Quantum critical transitions and interference effects in double quantum dot Kondo systems

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Talk Outline

- Kondo physics: a brief review.
- Kondo effect in *double* quantum dots
  - Numerical Renormalization Group methods.
  - Zero-field splitting of the Kondo resonance: interference and “band filtering” effects.
  - Quantum critical transition in DQDs: an effective pseudogapped host.
- Conclusions
Kondo effect

- 30’s - Resistivity measurements: minimum in $\rho(T)$;
- $T_{\text{min}}$ depends on $c_{\text{imp}}$.
- 60’s - Correlation between the existence of a Curie-Weiss component in the susceptibility ($magnetic\ moment$) and resistance minimum.

Kondo’s explanation for $T_{\text{min}}$ (1964)

$$H_{s-d} = J \sum_{k,k'} S^+ c_{k\downarrow}^\dagger c_{k'\uparrow} + S^- c_{k\uparrow}^\dagger c_{k\downarrow}$$

$$+ S_z \left( c_{k\uparrow}^\dagger c_{k'\uparrow} - c_{k\downarrow}^\dagger c_{k\downarrow} \right)$$

$$+ \sum_k e_k c_{k\sigma}^\dagger c_{k\sigma}$$

- **Many-body** effect: virtual bound state near the **Fermi energy**.
- AFM coupling ($J>0$) → “spin-flip” scattering
- **Kondo problem**: s-wave coupling with spin impurity (s-d model):
Kondo’s explanation for $T_{\text{min}}$ (1964)

- **Perturbation theory in $J^3$:**
  - Kondo calculated the conductivity in the linear response regime.

\[
R_{\text{imp}}^{\text{spin}} \propto J^2 \left[ 1 - 4J \rho_0 \log \left( \frac{k_B T}{D} \right) \right]
\]

\[
R_{\text{tot}}(T) = aT^5 - c_{\text{imp}} R_{\text{imp}} \log \left( \frac{k_B T}{D} \right)
\]

\[
T_{\text{min}} = \left( \frac{R_{\text{imp}} D}{5ak_B} \right)^{1/5} c_{\text{imp}}^{1/5}
\]

- **Only one free parameter:** the Kondo temperature $T_K$
  - Temperature at which the perturbative expansion diverges.

\[
k_B T_K \sim D e^{-1/2J \rho_0}
\]
A little bit of Kondo history:

- Early ‘30s: Resistance minimum in some metals
- Early ‘50s: Theoretical work on impurities in metals “Virtual Bound States” (Friedel)
- 1961: Anderson model for magnetic impurities in metals
- 1964: s-d model and Kondo’s solution (PT)
- 1970: Anderson’s “Poor’s man scaling” approach
- 1974-75: Wilson’s Numerical Renormalization Group (non-perturbative) solution
History of Kondo Phenomena

- Resistance minimum observed in the ‘30s…
- …and explained in the ‘60s (Kondo)
- Log divergence problem: Wilson’s NRG ‘70s
- Bethe-Ansatz solution (essentially exact): ‘80s

So, what’s new about it?

Kondo signatures in electronic transport observed in many different set ups:

- Quantum dots (experimental control of the parameters)
- STM measurements of magnetic structures on metallic surfaces (e.g., single atoms, molecules. “Quantum mirage”)
- New insights: multi-impurity systems, spin interactions,...
Kondo Effect in Quantum Dots

- $T > T_K$: Coulomb blockade (low $G$)
- $T < T_K$: Kondo singlet formation
- Kondo resonance at $E_F$ (width $T_K$).
- New conduction channel at $E_F$: Zero-bias enhancement of $G$
Kondo Effect in CB-QDs

$N_{ODD}$ valley: Conductance rises for low $T$ (Kondo effect)

Kondo Temperature $T_k$: only scaling parameter ($\sim 0.5K$, depends on $V_g$)


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Kondo Effect in *Double* QDs

