Spin versus charge noise from Kondo traps

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Magnetic and charge noise have a common microscopic origin in solid-state devices, as described by a universal electron trap model. In spite of this common origin, magnetic (spin) and charge noise spectral densities display remarkably different behaviors when many-particle correlations are taken into account, leading to the emergence of the Kondo effect. We derive exact frequency sum rules for trap noise and perform numerical renormalization-group calculations to show that while spin noise is a universal function of the Kondo temperature, charge noise remains well described by single-particle theory even when the trap is deep in the Kondo regime. We obtain simple analytical expressions for charge and spin noise that account for Kondo screening in all frequency and temperature regimes, enabling the study of the impact of disorder and the emergence of magnetic $1/f$ noise from Kondo traps. We conclude that the difference between charge and spin noise survives even in the presence of disorder, showing that noise can be more manageable in devices that are sensitive to magnetic (rather than charge) fluctuations and that the signature of the Kondo effect can be observed in spin noise spectroscopy experiments.

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I. INTRODUCTION

The tunneling of conduction electrons into local charge traps is a prevalent phenomenon in solid-state physics. Traps can be realized by artificial structures such as quantum dots [1] or by natural "unwanted" defects such as dangling bonds [2] and bound states in metal-oxide interfaces [3]. It has long been recognized that trap fluctuation causes charge noise in electronic devices, with the signature of individual traps being observed with a Lorentzian $1/f^2$ noise spectral density in small structures [4,5] and an ensemble of them causing $1/f$ noise in large structures [6]. Here we address the fundamental question of how the electron *spin* alters trap noise.

One of the greatest developments of interacting electron physics was the discovery that a local trap interacting with a Fermi sea gives rise to the Kondo effect, the formation of a many-body singlet with conduction-electron spins screening out the local trap spin [7]. The signatures of the Kondo effect in transport phenomena are well studied, but key issues related to dynamics have been addressed only recently with the emergence of modern numerical renormalization group (NRG) algorithms [8]. It is particularly interesting to find out whether trap noise will impact devices that are sensitive to *magnetic fluctuations* as opposed to *charge*, e.g., spin-based or spintronic devices [9,10], in the same way that it affects conventional charge-based devices. Recent measurements of intrinsic magnetic flux noise in superconducting quantum interference devices do indeed confirm that trap spin fluctuation is the dominant source of noise [11–13]. Moreover, novel developments in spin noise spectroscopy [14] open several possibilities for the detection of correlated spin fluctuations in quantum-dot systems.

Given these interesting prospects, the question that we address here is the qualitative difference between pure charge and spin noise of a "Kondo trap" interacting with a Fermi sea, which we define as a local charge trap in the Kondo regime.

The interplay of Kondo physics and noise has been explored mostly in the context of transport through quantum-dot systems, with the Kondo trap right inside the transport path. In this case trap charge and spin fluctuation are intertwined in a nontrivial way. Calculations of the shot noise and current noise in different setups such as single [15–19] and double quantum dots [20–22] in the Kondo regime have been reported. Much less studied is the role of the Kondo state in *spin* noise. The case of spin-current noise was considered in Refs. [23,24], and qualitative differences between spin-current and charge-current noise were found to exist.

In this paper, we show that focusing on *pure spin/charge trap noise* (i.e., finite-frequency trap occupation noise) allows for a different perspective on the problem of Kondo trap dynamics: it enables a clear separation between the contributions of single-particle excitations and the many-particle processes connected to the formation of the Kondo singlet state. Moreover, considering pure spin (charge) trap noise is important for describing transport experiments with traps outside the transport channel. In this case, trap fluctuations produce bias magnetic (electric) noise that in turn may dominate the spin-current (charge-current) noise.

Our paper is organized as follows. In Sec. II we outline our model for pure spin/charge trap noise, and establish its connection to the usual spin/charge susceptibilities. We demonstrate six exact results: four sum rules and two Shiba relations. In Sec. III we describe our Hartree-Fock (HF) or mean-field approximation, which mainly accounts for single-particle processes. In Sec. IV we present our nonperturbative NRG calculations, which account for single-particle and many-particle processes on the same footing. The NRG results show that finite-frequency spin and charge noises have quite distinct behaviors and are dominated by completely different processes. In Sec. V we use NRG and the sum rules to obtain an analytic approximation to spin noise in the Kondo regime, and in Sec. VI we use this analytic approximation to study the interplay between disorder and Kondo correlations in an ensemble of Kondo traps. We show that, in the presence of disorder, the spin noise displays a temperature-dependent

$1/f$ noise that is qualitatively distinct from the temperature-independent charge $1/f$ noise. Finally, Sec. VII presents our concluding remarks, with a discussion of the impact of our results in the effort to detect Kondo correlations in spin noise spectroscopy experiments, and our prediction of qualitatively different $1/f$ noise impacting spin-based and charge-based devices.

II. CHARGE TRAP MODEL AND EXACT SUM RULES

Our starting point is the Anderson model [25] for a trapping center interacting with a Fermi sea,

$$H = \mathcal{H}_{\mathrm{band}} + \mathcal{H}_{\mathrm{hyb}} + \mathcal{H}_{\mathrm{trap}}, \tag{1}$$

with

$$\mathcal{H}_{\mathrm{band}} = \sum_{k,\sigma} \epsilon_{k\sigma} n_{k\sigma}, \tag{2a}$$

$$\mathcal{H}_{\mathrm{hyb}} = \sum_{k,\sigma} V_{dk}(c^{\dagger}_{k\sigma} d_{\sigma} + d^{\dagger}_{\sigma} c_{k\sigma}), \tag{2b}$$

$$\mathcal{H}_{\mathrm{trap}} = \epsilon_d (n_{\uparrow} + n_{\downarrow}) + U n_{\uparrow} n_{\downarrow}. \tag{2c}$$

In the above equations, $c^{\dagger}_{k\sigma}$ ($c_{k\sigma}$) is a creation (destruction) operator for a conduction electron with wave vector k and spin $\sigma = \uparrow,\downarrow$, and $n_{k\sigma} = c^{\dagger}_{k\sigma} c_{k\sigma}$ counts the number of band electrons in state k,σ with energy $\epsilon_{k\sigma}$. Similarly, the operators $d^{\dagger}_{\sigma}$ and d_{σ} create and destroy a trap electron with spin σ, respectively, with $n_{\sigma} = d^{\dagger}_{\sigma} d_{\sigma}$ being the number operator for electrons with spin σ occupying the trap state with energy ϵ_d. Finally, U is the Coulomb repulsion energy for the trap, with $\epsilon_d + U$ being the energy required to add a second electron to a trap site that already contains one electron.

