theory.

Dynamics of open quantum systems

Lindblad dynamics

- Most widely used tool to describe experiments in Quantum Information setups.

$$\frac{d\rho}{dt} = -i[H, \rho] + D(\rho)$$

$$D(\rho) = \sum_{\alpha} L_\alpha \rho L_\alpha^\dagger - \frac{1}{2}\{L_\alpha^\dagger L_\alpha, \rho\}$$

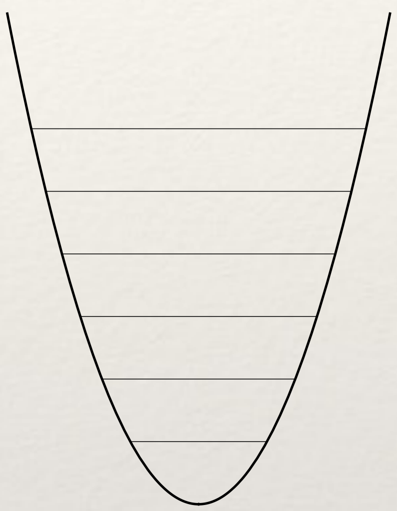
- Idea: the most general evolution of a closed system is a Unitary. The most general evolution of an open system is a *Kraus map*:

$$\rho \to \sum_k M_k \rho M_k^\dagger, \qquad \sum_k M_k^\dagger M_k = 1$$

- Lindblad's theorem: if such a map is also Markovian (forms a semi-group), then it can be expressed as a Lindblad master equation.

Open quantum harmonic oscillator

- We revisit this problem using the simplest model in quantum mechanics: the harmonic oscillator:



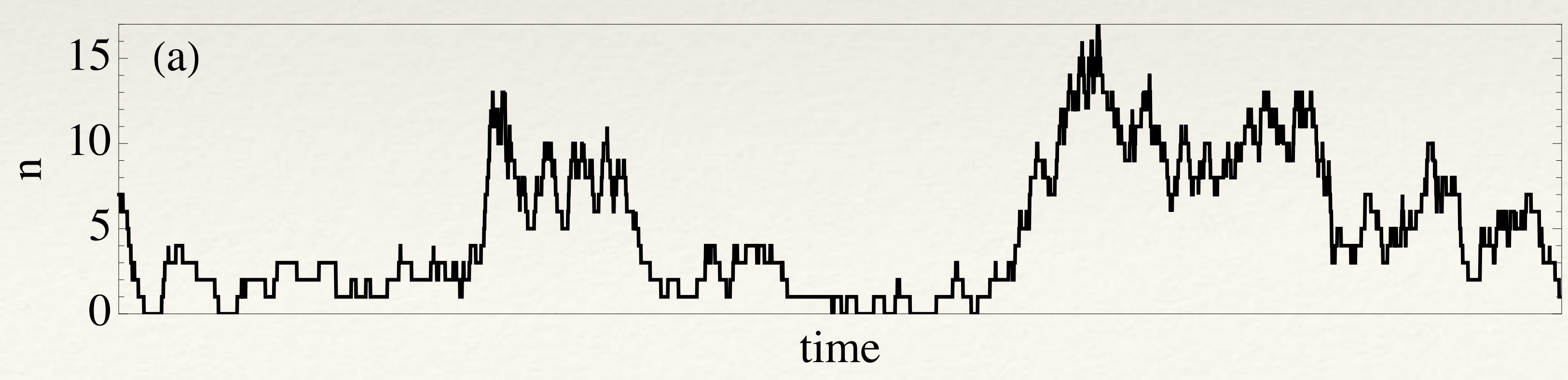
$$H = \omega(a^\dagger a + 1/2)$$

- The dissipator describing the contact with a thermal bath is

$$D(\rho) = \gamma(\bar{n}+1)\left[a\rho a^\dagger - \frac{1}{2}\{a^\dagger a, \rho\}\right] + \gamma\bar{n}\left[a^\dagger \rho a - \frac{1}{2}\{a a^\dagger, \rho\}\right]$$

$$\bar{n} = \frac{1}{e^{\beta\omega} - 1}$$

Classical dynamics describes emission and absorption of quanta.
But also captures quantum features.