Series configuration

Jeong, Chang, Melloch *Science* **293** 2222 (2001)

Kondo Effect in *Double* QDs

Parallel configuration


![Image of double quantum dot configuration with voltages V1, V2, V3, V4, V5 labeled.](image)

Kondo signal at zero bias

![Graph showing dI/dV vs. VSD with a peak at zero bias.](image)
Kondo Effect in *Double* QDs

Double Quantum Dots:
- Allow controlled studies of both *intradot* and *interdot* correlations
- Interference and phase measurements.
- RKKY interactions
- Quantum phase transitions.
- Prospects in quantum information processing.
DQD theory: different regimes

- Non-identical dots coupled to leads and to each other.
- For $V_{iR}=V_{iL}$; coupling to the symmetric channel only.
- Dot 2: effectively non-interacting.

\[
\begin{align*}
\varepsilon_1 &\pm U_1 \\
\varepsilon_2 &\pm U_2 \\
\Delta_1 &
\end{align*}
\]
DQD: theoretical description

- Non-identical dots coupled to leads and to each other.
- For $V_{iR} = V_{iL}$; coupling to the symmetric channel only.
- Dot 2: mixed-valence regime.

$$H_{\text{Leads}} = \sum_{k,j=L,R} \epsilon_k c_j^\dagger c_{j\sigma} \ ,$$

$$H_{i=1,2} = \epsilon_i a_{i\sigma}^\dagger a_{i\sigma} + U_i n_i n_{i\uparrow} \ ,$$

$$H_{\text{Dot-Leads}} = \sum_{k,j=L,R} V_{ij} a_{i\sigma}^\dagger c_{j\sigma} + \text{h.c.} \ ,$$

$$H_{\text{Dot-Dot}} = \lambda \left(a_{1\sigma}^\dagger a_{2\sigma} + \text{h.c.}\right)$$
Green’s function for dot 1: **effective** DoS

- **Constant DoS** in the leads $\rho(\epsilon) = \rho_0$
- Large band limit ($\epsilon << D$).

\[
[G_{11}(\epsilon^+)]^{-1} = \epsilon^+ - \epsilon_1 - \Sigma^*(\epsilon) - \Lambda(\epsilon) + i\Delta(\epsilon)
\]

\[
\Delta(\epsilon) = \Delta_1 + \pi \bar{\rho}(\epsilon) \left[ \lambda^2 - \Delta_1 \Delta_2 + 2\lambda \sqrt{\Delta_1/\Delta_2} (\epsilon - \epsilon_2)^2 \right]
\]

With

\[
\Delta_1 = \pi V_1^2 \rho_0
\]
\[
\Delta_2 = \pi V_2^2 \rho_0
\]

\[
\bar{\rho}(\epsilon) = \frac{\Delta_2/\pi}{(\epsilon^+ - \epsilon_2)^2 + \Delta_2^2}
\]
Green’s function for dot 1: effective DoS

\[
\begin{align*}
[G_{11}(\epsilon^+)]^{-1} &= \epsilon^+ - \epsilon_1 - \Sigma^*(\epsilon) \\
&\quad - \Lambda(\epsilon) + i\Delta(\epsilon)
\end{align*}
\]

\[
\Delta(\epsilon) = \Delta_1 + \pi \bar{\rho}(\epsilon) \left[ \lambda^2 - \Delta_1 \Delta_2 + 2\lambda \sqrt{\Delta_1/\Delta_2} (\epsilon - \epsilon_2)^2 \right]
\]

\[
\bar{\rho}(\epsilon) = \frac{\Delta_2/\pi}{(\epsilon^+ - \epsilon_2)^2 + \Delta_2^2}
\]

Diagram showing the effective DoS with peaks and minimum points.
Wilson’s NRG method:
- Logarithmic discretization of the hybridization function: Lanczos method.
- Iterative numerical solution of the many-body Hamiltonian.
- Calculation of the spectral function: $A_{11} = -\frac{1}{\pi} \text{Im}[G_{11}]$. 