Our goal is to calculate the trap *spin* $S_s(\omega,T)$ and *charge* $S_c(\omega,T)$ noise spectral densities, defined by

$$S_{i=s,c}(\omega,T) = \frac{1}{2\pi}\int_{-\infty}^{\infty} dt\, e^{i\omega t}\langle \delta\hat{\mathcal{O}}_i(t)\delta\hat{\mathcal{O}}_i(0)\rangle, \tag{3}$$

where $\delta\hat{\mathcal{O}}_i(t) = \hat{\mathcal{O}}_i(t) - \langle\hat{\mathcal{O}}_i\rangle$, with trap spin and charge operators given by $\hat{\mathcal{O}}_s = S_z = (n_{\uparrow} - n_{\downarrow})/2$ and $\hat{\mathcal{O}}_c = (n_{\uparrow} + n_{\downarrow})$, respectively, and $\langle\cdot\rangle$ denoting the thermal equilibrium average.

We write an exact expression for the spin and charge noise by performing a spectral decomposition of Eq. (3) in the basis of energy eigenstates:

$$S_i(\omega) = \sum_{m,n} \frac{e^{-E_m/T}}{Z} |\langle n|\hat{\mathcal{O}}_i|m\rangle|^2 \delta(\omega - E_{nm}) - \langle\hat{\mathcal{O}}_i\rangle^2\delta(\omega)\ , \tag{4}$$

where Z is the partition function, $|m\rangle$ are (many-body) eigenstates of the Hamiltonian (1) with energy E_m ($E_{nm} \equiv E_n - E_m$), and $\langle n|\hat{\mathcal{O}}_i|m\rangle$ are the many-body matrix elements of the local operator $\hat{\mathcal{O}}_i$. For simplicity, we set $\hbar = k_B = 1$. Note that Eq. (4) implies that $S_i(\omega,T) \geqslant 0$ and $S_i(-\omega,T) = e^{-\omega/T} S_i(\omega,T)$, as required by our assumption of thermal equilibrium.

The noise spectra are closely related to the dynamical susceptibility associated with the operator $\hat{\mathcal{O}}_i$. We shall explore this connection in order to derive the exact frequency sum rules and Shiba relations [26] for $S_i(\omega,T)$. These relationships will be used in Sec. V to obtain analytical approximations for the noise spectra.

Assuming that an external field $F_i(t)$ couples to $\hat{\mathcal{O}}_i$ through $\mathcal{H}_{\mathrm{ext}} = -\hat{\mathcal{O}}_i F_i(t)$, the linear response of $\hat{\mathcal{O}}_i$ to F_i will be $\langle\hat{\mathcal{O}}_i(t)\rangle_{F\neq 0} - \langle\hat{\mathcal{O}}_i\rangle_{F=0} = 2\pi\int d\omega e^{-i\omega t}\chi_i(\omega,T)F_i(\omega)$, where $\chi_i(\omega,T)$ is the dynamical susceptibility given by [27]

$$\chi_i(\omega,T) = \frac{i}{2\pi}\int_0^{\infty} dt\ e^{i\omega t}\langle[\hat{\mathcal{O}}_i(t),\hat{\mathcal{O}}_i(0)]\rangle. \tag{5}$$

Performing a spectral decomposition of Eq. (5) and comparing it to Eq. (4) lead to the following Lehmann representation:

$$\chi_i(\omega,T) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{d\omega'}{\omega - \omega' + i\eta}[S_i(-\omega',T) - S_i(\omega',T)], \tag{6}$$

with $\eta \to 0^+$. Separating the susceptibility into real and imaginary parts, $\chi_i = \chi_i' + i\chi_i''$, using $S_i(-\omega,T) = e^{-\omega/T}S_i(\omega,T)$, and taking the imaginary part of Eq. (6) lead to

$$\chi_i''(\omega,T) = \frac{1 - e^{-\omega/T}}{2} S_i(\omega,T), \tag{7}$$

which is known as the fluctuation-dissipation theorem. Moreover, taking the real part of Eq. (6) yields

$$\chi_i'(\omega,T) = \frac{1}{2\pi}\mathcal{P}\int_{-\infty}^{\infty}\frac{d\omega'}{\omega' - \omega}(1 - e^{-\omega'/T})S_i(\omega',T), \tag{8}$$

which is the Kramers-Kronig causality relation.

We now derive the frequency sum rules. The first one is obtained by direct integration of Eq. (4) over all frequencies:

$$\int_{-\infty}^{\infty} S_i(\omega,T)\, d\omega = \langle\hat{\mathcal{O}}_i^2\rangle - \langle\hat{\mathcal{O}}_i\rangle^2. \tag{9}$$

We call this the *spin* or the *charge* sum rule depending on whether $i = s$ or $i = c$. Another sum rule is obtained by setting $\omega = 0$ in Eq. (8) and noting that Eq. (6) implies $\chi_i(\omega = 0,T) = \chi_i'(\omega = 0,T)$:

$$\int_{-\infty}^{\infty}\frac{1 - e^{-\omega'/T}}{2\pi\omega'}S_i(\omega,T)d\omega' = \chi_i(\omega = 0,T). \tag{10}$$

Accordingly, we call this the spin or charge *susceptibility* sum rule. Altogether Eqs. (9) and (10) form a set of four exact sum rules that are valid at any temperature T.

