Quantum critical transitions and interference effects in double quantum dot Kondo systems

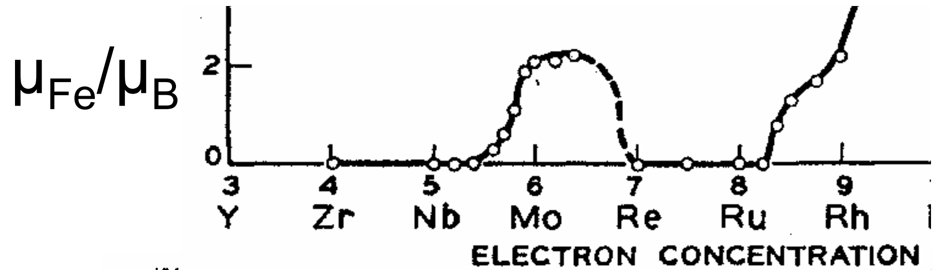
Luis Dias - Ohio University

Talk Outline

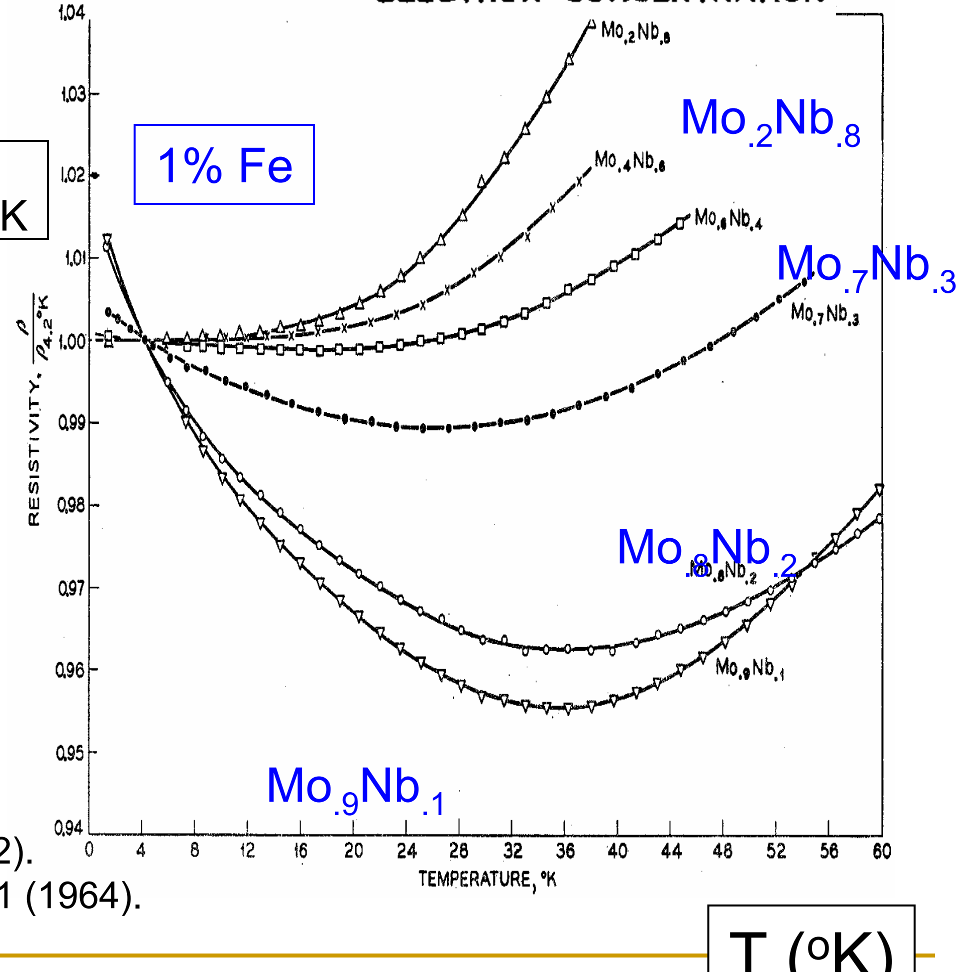
Mr. Jun Kondo

- Kondo physics: a brief review.
- Kondo effect in *double* quantum dots
  - Numerical Renormalization Group methods.
  - Zero-field splitting of the Kondo resonance: interference and "band filtering" effects.
  - Quantum critical transition in DQDs: an effective pseudogapped host.
- Conclusions

Kondo effect



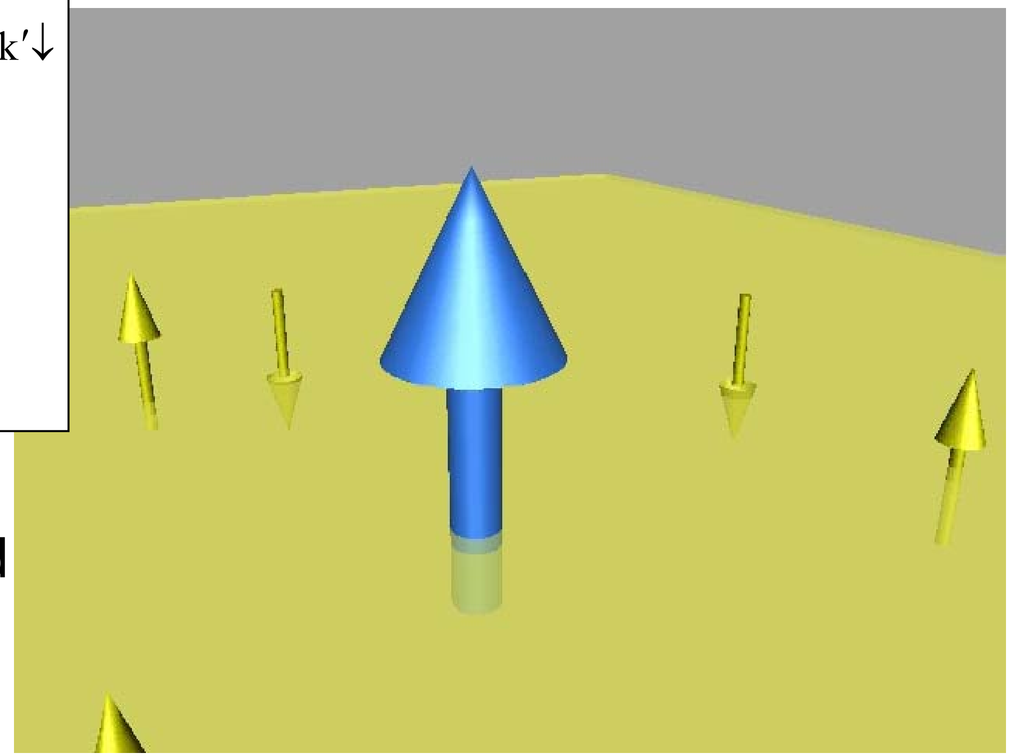
- 30's - Resistivity measurements: minimum in ρ(T);
- T_{min} depends on $c_{imp.}$
- 60's - Correlation between the existence of a Curie-Weiss component in the susceptibility (*magnetic moment*) and resistance minimum .