\[ \gamma_n \sim \Lambda^{-n/2} \]
Side Dot: “Filtering” of the effective DoS.

\[
\left[ G_{11}(\epsilon^+) \right]^{-1} = \epsilon^+ - \epsilon_1 - \Sigma^*(\epsilon) - \Lambda(\epsilon) + i\Delta(\epsilon)
\]

\[
\Delta(\epsilon) = \pi \bar{\rho}(\epsilon) \lambda^2
\]

\[
\bar{\rho}(\epsilon) = \frac{\Delta_2 / \pi}{(\epsilon^+ - \epsilon_2)^2 + \Delta_2^2}
\]

Lorentzian peak

\(\epsilon_2 = 0\)
Splitting in the Kondo peak

- Resonance in the DoS at $\varepsilon_2 = 0$:
  - Kondo peak splits for $T_K > \Delta_2$
  - Kondo screening is preserved.
  - Diamagnetic phase for $\Delta_2 < T < T_K$

Consistent with narrow-band results:
Hofstetter, Kehrein, PRB 59 R112732 (1999)
Side-dot: Splitting the Kondo peak

- Large $\lambda$: $T_K \sim \lambda$.
- $T_K$ decreases with $\Delta_2$.
- Splitting $\sim T_K$.

Parallel Dot: Pseudo-Gapped effective DoS

\[ \left[ G_{11}(\epsilon^+) \right]^{-1} = \epsilon^+ - \epsilon_1 - \Sigma^*(\epsilon) - \Lambda(\epsilon) + i\Delta(\epsilon) \]

\[ \Delta(\epsilon) = \Delta_1 \left[ 1 - \pi \bar{\rho}(\epsilon) \Delta_2 \right] \]

\[ \bar{\rho}(\epsilon) = \frac{\Delta_2}{\pi (\epsilon^+ - \epsilon_2)^2 + \Delta_2^2} \]
Parallel dots: p-h symmetric case

- $\varepsilon_2 = 0$; Pseudogap: $\Delta(\varepsilon) \sim |\varepsilon - E_F|^2$.
- Particle-hole symmetry ($\varepsilon_1 = -U/2$)
  - No Kondo state for any $\Delta_1$, $\Delta_2$.
  - No local moment screening.

Parallel dots: quantum critical transition

In between: Quantum critical transition at $\varepsilon_1=\varepsilon^*$: 
$\mu^2 \rightarrow 1/6$

Fritz, Vojta, PRB 70 214427 (2004).


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Local moment
Kondo

Pseudogap
Parallel case: quantum critical transition

No p-h symmetry ($\varepsilon_1 \neq -U/2$)
- Critical transition at $\varepsilon_1 = \varepsilon_1^*$.
- Kondo screening for $\varepsilon_1 > \varepsilon_1^*$.

$\varepsilon_1 = \varepsilon^* = -0.0086524U$

General case: Kondo splitting + Pseudo-Gap

- By changing the gate voltage in Dot 2 (therefore $\varepsilon_2$), the three regimes can be reached.
- $\Delta(\varepsilon=0)$ determines the Kondo temperature.

$LDS \ et \ al. \ Phys. \ Rev. \ Lett. \ 97 \ 096603 \ (2006)$
Conclusions

- **DQD systems**: experimental realization of a Kondo impurity coupled to an effective DoS with resonances and pseudo-gaps.

- **Kondo filtering**: resonances in the effective DoS lead to splittings in the Kondo peak at $E_F$.

- Kondo singlet remains and $T_K$ increases for larger interdot hybridization.

- **Quantum critical transition**: Coherent coupling through the leads induces a pseudo-gap in the effective DoS: quantum critical transition to non-Kondo regime.

- Different regimes can be achieved by tuning the gate voltage in Dot 2.
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