Finally, there are two additional exact relationships between noise and susceptibility that apply only at $T = 0$. These are the so-called Shiba relations [26]:

$$\lim_{\omega\to 0^+}\frac{S_s(\omega,T = 0)}{8\pi^2\omega} = [\chi_s(\omega = 0,T = 0)]^2, \tag{11a}$$

$$\lim_{\omega\to 0^+}\frac{S_c(\omega,T = 0)}{2\pi^2\omega} = [\chi_c(\omega = 0,T = 0)]^2. \tag{11b}$$

They imply that $S_i(\omega,T)$ is Ohmic (linear in ω) at $T = 0$, with a slope related to the static susceptibility $\chi_i(\omega{=}0,T{=}0)$.

III. HARTREE-FOCK APPROXIMATION

As a first approximation we calculate the noise spectral densities using Hartree-Fock (HF) decomposition based on writing expectation values into products of spectral functions

[25]. The advantage of HF is that it becomes exact in the $U=0$ noninteracting limit [5,28]. The result for charge noise is

$$S_c^{\mathrm{HF}}(\omega,T)=\sum_{\sigma=\uparrow,\downarrow}\int d\epsilon A_{\sigma\sigma}(\epsilon)A_{\sigma\sigma}(\epsilon-\omega)\times[1-f(\epsilon)]f(\epsilon-\omega), \tag{12}$$

and for the spin noise we get simply $S_s^{\mathrm{HF}}(\omega,T)=\frac{1}{4}S_c^{\mathrm{HF}}(\omega,T)$; that is, in the HF approximation magnetic noise is simply $\frac{1}{4}$ times the charge noise. In Eq. (12) $f(\epsilon)=1/\{\exp[(\epsilon-\epsilon_F)/T]+1\}$ is the Fermi function, and

$$A_{\uparrow\uparrow}(\epsilon)=\frac{\Gamma/\pi}{(\epsilon-\epsilon_d)^2+\Gamma^2}, \tag{13a}$$

$$A_{\downarrow\downarrow}(\epsilon)=\frac{\Gamma/\pi}{(\epsilon-\epsilon_d-U)^2+\Gamma^2} \tag{13b}$$

are HF local densities of states for the trap with spin $\uparrow$ and $\downarrow$, respectively. The energy scale $\Gamma\equiv\pi\rho V_d^2$ models the rate for escape of a trap electron into the Fermi sea, with ρ being the energy density at the Fermi level and $V_{dk}\equiv V_d$ being a k-independent coupling between the trap and Fermi sea. Note that Eqs. (13a) and (13b) break the local spin symmetry by assuming the energy for the $\uparrow$ and $\downarrow$ trap states are ϵ_d and ϵ_d+U, respectively. This result is well known to be incorrect in that it misses Kondo physics, i.e., the screening of trap spin by the electron gas spins.

IV. NRG CALCULATIONS

We shall compare the Hartree-Fock approach to nonperturbative NRG calculations of the noise spectra that take into account local spin symmetry and the formation of the Kondo singlet. The NRG algorithm calculates, within some well-controlled approximations [8], the many-body spectrum for the Anderson model [8,29]. Conduction electrons are assumed to have a continuum spectrum, forming a metallic band with a half bandwidth D.

At zero temperature, the first term in Eq. (4) can be computed from the NRG spectral data [8,30,31] down to arbitrarily small nonzero frequencies $|\omega|>0$. The spectral weight at $\omega=0$ and the fulfillment of the sum rules can be obtained by calculating the expectation values $\langle\hat{O}_i\rangle$ and $\langle\hat{O}_i^2\rangle$ with NRG. Since we will be interested in the large-frequency regime and our spectral functions obey well-defined sum rules, we have chosen to use the "complete Fock space" (CFS) approach [32,33] to calculate $S_i(\omega>0)$ at zero temperature. As discussed in Appendix A, this choice has two important features: (i) the $T=0$ spectral functions are sum rule conserving by construction, and (ii) broadening artifacts in the high-frequency regime, which can mask the correct power-law behavior, are minimized.

Figure 1 shows the calculated charge noise in the case $\epsilon_d=-U/2$ for $\Gamma=10^{-4}D$ and several different U. Remarkably, HF remains a good approximation to charge noise even at large U. We interpret this result to be evidence that charge noise is dominated by single-particle processes even when the trap is *deep* in the Kondo regime ($U\gg\Gamma$ for $T=0$).

The situation is drastically different for magnetic noise, as shown in Fig. 2. While NRG and HF agree with each other in the $U=0$ limit (when HF is exact), as soon as U becomes nonzero, the two methods show opposite results. As U increases, the single-particle noise (HF) decreases, while the many-body noise (NRG) increases. The low-frequency NRG results can be better visualized in Fig. 3. We find [Figs. 3(a) and 3(c)] that the magnetic noise for a single trap is Ohmic at low frequencies, with a peak at $\omega\approx T_K$, where T_K is the Kondo temperature. The magnetic noise spectral densities all collapse in the same universal curve and scale as an anomalous power law $\propto T_K/[\omega\ln^2(\omega/T_K)]$ in the $T_K\ll\omega\ll U$ frequency range [Fig. 3(b)], consistent with

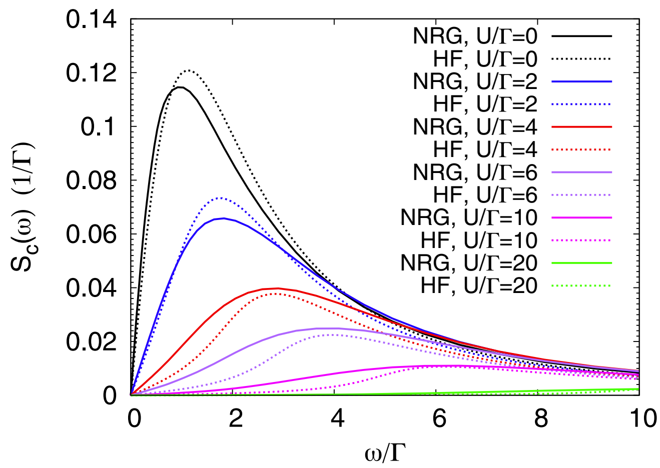


FIG. 1. (Color online) Charge noise as a function of frequency for the trap in the symmetric case with $\epsilon_d=-U/2$. NRG calculations are shown to be well approximated by a mean-field Hartree-Fock decomposition (HF) even when U/Γ is large and the trap is deep in the Kondo regime. This shows that charge noise is well described by single-particle excitations.