Top: A.M. Clogston *et al* Phys. Rev. **125** 541(1962).
Bottom: M.P. Sarachik *et al* Phys. Rev. **135** A1041 (1964).

Kondo's explanation for T_{min} (1964)

$$H_{\text{s-d}} = J \sum_{k,k'} S^{+} c^{\dagger}_{k\downarrow} c_{k'\uparrow} + S^{-} c^{\dagger}_{k\uparrow} c_{k'\downarrow} + S_z \left(c^{\dagger}_{k\uparrow} c_{k'\uparrow} - c^{\dagger}_{k\downarrow} c_{k'\downarrow} \right) + \sum_{k} e_k c^{\dagger}_{k\sigma} c_{k\sigma}$$



- **Many-body** effect: virtual bound state near the Fermi energy.
- AFM coupling (J>0)→ "spin-flip" scattering
- Kondo problem: s-wave coupling with spin impurity (s-d model):

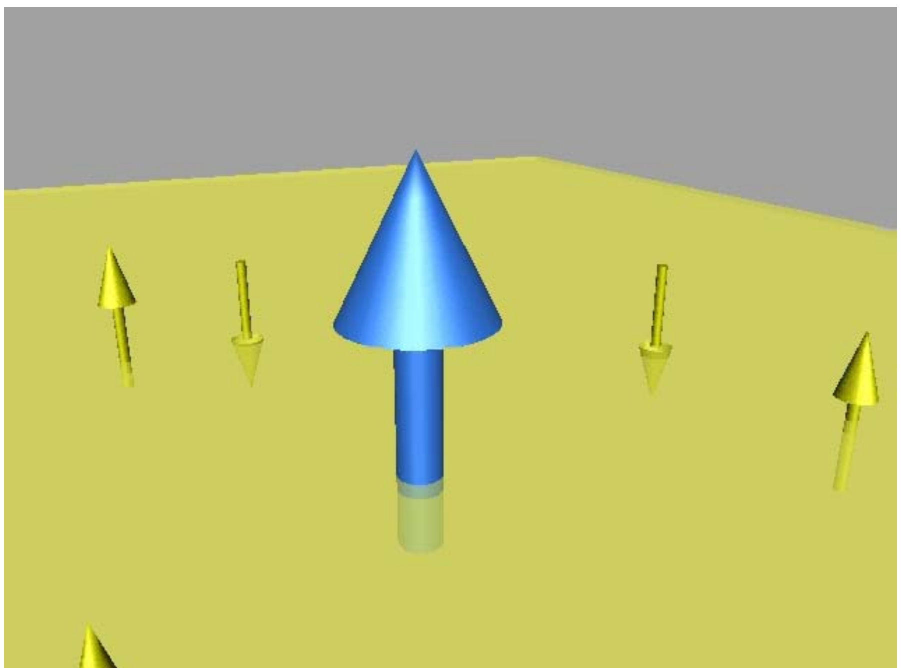
Kondo's explanation for T_{min} (1964)

- Perturbation theory in J^3:
  - Kondo calculated the conductivity in the linear response regime

$$R_{\text{imp}}^{\text{spin}} \propto J^2 \left[1 - 4J\rho_0 \log\left(\frac{k_B T}{D} \right) \right]$$

$$R_{\text{tot}}(T) = aT^5 - c_{\text{imp}} R_{\text{imp}} \log\left(\frac{k_B T}{D} \right)$$

$$T_{\min} = \left(\frac{R_{\text{imp}} D}{5 a k_B} \right)^{1/5} c_{\text{imp}}^{1/5}$$



- Only *one* free paramenter: the Kondo temperature T_K
  - Temperature at which the perturbative expansion ***diverges.***

$$k_B T_K \sim D e^{-1/2J\rho_0}$$

A little bit of Kondo history:

- Early ‘30s : Resistance minimum in some metals
- Early ‘50s : theoretical work on impurities in metals “Virtual Bound States” (Friedel)
- 1961: Anderson model for magnetic impurities in metals
- 1964: s-d model and Kondo’s solution (PT)
- 1970: Anderson’s “Poor’s man scaling” approach
- 1974-75: Wilson’s Numerical Renormalization Group (non-perturbative) solution
- 1980 : Andrei and Wiegmann’s Bethe-Ansatz solution.

History of Kondo Phenomena

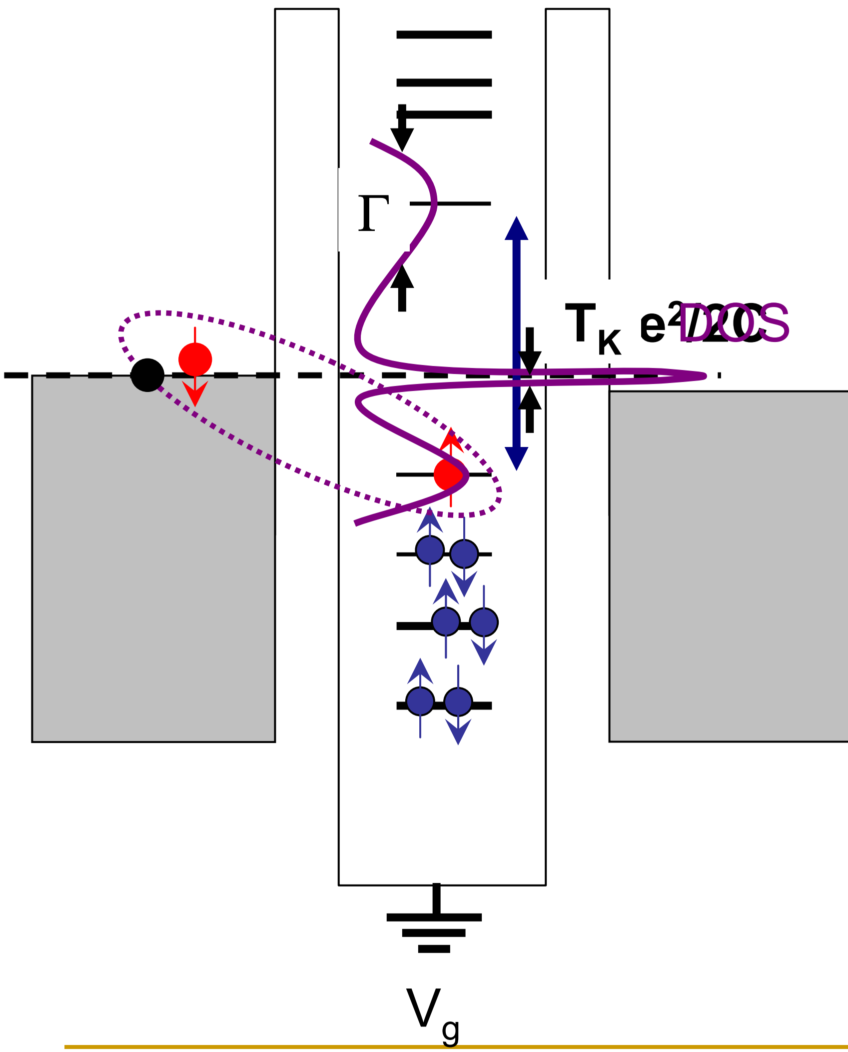
- Resistance minimum observed in the ‘30s…
- …and explained in the ‘60s (Kondo)
- Log divergence problem: Wilson’s NRG ‘70s
- Bethe-Ansatz solution (essentially exact): ‘80s