PRL **118,** 220601 (2017) PHYSICAL REVIEW LETTERS week ending 2 JUNE 2017

Wigner Entropy Production Rate

Jader P. Santos,[1] Gabriel T. Landi,[2] and Mauro Paternostro[3]
[1]*Universidade Federal do ABC, 09210-580 Santo André, Brazil*
[2]*Instituto de Física da Universidade de São Paulo, 05314-970 São Paulo, Brazil*
[3]*Centre for Theoretical Atomic, Molecular and Optical Physics, School of Mathematics and Physics, Queen's University Belfast, Belfast BT7 1NN, United Kingdom*
(Received 6 March 2017; revised manuscript received 10 April 2017; published 1 June 2017)

Phase space

- Instead of using wavefunctions or density matrices, we work in *phase space* using the *Wigner function:*

$$W(\alpha,\alpha^*) = \frac{1}{\pi^2}\int d^2\lambda e^{-\lambda\alpha^*+\lambda^*\alpha}\mathrm{tr}\left\{\rho e^{\lambda a^\dagger-\lambda^* a}\right\}$$

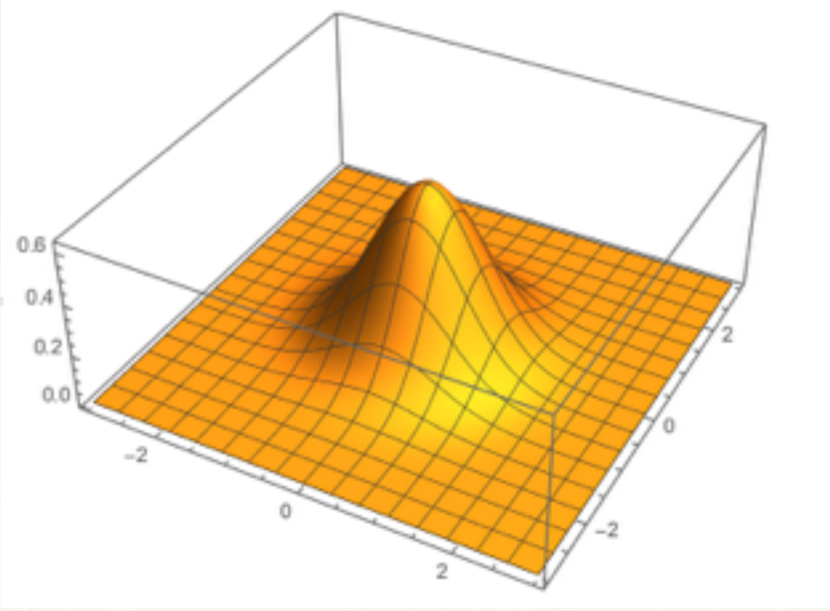
- Phase space is now the complex plane, with:

$$x = \sqrt{2}\mathrm{Re}(\alpha), \qquad p = \sqrt{2}\mathrm{Im}(\alpha)$$

- Thermal equilibrium is a Gaussian

$$W_{\mathrm{eq}} = \frac{1}{\pi(\bar{n}+1/2)}\exp\left\{-\frac{|\alpha|^2}{\bar{n}+1/2}\right\}$$

- For T = 0 this gives the vacuum state, which still has a non-zero width: *quantum fluctuations.*



Rényi-2 and Wigner entropy

The second laws of quantum thermodynamics

Fernando Brandão[a,1], Michał Horodecki[b], Nelly Ng[c], Jonathan Oppenheim[c,d,2], and Stephanie Wehner[c,e]

PNAS | **March 17, 2015** | vol. 112 | no. 11 | **3275–3279**

- The authors of this paper showed that for quantum systems all Rényi entropies have thermodynamic significance.