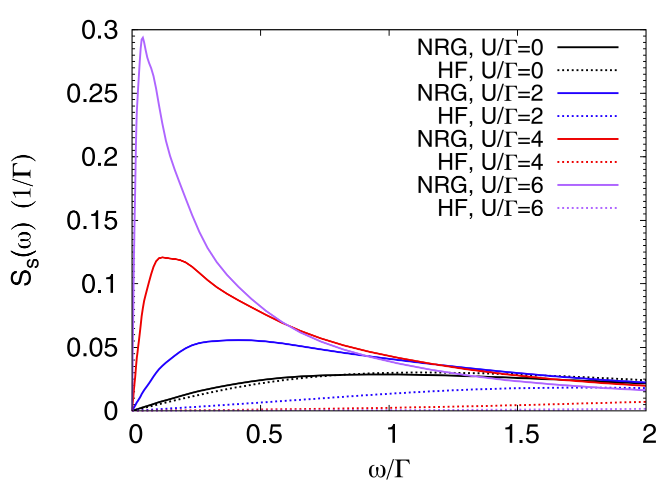


FIG. 2. (Color online) Spin noise as a function of frequency for the trap in the symmetric case with $\epsilon_d=-U/2$. The NRG results agree with HF only at $U=0$. As U increases, NRG shows that the magnetic noise increases, developing a peak at $\omega\approx T_K$. In contrast, the single-particle contributions described by HF decrease dramatically as U increases. This shows that magnetic noise is dominated by many-body processes.

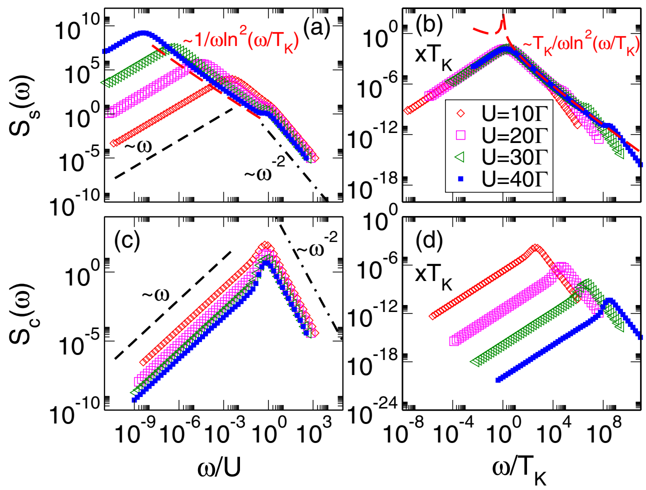


FIG. 3. (Color online) Universal scaling for spin noise in the Kondo regime for $\epsilon_d = -U/2$. (a) and (b) NRG results for spin noise $S_s(\omega)$. Note how all the curves collapse into a single scaling relation when the noise is written as a function of ω/T_K. For $\omega \lesssim T_K$, the magnetic noise scales linearly with ω (Ohmic noise), and for $T_K \lesssim \omega < U$ it decreases with an anomalous power of frequency $\propto 1/[\omega \ln^2(\omega/T_K)]$. For $\omega > U$, spin noise is cut off $\propto 1/\omega^2$. (c) and (d) NRG results for charge noise $S_c(\omega)$ *do not* show universal Kondo scaling and behave just like the single-particle approximation (HF) with noise peaked at $\omega \approx \text{Max}\{\Gamma, U\}$ with a smooth cutoff $1/\omega^2$ at $\omega > U$.

previous results for the dynamical spin susceptibility [34–38] and the spin-current noise [24] in the Kondo regime.

V. ANALYTICAL APPROXIMATION FOR SPIN NOISE IN THE KONDO REGIME

While the HF approximation [Eq. (12)] failed to describe spin noise, it was shown to give a good description of charge noise at $T = 0$ (Fig. 1). In Appendix B we show that HF actually provides a *good* approximation for charge noise at $T \geqslant 0$, in the sense that it approximately satisfies the sum rules and Shiba relations demonstrated in Sec. II. The goal of the current section is to use our NRG calculations, sum rules, and Shiba relations to obtain an analytical approximation for spin noise at $T \geqslant 0$ in the Kondo regime.

It is well known [8] that NRG has difficulty in calculating spectral features at frequencies $\omega < T$. Here we propose an alternate approach to evaluate the spin noise for a broader ω/T range.

Motivated by the susceptibility sum rule Eq. (10) and the property $S_s(-\omega,T) = e^{-\omega/T} S_s(\omega,T)$, we propose the following fit function:

$$S_s^{\text{Fit}}(\omega,T) = \frac{2\omega\chi_s(\omega=0,T)}{1-e^{-\omega/T}} \frac{\Gamma_s}{\omega^2+\Gamma_s^2}, \tag{14}$$

with the $\omega = 0$ susceptibility given by a continuous fit to the NRG result [39]

$$\chi_s(\omega=0,T) = \begin{cases} \frac{\mathcal{W}}{8\pi T_K} & \text{for } T \leqslant 0.23T_K, \\ \frac{0.68}{8\pi(T+\sqrt{2}T_K)} & \text{for } 0.23T_K < T \leqslant 28.59T_K, \\ \frac{1}{8\pi T}\left[1 - \frac{1}{\ln(T/T_K)} - \frac{\ln[\ln(T/T_K)]}{2\ln^2(T/T_K)}\right] & \text{for } T > 28.59T_K, \end{cases} \tag{15}$$

where $\mathcal{W} = 0.4128$ is the Wilson number.