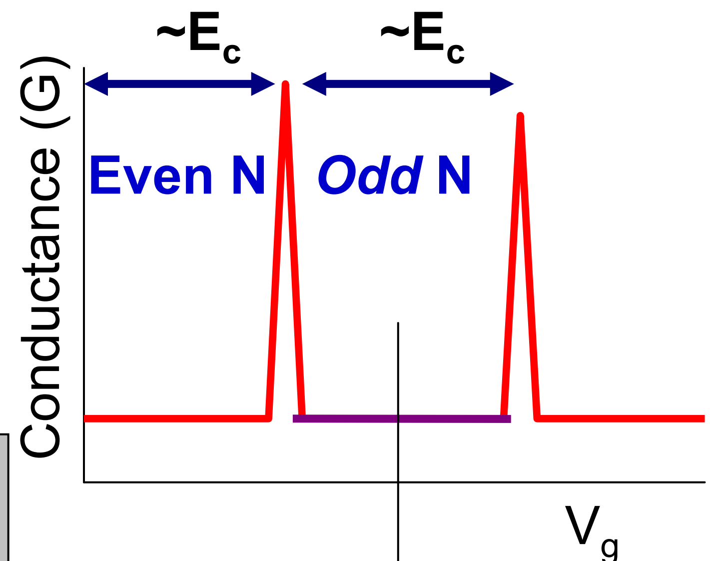
So, what’s new about it?

Kondo signatures in electronic transport observed in many different set ups:

- Quantum dots (experimental control of the parameters)
- STM measurements of magnetic structures on metallic surfaces (e.g., single atoms, molecules. “Quantum mirage”)
- New insights: multi-impurity systems, spin interactions,...