$$S_\alpha = \frac{1}{1-\alpha} \ln \mathrm{tr} \rho^\alpha$$

- The simplest one to use is the Rényi-2 entropy:

$$S_2 = -\ln \mathrm{tr} \rho^2$$

- In *PRL* **109**, 190502 (2012) the authors showed that for Gaussian states, this actually coincides with the *Wigner entropy*

$$S = -\int d^2\alpha W(\alpha, \alpha^*) \ln W(\alpha, \alpha^*)$$

Quantum Fokker-Planck equation

$$\frac{d\rho}{dt} = -i[H,\rho] + D(\rho)$$

- In terms of the Wigner function, the Lindblad equation becomes a quantum Fokker-Planck equation:

$$\partial_t W = -i\omega\Big[\partial_{\alpha^*}(\alpha^* W) - \partial_\alpha(\alpha W)\Big] + \mathcal{D}(W)$$

$$\mathcal{D}(W) = \partial_\alpha J(W) + \partial_{\alpha^*} J^*(W)$$

$$J(W) = \frac{\gamma}{2}\Big[\alpha W + (\bar{n}+1/2)\partial_{\alpha^*} W\Big]$$

- This is a continuity equation and J(W) is the irreversible component of the probability current.

$$J(W_{\rm eq}) = 0$$

U. Seifert, *Rep. Prog. Phys.* **75**, 126001 (2012)

Wigner entropy production and flux

- We use 3 different methods to show that the Wigner entropy production for a harmonic oscillator will be:

$$\Pi = \frac{4}{\gamma(\bar{n}+1/2)} \int d^2\alpha \frac{|J(W)|^2}{W}$$

- The entropy flux rate then becomes

$$\Phi = \frac{\gamma}{\bar{n}+1/2}\Big[\langle a^\dagger a\rangle - \bar{n}\Big] = \frac{\Phi_E}{\omega(\bar{n}+1/2)}$$

- At high temperatures $\omega(\bar{n}+1/2) \simeq T$ so we get

$$\Phi \simeq \frac{\Phi_E}{T}$$

- Now both Π and Φ remain finite at T = 0.

Stochastic trajectories and fluctuation theorems

- We can also arrive at the same result using a completely different method.
  - We analyze the stochastic trajectories in the complex plane.
- The quantum Fokker-Planck equation is equivalent to a Langevin equation in the complex plane:

$$\frac{dA}{dt} = -i\omega A - \frac{\gamma}{2}A + \sqrt{\gamma(\bar{n}+1/2)}\xi(t)$$

$$\langle \xi(t)\xi(t')\rangle = 0, \qquad \langle \xi(t)\xi^*(t')\rangle = \delta(t-t')$$

- We can now define the entropy produced in a trajectory as a functional of the path probabilities for the forward and reversed trajectories:

$$\Sigma[\alpha(t)] = \ln \frac{\mathcal{P}[\alpha(t)]}{\mathcal{P}_R[\alpha^*(\tau - t)]}$$