In Eq. (14) $\Gamma_s \equiv \Gamma_s(\omega,T)$ is a fit function of frequency and temperature that will be determined by the exact sum rules and the Shiba relations. We recall that previous relaxational fits for Γ_s assume no frequency dependence [40]. Here we allow $\Gamma_s(\omega,T)$ to vary in frequency so that the logarithmic frequency decay discussed in Sec. IV is properly accounted for.

For $T \gg T_K$, the perturbative method of Suhl and Nagaoka [41,42] yields the high-temperature limit (the Korringa law):

$$\Gamma_s(\omega,T \gg T_K) \approx \frac{1}{4\pi} \frac{T}{1+\frac{4}{3\pi^2}\ln^2\left(\frac{T}{T_K}\right)}. \tag{16}$$

At $T = 0$ the Shiba relation (11a) applied to Eq. (14) implies [40]

$$\Gamma_s(\omega=0,T=0) = \frac{1}{4\pi^2\chi_s(0,0)} = \frac{2T_K}{\pi\mathcal{W}}, \tag{17}$$

where we used the NRG result $\chi_s(0,0) = \mathcal{W}/(8\pi T_K)$.

In order to interpolate between Eqs. (16) and (17) we propose the following expression:

$$\Gamma_s(\omega,T) = \frac{1}{4\pi} \frac{T+\frac{8}{\mathcal{W}}T_K}{1+\frac{1}{3\pi^2}\ln^2\left[1+\left(\frac{T}{T_K}\right)^2+\left(\frac{\omega}{\alpha T_K}\right)^2\right]}, \tag{18}$$

where α is a fit parameter to be determined by the spin sum rule [Eq. (9)]:

$$\text{Sum}_s(T) = 4\int_{-\infty}^{\infty} d\omega S_s^{\text{Fit}}(\omega,T). \tag{19}$$

This sum rule is most sensitive to α at $T = 0$, and we find that the optimal fit value is quite close to $\alpha = 3$ when $\text{Sum}_s(T=0) = 0.9994$. As an independent check, we evaluate the spin sum rule at $T > 0$ and the susceptibility sum rule at $T \geqslant 0$:

$$\text{Sum}_{\chi_s}(T) = \frac{1}{\chi_s(0,T)}\int_{-\infty}^{\infty} d\omega \frac{1-e^{-\omega/T}}{2\pi\omega} S_s^{\text{Fit}}(\omega,T). \tag{20}$$

In all cases, we obtain agreement within 36%. A few examples are shown in Table I. Moreover, we find that Eqs. (14) and (18) with $\alpha = 3$ provide an excellent fit of our NRG results at $T = 0$, as shown in Fig. 4.

Note that the choice of Eq. (18) implies that $T_K S_s^{\text{Fit}}(\omega,T)$ is a universal function of ω/T_K and T/T_K and that the presence of the temperature-dependent functions $\chi_s(0,T)$ and $\Gamma_s(\omega,T)$ suggest that spin noise has a much stronger temperature dependence than charge noise. In particular, Eq. (14) fully accounts for the Kondo screening for $T < T_K$ through $\chi_s(\omega = 0,T)$.

TABLE I. Sum rules [Eqs. (19) and (20)] applied to our analytical fit of spin noise, Eqs. (14) and (18), with $\alpha = 3$. For the spin sum rules we used analytical approximations for $\chi_s(0,T)$ obtained by NRG [Eqs. (4.53) and (4.60) in Ref. [39]]. In all cases we find that the sum rules are satisfied within 30%.

T/T_K	Sum_s	Sum_{χ_s}
0	0.9994	0.9518
0.5	0.9247	0.9502
1	0.8245	0.9503
10	0.6910	0.9875
100	0.7777	0.9979

VI. SPIN NOISE IN THE PRESENCE OF DISORDER

In the case of an ensemble of N Kondo traps, the noise will be affected by disorder. The usual model for trap disorder (the one that gives rise to ubiquitous charge $1/f$ noise) [6] is to assume trap tunneling rate $\Gamma = \Gamma_0 e^{-\lambda}$, where λ models the tunneling distance between the trap and Fermi sea. The model assumes λ uniformly distributed with density $P'(\lambda) = N/\lambda_{\max}$ for $\lambda \in [0,\lambda_{\max}]$ and $P'(\lambda) = 0$ for λ outside this interval, resulting in $P(\Gamma) = (N/\lambda_{\max})/\Gamma$ and the corresponding $1/f$ frequency dependence for trap charge noise. As we shall show, this same model applied to Kondo traps gives rise to a much broader distribution of Kondo temperatures that we denote $P(T_K)$.

For definiteness, we assume all Kondo traps have fixed ϵ_d and U, with the disorder solely affecting the parameter $\Gamma(\lambda)$. The dependence of the Kondo temperature with λ is given by [43]

$$T_K(\lambda) = \sqrt{\frac{\Gamma(\lambda)U}{2\pi}}\, e^{\frac{\sqrt{3}\epsilon_d(\epsilon_d+U)}{U}\frac{1}{\Gamma(\lambda)}} = T_K^{\max} e^{-[\frac{\lambda}{2}+\kappa(e^{\lambda}-1)]}. \tag{21}$$