Kondo Effect in Quantum Dots

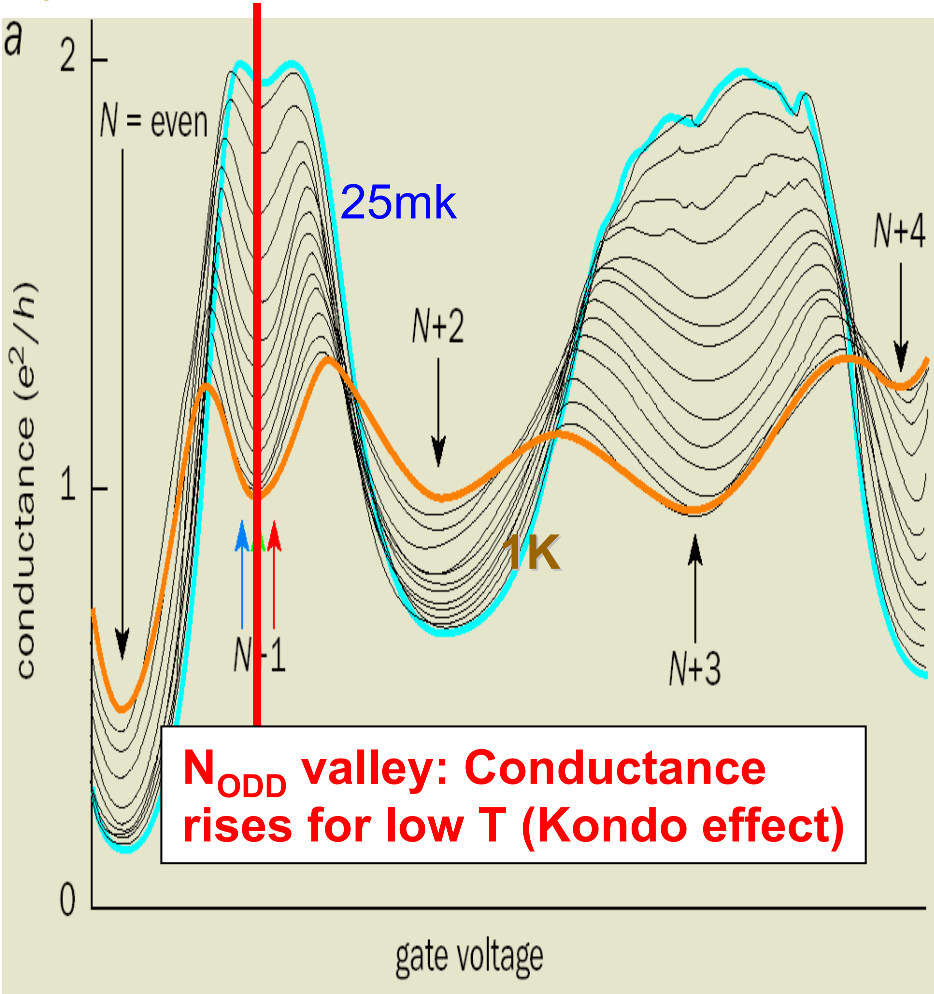


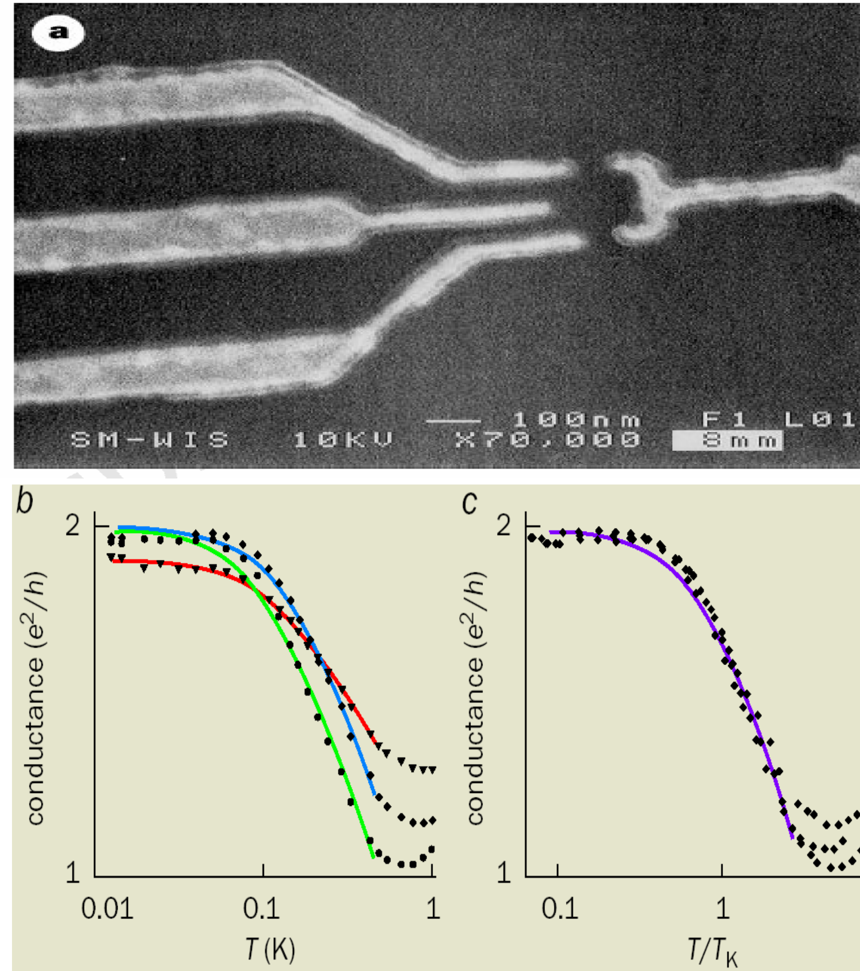


- $T>T_K$: Coulomb blockade (low G)
- $T<T_K$: Kondo singlet formation
- Kondo resonance at E_F (width T_K).
- New conduction channel at E_F:

Zero-bias enhancement of G

Kondo Effect in CB-QDs





Kondo Temperature T_k : only scaling parameter (~0.5K, depends on V_g)

Kowenhoven and Glazman *Physics World* – Jan. 2001.

From: Goldhaber-Gordon *et al. Nature* **391** 156 (1998)

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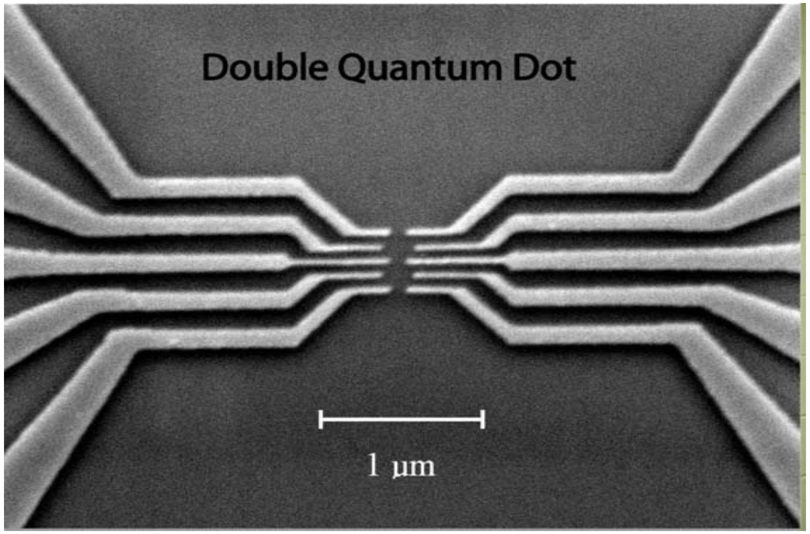
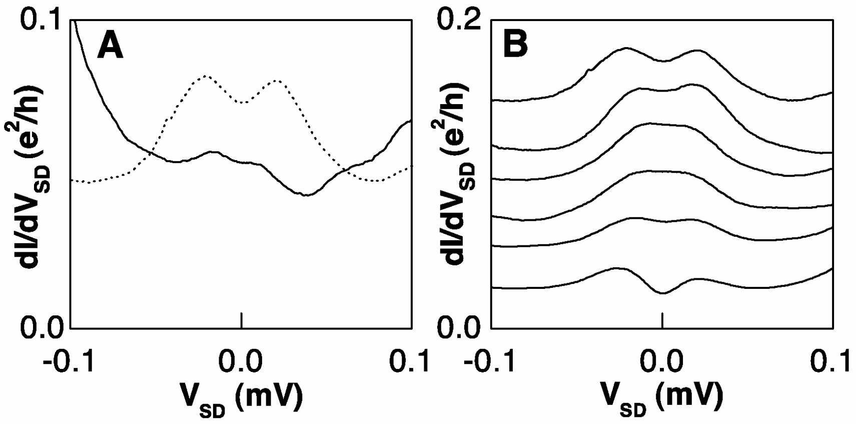
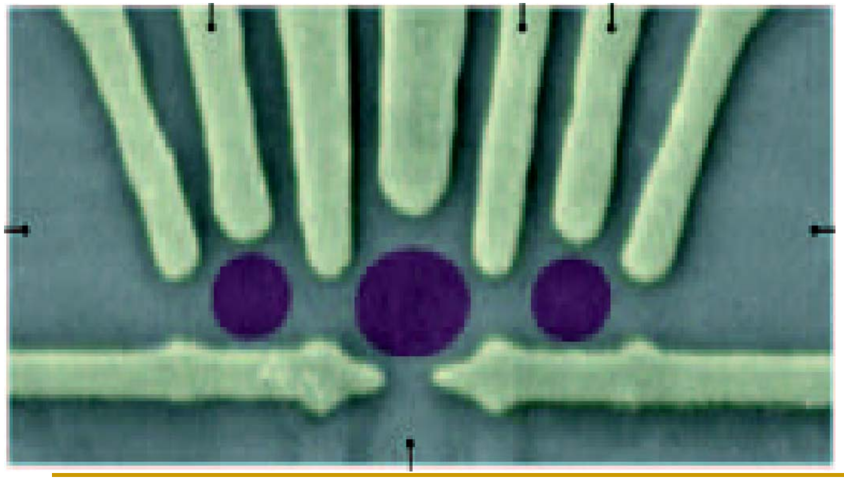
Kondo Effect in *Double* QDs

Series configuration

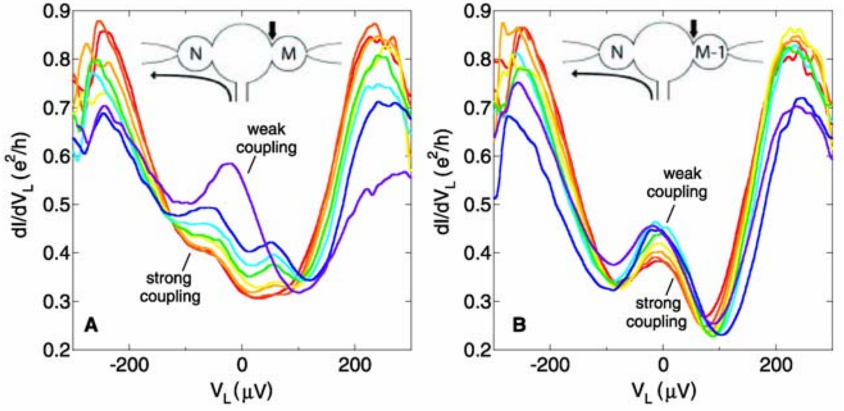


Jeong, Chang, Melloch *Science* **293** 2222 (2001)