- This quantity satisfies a fluctuation theorem

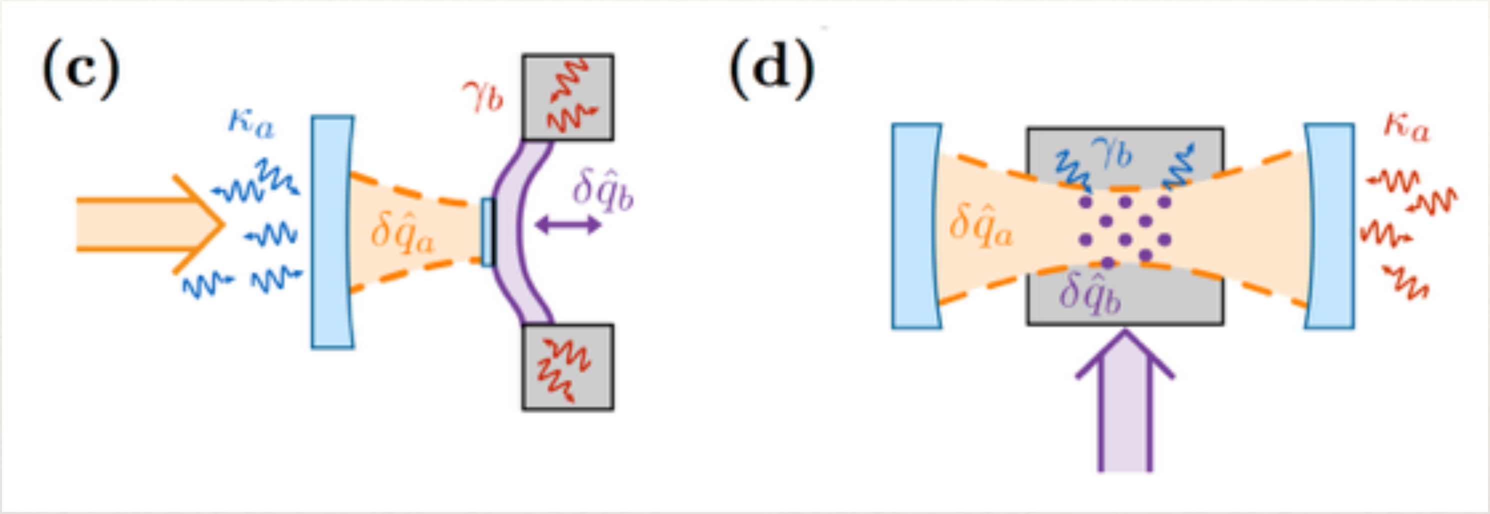
$$\langle e^{-\Sigma} \rangle = 1$$

- We show that we can obtain exactly the same formula for the entropy production rate if we define it as

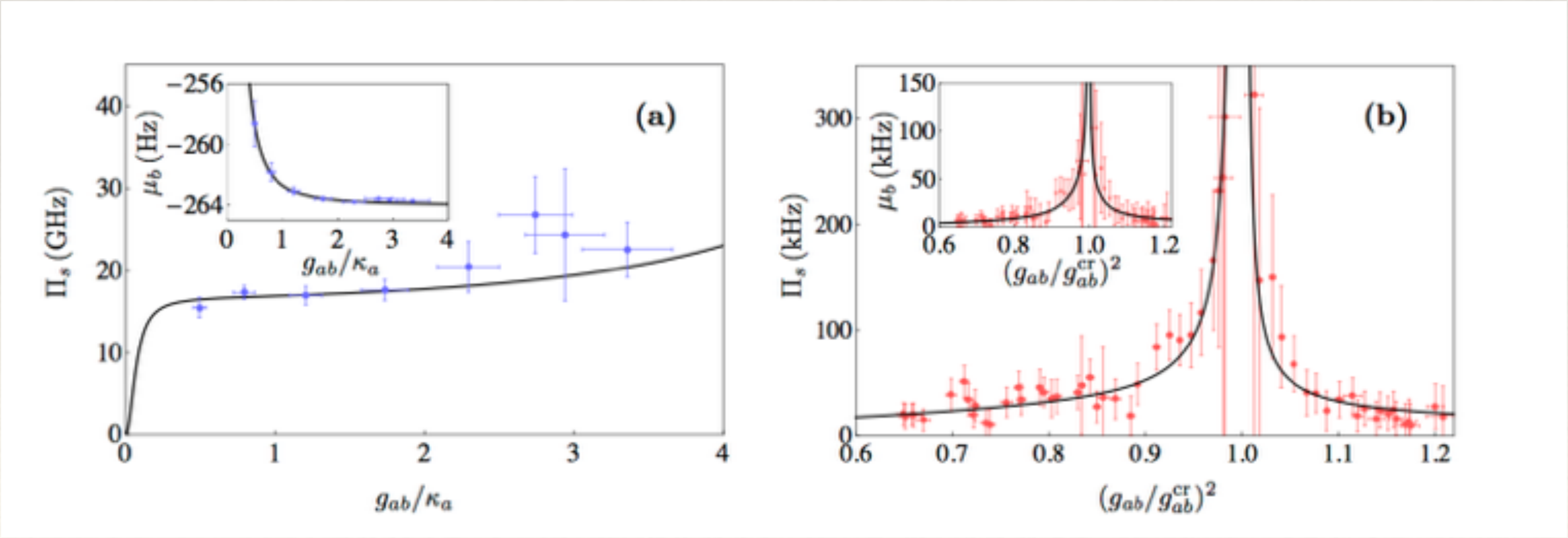
$$\Pi = \frac{\langle d\Sigma[A(t)] \rangle}{dt}$$

Experiments

Optomechanical system (Vienna)



BEC in a high-finesse cavity (ETH)



Generalizations

Spins and qubits

- We would like to have a similar framework for spin systems.