Here $\kappa = -\sqrt{3}\epsilon_d(\epsilon_d + U)/(U\Gamma_0) > 0$ characterizes the type of trap. We shall assume $\kappa \gg (\lambda_{\max} + 1)/2$, a limit that is typically satisfied by Kondo traps with $U \gg \Gamma$. The maximum and minimum Kondo temperatures of the distribution are given by $T_K^{\max} = T_K(\lambda = 0)$ and $T_K^{\min} = T_K(\lambda = \lambda_{\max})$, respectively; for $T_K \in [T_K^{\min},T_K^{\max}]$ the trap density becomes

$$P(T_K) = \frac{P'(\lambda)}{\left|\frac{dT_K}{d\lambda}\right|} \approx \frac{\frac{N}{\lambda_{\max}}}{T_K\left[\kappa - \ln\left(\frac{T_K}{T_K^{\max}}\right)\right]}, \tag{22}$$

with $P(T_K) = 0$ for $T_K \notin [T_K^{\min},T_K^{\max}]$. Note how $P(T_K)$ is exponentially broader than $P(\Gamma)$: we have $T_K^{\max}/T_K^{\min} \approx \exp[\kappa \exp(\lambda_{\max})]$, in contrast to $\Gamma_{\max}/\Gamma_{\min} = \exp(\lambda_{\max})$. In spite of this difference, the normalization condition $\int dT_K P(T_K) \approx N$ still holds since the logarithm in Eq. (22) makes $P(T_K)$ flatter than an $\sim 1/T_K$ distribution, thereby making the integral finite. We remark that our $P(T_K)$ is appropriate to describe highly disordered traps, such as traps randomly distributed at an insulator close to the metal-insulator interface. This situation is quite different from Kondo impurities in bulk alloys, whose $P(T_K)$ is considerably less broad [40,44].

Applying this averaging prescription to our spin noise Eq. (14) yields

$$\langle S_s(\omega)\rangle = \int_{T_K^{\min}}^{T_K^{\max}} dT_K\, P(T_K) S_s^{\mathrm{Fit}}(\omega,T). \tag{23}$$

The results are shown in Figs. 5(a) and 5(b). At low temperatures ($T < T_K^{\max}$) the noise shows $1/f$ behavior up to frequencies of the order of $T_K^{\max}$; at larger frequencies, Kondo-enhanced exchange processes lead to a $1/[f \ln^2(f)]$ behavior. For higher temperatures ($T > T_K^{\max}$) the noise saturates in the low-frequency region, and the $1/[f \ln^2(f)]$ behavior gets washed out of the high-frequency region.

Interestingly, the frequency range with $1/f$ behavior gets *reduced* as the temperature increases. This shows that spin $1/f$ noise behavior is strongly temperature dependent, in marked contrast to the usually temperature-independent charge $1/f$ noise. The additional temperature dependence implies that temperature actually competes against disorder, converting the spin $1/f$ noise into a Lorentzian.

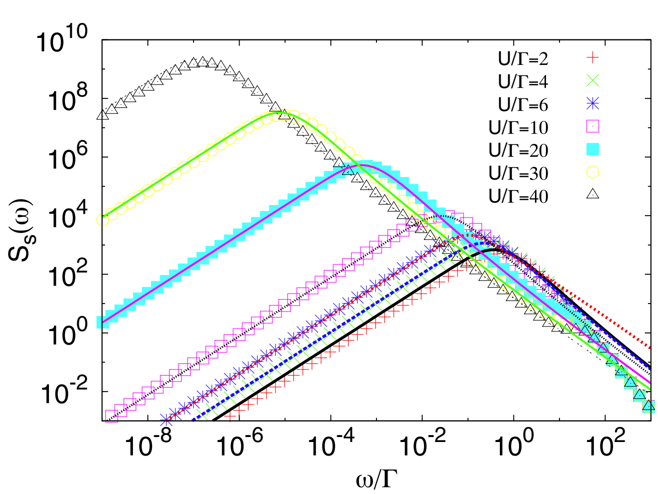
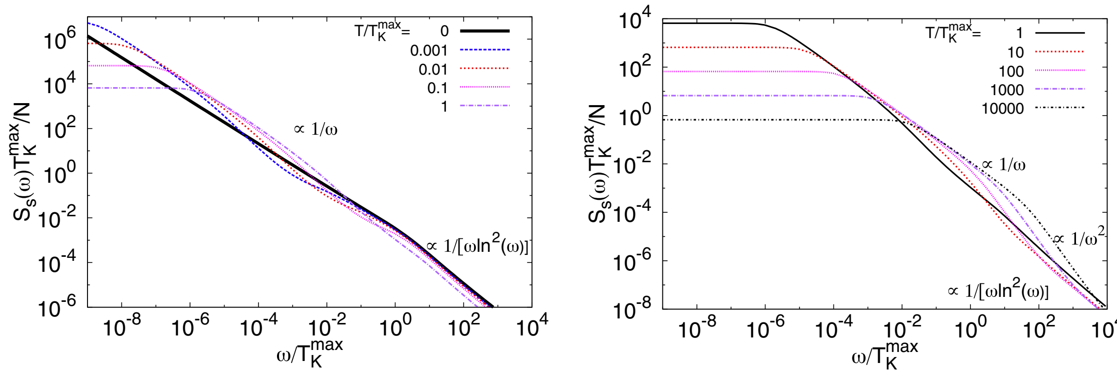


FIG. 4. (Color online) Comparison of the spin noise fit $S_s^{\mathrm{Fit}}(\omega)$ [Eqs. (14) and (18) with $\alpha = 3$; lines] with the NRG results (symbols) at $T = 0$.

VII. CONCLUDING REMARKS

In conclusion, we presented a theory of charge and spin noise of a Kondo trap interacting with a Fermi sea. We showed that trap spin noise is qualitatively different from charge noise in that the former occurs due to many-body scattering processes, while the latter is mainly dominated by single-particle tunneling. This difference implies that spin noise has a stronger temperature dependence than charge noise and that it is controllable by tuning Kondo temperature T_K rather than trap tunneling rate Γ.