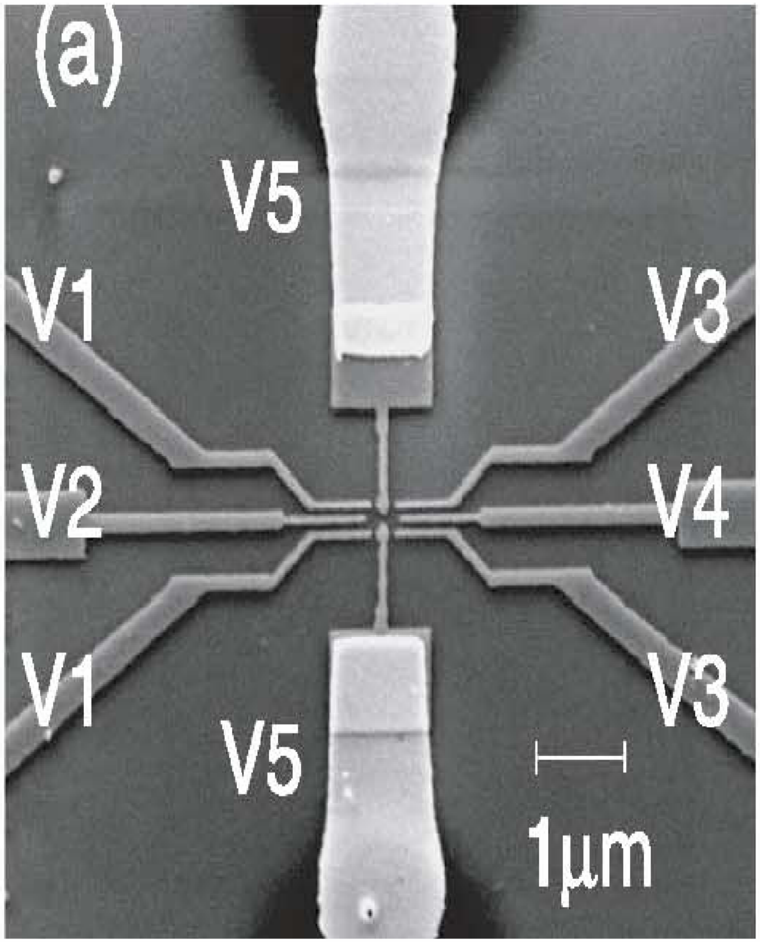
Craig et al., *Science* **304** 565 (2004)

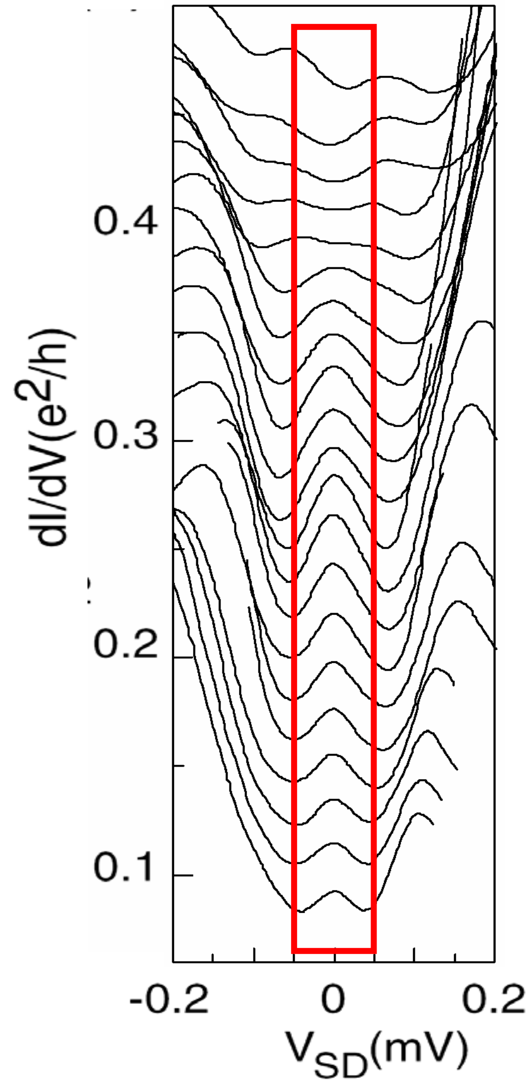


Kondo Effect in *Double* QDs

Parallel configuration

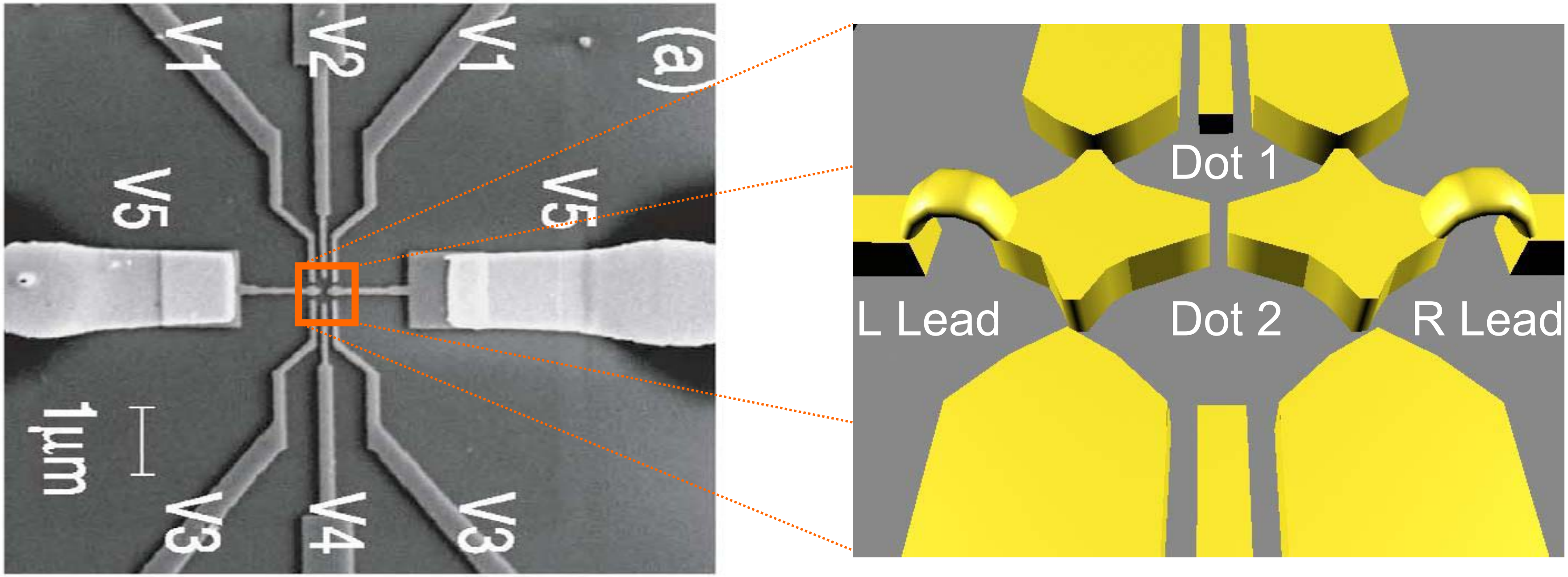
Chen, Chang, Melloch, *PRL* **92** 176801 (2004)