Spin coherent states: $$|\Omega\rangle = e^{-\phi J_z} e^{-\theta J_y} |J, J\rangle$$

Husimi-Q function: $$\mathcal{Q}(\Omega) = \langle \Omega | \rho | \Omega \rangle$$

Wehrl entropy: $$\Sigma = -\int d\Omega \mathcal{Q}(\Omega) \ln \mathcal{Q}(\Omega)$$

- The Quantum Fokker-Planck equation is now written in terms of orbital angular momentum operators.

$$-i[J_z, \rho] \quad \rightarrow \quad \mathcal{J}_z(\mathcal{Q}) = -i \frac{\partial}{\partial \phi} \mathcal{Q}$$

Dephasing and amplitude damping

- The dephasing bath induces no population changes, only decoherence:

$$D(\rho) = -\frac{\lambda}{2}[J_z,[J_z,\rho]]$$

- It leads to no entropy flux, only an entropy production:

$$\Pi = \frac{\lambda}{2}\int d\Omega \frac{|\mathcal{J}_z(\mathcal{Q})|^2}{\mathcal{Q}}$$

- We compare this with the amplitude damping:

$$D(\rho) = \gamma(\bar{n}+1)\left[J_-\rho J_+ - \frac{1}{2}\{J_+J_-,\rho\}\right] + \gamma\bar{n}\left[J_+\rho J_- - \frac{1}{2}\{J_-J_+,\rho\}\right]$$

- We now see the separation of a contribution from population changes and a contribution from decoherence.

$$\Pi = \frac{\gamma}{2}\int \frac{d\Omega}{\mathcal{Q}}\left\{\frac{[2J\mathcal{Q}\sin\theta + (\cos\theta - (2\bar{n}+1))\partial_\theta \mathcal{Q}]^2}{(2\bar{n}+1)-\cos\theta} + |\mathcal{J}_z(\mathcal{Q})|^2\left[(2\bar{n}+1)\cos\theta - 1\right]\frac{\cos\theta}{\sin^2\theta}\right\}$$

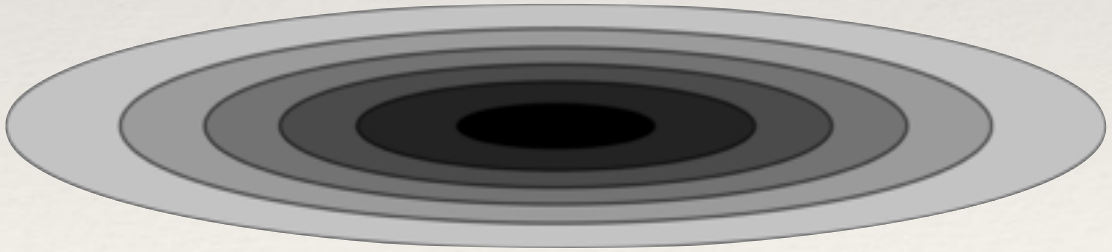
Squeezed baths

- Example of a non-equilibrium reservoir.