Kondo trap dynamics displays two quite distinct behaviors depending on which property is probed. The experimental methods of charge [1] and spin [14] noise spectroscopy use optical absorption to detect noise via the fluctuation-dissipation theorem [optical absorption at frequency ω is directly proportional to $\chi_i''(\omega,T)$ and to noise as in Eq. (7)]. Our results elucidate how Kondo correlations can be observed with these methods. Pure charge absorption does not enable the detection of the Kondo effect; in Ref. [1] the formation of the exciton state mixes charge and spin fluctuation, and



(a) Low temperature behavior. (b) High temperature behavior.

FIG. 5. (Color online) Spin noise in the presence of trap disorder. The calculated noise for N traps was averaged according to the prescription $\Gamma = \Gamma_0 e^{-\lambda}$, with λ being the tunneling distance between the trap and the Fermi gas uniformly distributed in the interval $[0,\lambda_{\max}]$. This gives rise to the broad distribution of Kondo temperatures shown in Eq. (22). The resulting noise, shown here for $\kappa = 10$ and $\lambda_{\max} = 5$, displays $1/f$ behavior over a frequency range that decreases as the temperature increases. This is in contrast to the temperature-independent charge $1/f$ noise described in the literature [6].

this feature was critical in enabling the authors' observation of the Kondo effect. For spin noise spectroscopy, universal scaling with Ohmic behavior at $T < \omega < T_K$ coupled with a $1/[\omega \ln^2(\omega/T_K)]$ tail for $T_K \ll \omega \ll U$ can be taken as the signature of the Kondo effect, allowing the extension of this technique to probe Kondo correlations. However, in the presence of strong disorder over a range of Kondo temperatures $T_K \in [T_K^{\min}, T_K^{\max}]$, we find that the Ohmic behavior is washed out, and the signature of Kondo correlations is visible only for $\omega > T_K^{\max}$ and $T < 10T_K^{\max}$ [see Fig. 5(b)].

The qualitative difference between spin and charge noise survives even in the presence of disorder and high temperatures (namely, $T_K \gg T_K^{\max}$). As the temperature increases, the range of $1/f$ behavior for spin noise decreases, while the range of $1/f$ charge noise remains essentially unaltered. The additional temperature dependence for spin noise implies that temperature actually competes against disorder, converting the spin noise $1/f$ behavior into a Lorentzian-like dependence. Given that $1/f$ noise is notoriously difficult to control [45], we reach the conclusion that ubiquitous trap noise can be more manageable in spin- or flux-based devices that are sensitive to magnetic fluctuations rather than charge.

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APPENDIX A: DETAILS OF THE NRG CALCULATIONS

As we argue in the main text, our choice of the complete Fock space (CFS) procedure [32] (or, equivalently, the full density matrix NRG method [33] at $T = 0$) in the NRG calculations presents some advantages for the calculations of the correlation functions listed in Eq. (4). To illustrate this point, we compared results obtained using CFS and the earlier "density matrix NRG" (DM-NRG) method [46].

The main panel in Fig. 6 presents NRG data for the spin noise $S_s(\omega)$ using DM-NRG (open circles) and CFS (solid squares) for $U = 40\Gamma$ and other parameters set as in Fig. 3. In both cases, the NRG calculations were performed using a discretization parameter $\Lambda = 2.5$ retaining up to 1000 states at each NRG step, which ensures convergence for the single-trap Anderson model. The spectral data were broadened using the usual logarithmic Gaussian functions [Eq. (74) in Ref. [8]] with a broadening parameter $b = \ln(\sqrt{\Lambda}) \approx 0.46$. (We have used z averaging for some of the data presented, particularly the data presented in Fig. 1).

Clearly, DM-NRG underestimates the peak at $\omega = T_K$ in comparison with CFS. More importantly, it misses

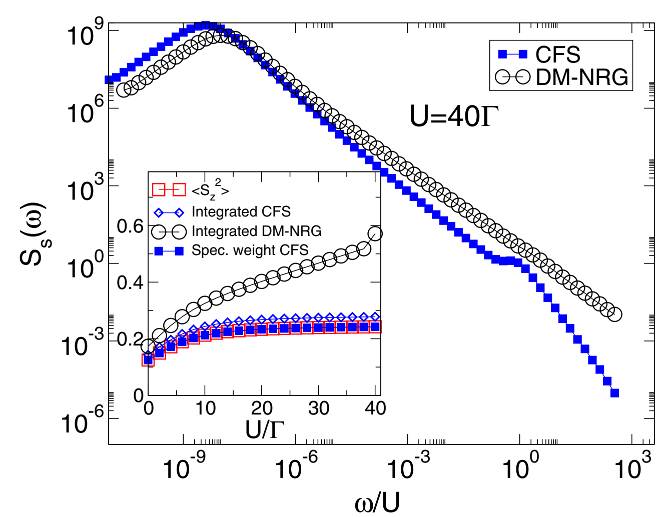

FIG. 6. (Color online) NRG data for the spin noise $S_s(\omega)$ at $T = 0$ for $U = 40\Gamma$ calculated with DM-NRG and CFS procedures. Inset: A check of the spin sum rule given by Eq. (A1) for the different approaches shows that the CFS fulfils the spin sum rule apart from numerical integration errors.

the transition from the $S_s(T_K \ll \omega \ll U) \propto 1/[\omega \ln^2(\omega/T_K)]$ behavior to the $S_s(\omega \gg U) \propto \omega^{-2}$ behavior, which is one of the important features distinguishing the spin noise from the charge noise.

We have also compared both methods by checking the fulfillment of the spin sum rule in Eq. (9). In the absence of spin polarization (due to, e.g., magnetic fields or ferromagnetic couplings), $\langle \hat{S}_z \rangle = 0$ and the spin sum rule is given by

$$\int_{-\infty}^{\infty} S_s(\omega)\, d\omega = \langle \hat{S}_z^2 \rangle. \tag{A1}$$

To this end, we performed a static NRG calculation of $\langle \hat{S}_z^2 \rangle (T \to 0)$ (open squares in the inset of Fig. 6) and compared it with a numerical integral of $S_s(\omega)$. The agreement of the integrated CFS data (diamonds) is much better than the DM-NRG (circles), although the fulfillment of the spin sum rule is not perfect due to numerical errors coming from the integration procedure.