Kondo signal at zero bias

Kondo Effect in *Double* QDs

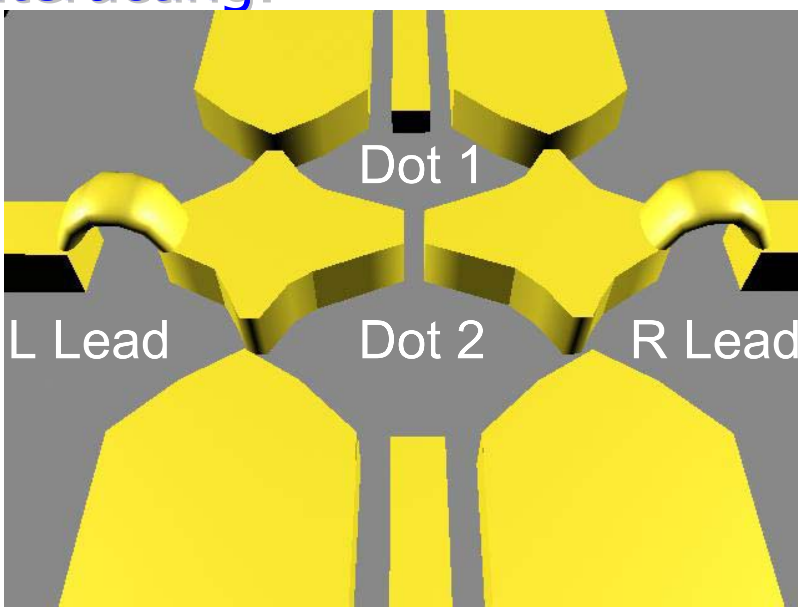


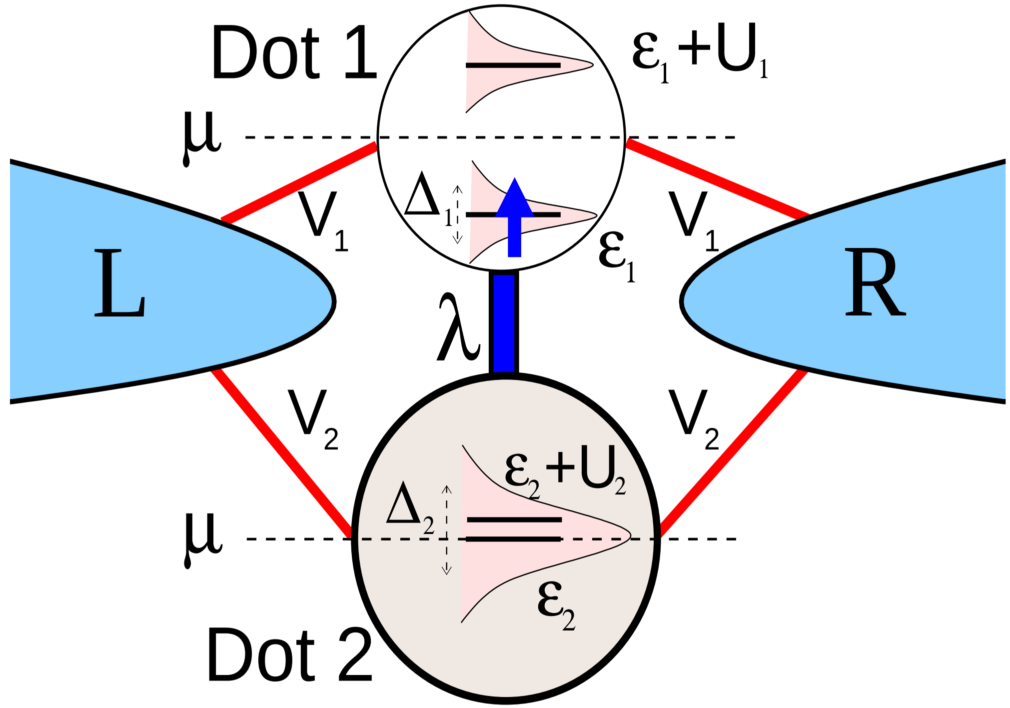
Double Quantum Dots:

- Allow controlled studies of both intradot and interdot correlations
- Interference and phase measurements.
- RKKY interactions
- Quantum phase transitions.
- Prospects in quantum information processing.

DQD theory: different regimes

- Non-identical dots coupled to leads and to each other.
- For $V_{iR}=V_{iL}$; coupling to the symmetric channel only.
- Dot 2: effectively non-interacting.