$$\mathcal{D}_z(\rho) = \gamma(N+1)\left[a\rho a^\dagger - \frac{1}{2}\{a^\dagger a, \rho\}\right]$$
$$+\gamma N\left[a^\dagger \rho a - \frac{1}{2}\{a a^\dagger, \rho\}\right]$$
$$-\gamma M\left[a^\dagger \rho a^\dagger - \frac{1}{2}\{a^\dagger a^\dagger, \rho\}\right]$$
$$-\gamma M^*\left[a\rho a - \frac{1}{2}\{aa, \rho\}\right]$$

$$J_E = \frac{d\langle a^\dagger a\rangle}{dt} = \gamma(N - \langle a^\dagger a\rangle)$$

$$J_S = \frac{d\langle aa\rangle}{dt} = \gamma(M - \langle aa\rangle)$$



$$N + 1/2 = (\bar{n} + 1/2)\cosh 2r$$

$$M = -(\bar{n} + 1/2)e^{i\theta}\sinh(2r)$$

- For the squeezed bath we find that the entropy production rate is given by

$$\Pi = \frac{4}{\gamma(\bar{n}+1/2)} \int \frac{d^2\alpha}{W} \Big| J_z \cosh r + J_z^* e^{i(\theta - 2\omega_s t)} \sinh r \Big|^2$$

$$J_z(W) = \frac{\gamma}{2}\Big[\alpha W + (N+1/2)\partial_{\alpha^*} W + M_t \partial_\alpha W \Big]$$

- The entropy flux rate is given by

$$\Phi = \frac{\gamma}{\bar{n}+1/2}\left[\cosh(2r)\langle a^\dagger a\rangle - \bar{n} + \sinh^2(r) - \frac{\mathrm{Re}[M_t^* \langle aa\rangle]}{\bar{n}+1/2} \right]$$

Onsager theory for squeezing

- Our formalism allows us to cast this problem within the same thermodynamic framework of Onsager's transport theory:
  - Joint transport of energy and squeezing.
  - We can even define a Squeezing Peltier and Squeezing Seebeck effect.

$$J_E = \mathcal{T}_{1,1}\delta\bar{n} + \mathcal{T}_{1,2}\delta r$$

$$J_S = \mathcal{T}_{2,1}\delta\bar{n} + \mathcal{T}_{2,2}\delta r$$

- Entropy production and flux can be written like in standard thermodynamics:

$$\Phi = \bar{f}_E J_E + \bar{f}_S J_S + \bar{f}_S^* J_S^*$$

$$\Pi = (\bar{f}_E - f_E)J_E + (\bar{f}_S - f_S)J_S + (\bar{f}_S^* - f_S^*)J_S^*$$

Conclusions

- Quantum Information Sciences: *understand and exploit the role of quantum resources, such as coherence and entanglement.*
- Quantum thermodynamics: *understand how these resources affect properties such as heat, work and entropy production.*
- Theory of irreversibility for open quantum systems is incomplete.
- We proposed an alternative for Gaussian states using the Wigner entropy.
    - This approach solves the $T = 0$ problem and is also useful to study engineered reservoirs.

Thank you.

Collaborators:

- **Jader. P. Santos** (post-doc)
- Mauro Paternostro (Queen's @ Belfast)
- Lucas Céleri (UFG)
- Dragi Karevski and Malte Henkel (Uni Lorraine @ Nancy)
- André Timpanaro and Fernando Semião (UFABC)
- Sascha Wald (SISSA @ Trieste)
- Cecilia Cormick (NUC @ Cordoba)
- Giovanna Morigi (Särbrucken)

Students:

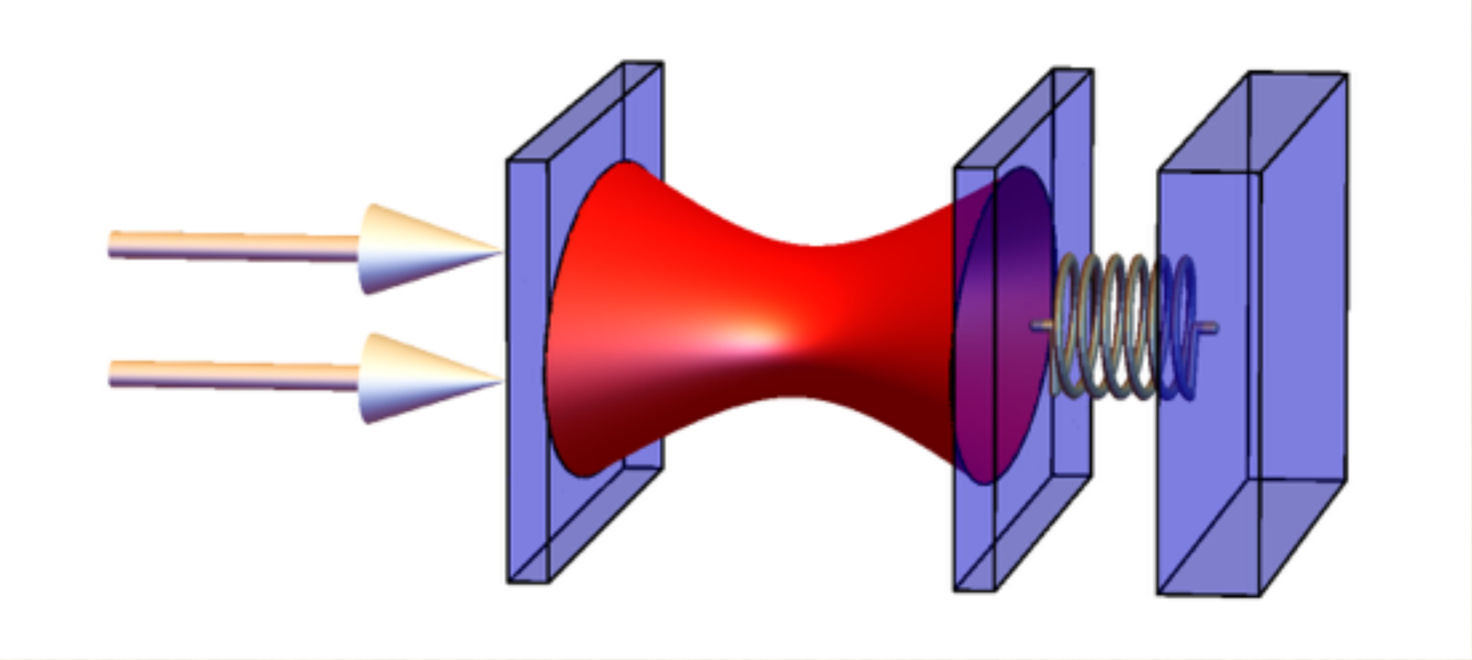
- **Wellington Ribeiro:** dephasing in fermionic systems.
- **William Malouf:** entropy production and mutual information.
- **Heitor Casagrande:** DMRG simulations of open quantum systems.
- **Pedro Portugal:** backflow of information in Non-Markovian dynamics.
- **Franklin Luis:** transport of squeezing in opto-mechanical systems.
- **Bruno Goes:** dynamics of dissipative quantum phase transitions.
- **Mariana Cipolla:** entropy production and entanglement in the spin-boson model.

Squeezed baths and gravitational waves

nature physics
LETTERS
PUBLISHED ONLINE: 28 MARCH 2016 | DOI: 10.1038/NPHYS3701

Observation of strong radiation pressure forces from squeezed light on a mechanical oscillator

Jeremy B. Clark, Florent Lecocq, Raymond W. Simmonds, José Aumentado and John D. Teufel*



Most used approaches

- Keldysh Green's functions (discussed in Altland's book on Cond. Mat. Field Theory).
- Quantum Fokker-Planck-Kramers equation.
  - M. J. de Oliveira, *PRE,* **94**, 012128 (2016)
- Quantum Brownian motion:
  - A. Caldeira and A. Leggett, *Physica A,* **121**, 587 (1983).
  - L. Pucci, M. Esposito and L. Peliti, *J. Stat. Mech.* **13**, P04005 (2013).

ARTICLE

Received 2 Feb 2015 | Accepted 22 May 2015 | Published 28 Jul 2015 DOI: 10.1038/ncomms8606 OPEN

Observation of non-Markovian micromechanical Brownian motion

S. Gröblacher[1,2], A. Trubarov[2], N. Prigge[3], G.D. Cole[2], M. Aspelmeyer[2,3] & J. Eisert[3]