Alternatively, the integral in Eq. (A1) can be done directly in Eq. (4), and it becomes a sum over the spectral weights $|\langle n|\hat{S}_z|m\rangle|^2$ provided that the set of many-body states $\{|m\rangle\}$ forms a complete set. In practice, this procedure can be done in the CFS scheme, as it retains matrix elements between "discarded" and "kept" NRG states, making the set of many-body states complete by construction [32,33]. In this case, free of numerical integration errors, the CFS data (solid squares) fulfill the spin sum rule down to machine precision, as shown in the inset of Fig. 6.

APPENDIX B: VALIDATION OF THE HARTREE-FOCK APPROXIMATION FOR CHARGE NOISE WHEN $T \geqslant 0$

The HF approximation Eq. (12) was shown to approximate charge noise at $T = 0$. Here we check its validity at $T \geqslant 0$ by direct evaluation of the Shiba relations and sum rules described in Sec. II.

The static ($\omega = 0$) charge susceptibility in the HF approximation is given by

$$\begin{aligned}\chi_c^{HF}(\omega = 0, T) &= \frac{1}{2\pi}\frac{\partial}{\partial \epsilon_F}\sum_\sigma \langle n_\sigma \rangle \\ &= \frac{1}{2\pi}\int d\epsilon \sum_\sigma A_{\sigma\sigma}(\epsilon)\frac{\partial f(\epsilon)}{\partial \epsilon_F} \\ &= \frac{1}{8\pi T}\int d\epsilon \frac{\sum_\sigma A_{\sigma\sigma}(\epsilon)}{\cosh^2\left(\frac{\epsilon-\epsilon_F}{2T}\right)}.\end{aligned} \tag{B1}$$

At $T = 0$ we get

$$\chi_c^{HF}(\omega = 0, T = 0) = \frac{1}{2\pi}\sum_\sigma A_{\sigma\sigma}(\epsilon_F). \tag{B2}$$

We start by checking the Shiba relation for charge noise, Eq. (11b). In the HF approximation we get

$$\lim_{\omega\to 0^+}\int_{\epsilon_F}^{\epsilon_F+\omega}\frac{d\epsilon}{\omega}\sum_\sigma A_{\sigma\sigma}(\epsilon)A_{\sigma\sigma}(\epsilon-\omega) = \sum_\sigma A_{\sigma\sigma}^2(\epsilon_F), \tag{B3}$$

which, according to the Shiba relation, should be equal to

$$2\pi^2\left[\chi_c^{HF}(0,0)\right]^2 = \frac{1}{2}\left[\sum_\sigma A_{\sigma\sigma}(\epsilon_F)\right]^2. \tag{B4}$$

The relation is satisfied exactly at $U = 0$; however, as U increases, Eq. (B3) becomes up to two times larger than Eq. (B4). This discrepancy can indeed be observed in the comparison with NRG; see the difference in slopes at $\omega = 0$ in Fig. 1. Nevertheless, the discrepancy is not too large.

The charge sum rule [Eq. (9) for $i = c$] in the HF approximation reads

$$\begin{aligned}\int_{-\infty}^{\infty} d\omega S_c^{HF}(\omega,T) &= \sum_\sigma \int d\epsilon A_{\sigma\sigma}(\epsilon)[1 - f(\epsilon)] \\ &\quad\times \int d\omega A_{\sigma\sigma}(\epsilon-\omega) f(\epsilon-\omega) \\ &= \sum_\sigma [1 - \langle n_\sigma \rangle]\langle n_\sigma \rangle \\ &= \langle \hat{\mathcal{O}}_c^2 \rangle_{HF} - \langle \hat{\mathcal{O}}_c \rangle_{HF}^2,\end{aligned} \tag{B5}$$

where $\langle \hat{\mathcal{O}}_c \rangle_{HF} = \sum_\sigma \int d\epsilon A_{\sigma\sigma}(\epsilon) f(\epsilon)$ and $\langle \hat{\mathcal{O}}_c^2 \rangle_{HF}$ is obtained by making the approximation $\langle n_\uparrow n_\downarrow \rangle \approx \langle n_\uparrow \rangle\langle n_\downarrow \rangle$. The last line of Eq. (B5) is expected to be a good approximation of the exact result, even in the Kondo regime, when charge fluctuations are strongly suppressed.

Finally, we verify the charge susceptibility sum rule [Eq. (10) for $i = c$] with explicit numerical calculations of the quantity

$$\mathrm{Sum}_{\chi_c} = \frac{1}{\chi_c^{HF}(0,T)}\int_{-\infty}^{\infty} d\omega \frac{1 - e^{-\omega/T}}{2\pi\omega} S_c^{HF}(\omega,T). \tag{B6}$$

As shown in Table II these values are very close to 1 for all tested parameters.

In conclusion, the HF approximation for charge noise is consistent with the exact relations of Sec. II for all parameters checked, indicating that it provides a good analytical approximation for charge noise even for $T > 0$.

TABLE II. Charge susceptibility sum rule in the HF approximation [Eq. (B6)]. The sum rule is seen to be satisfied ($\mathrm{Sum}_{\chi_c} = 1$) with high accuracy for several different parameters.

T/Γ	ϵ_d/Γ	U/Γ	Sum_{χ_c}
0.1, 1, 10	0	0	0.9997, 0.9999, 1.000
0.1, 1, 10	−2.5	5	0.9994, 0.9999, 1.000
0.1, 1, 10	−10	20	0.9915, 0.9978, 1.000
0.1, 1, 10	0	5	0.9996, 0.9999, 1.000
0.1, 1, 10	−10	10	0.9996, 0.9999, 1.000

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