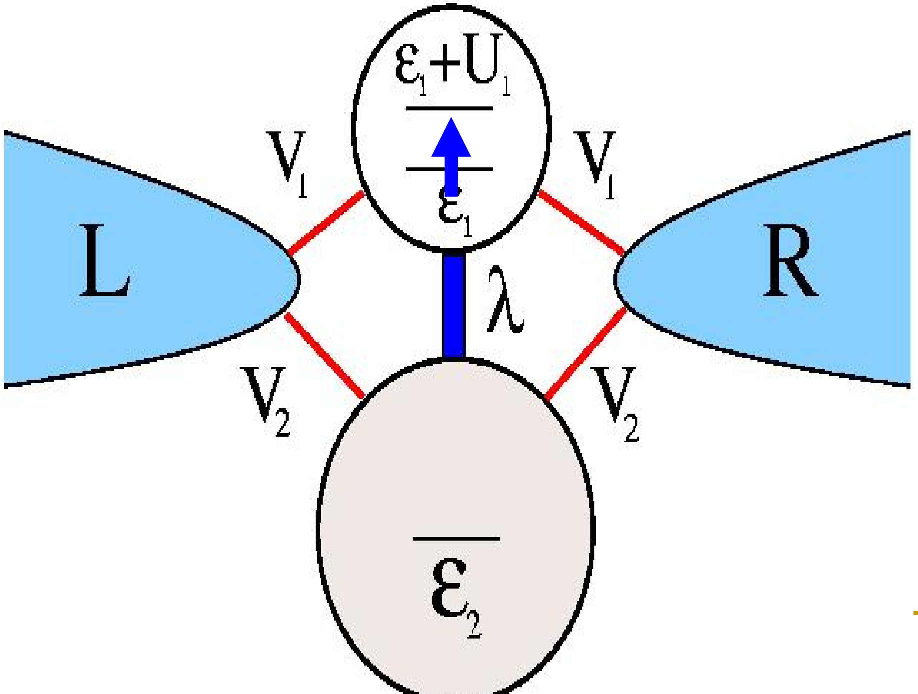


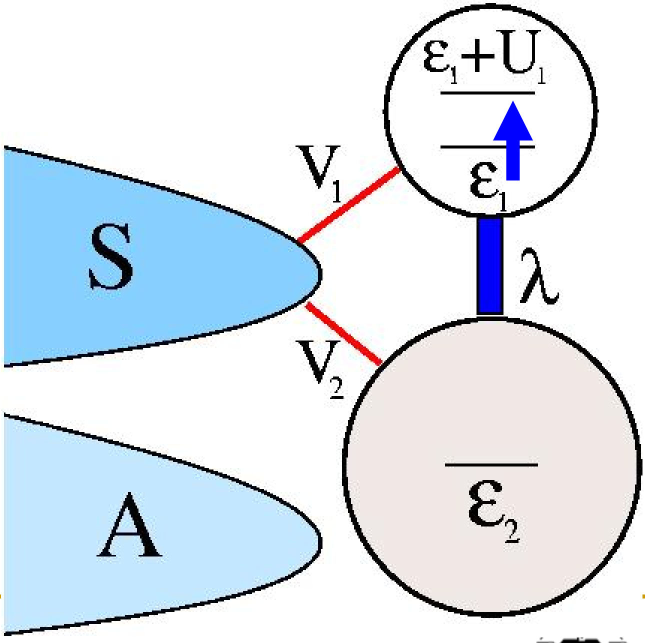


DQD: theoretical description

- Non-identical dots coupled to leads and to each other.
- For $V_{iR}=V_{iL}$; coupling to the symmetric channel only.
- Dot 2: mixed-valence regime.

$$
\begin{aligned}
H_{\text{Leads}} &= \sum_{\mathbf{k},j=L,R} \epsilon_k c^{\dagger}_{j\mathbf{k}\sigma} c_{j\mathbf{k}\sigma} \;, \\
H_{i=1,2} &= \epsilon_i a^{\dagger}_{i\sigma} a_{i\sigma} + U_i n_{i\uparrow} n_{i\downarrow} \\
H_{\text{Dot-Leads}} &= \sum_{\substack{\mathbf{k},j=L,R \\ i=1,2}} V'_{ij} a^{\dagger}_{i\sigma} c_{j\mathbf{k}\sigma} + \text{h.c.} \;, \\
H_{\text{Dot-Dot}} &= \lambda \left(a^{\dagger}_{1\sigma} a_{2\sigma} + \text{h.c.} \right)
\end{aligned}
$$





Green's function for dot 1: **effective** DoS

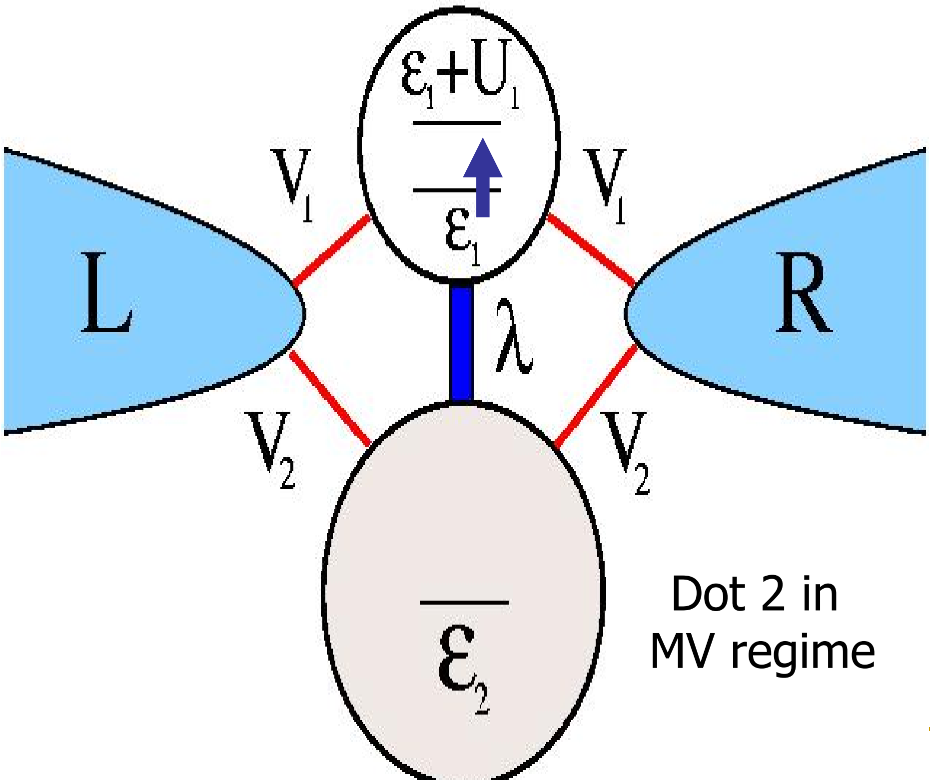
- Constant DoS in the leads

$\rho(\varepsilon) = \rho_0$

- Large band limit ($\varepsilon \ll D$).

$$\left[G_{11}(\epsilon^+)\right]^{-1} = \epsilon^+ - \epsilon_1 - \Sigma^*(\epsilon) - \Lambda(\epsilon) + i\Delta(\epsilon)$$

$$\Delta(\epsilon) = \Delta_1 + \pi\bar{\rho}(\epsilon)\left[\lambda^2 - \Delta_1\Delta_2 + 2\lambda\sqrt{\Delta_1/\Delta_2}(\epsilon - \epsilon_2)^2\right]$$



Dot 2 in MV regime

with

$$\Delta_1 = \pi V_1^2 \rho_0$$

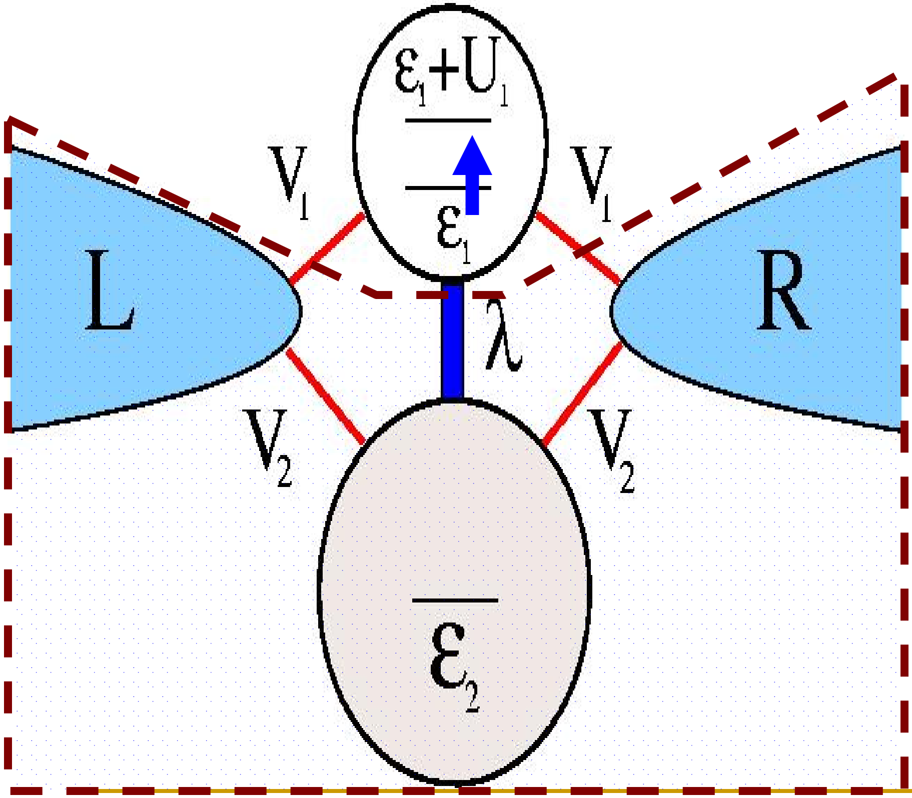
$$\Delta_2 = \pi V_2^2 \rho_0$$

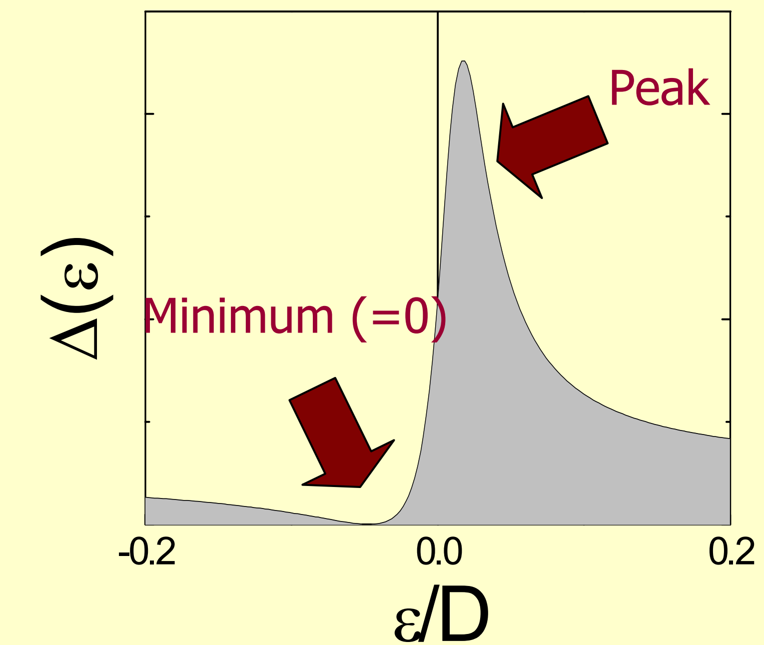
$$\bar{\rho}(\epsilon) = \frac{\Delta_2/\pi}{(\epsilon^+ - \epsilon_2)^2 + \Delta_2^2}$$

Green's function for dot 1: **effective** DoS

$$\left[G_{11}(\epsilon^+)\right]^{-1} = \epsilon^+ - \epsilon_1 - \Sigma^*(\epsilon) - \Lambda(\epsilon) + i\Delta(\epsilon)$$

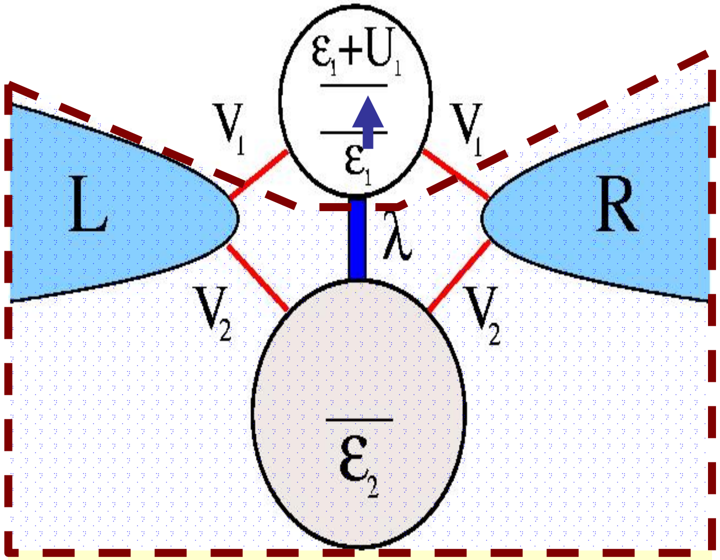
$$\Delta(\epsilon) = \Delta_1 + \pi\bar{\rho}(\epsilon)\left[\lambda^2 - \Delta_1\Delta_2 + 2\lambda\sqrt{\Delta_1/\Delta_2}(\epsilon - \epsilon_2)^2\right]$$

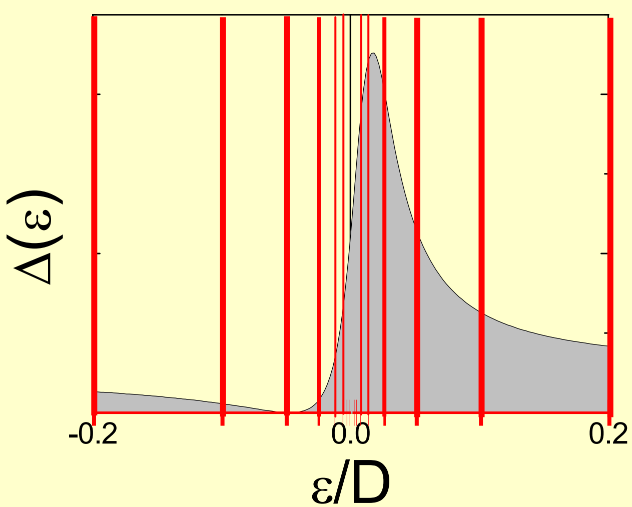


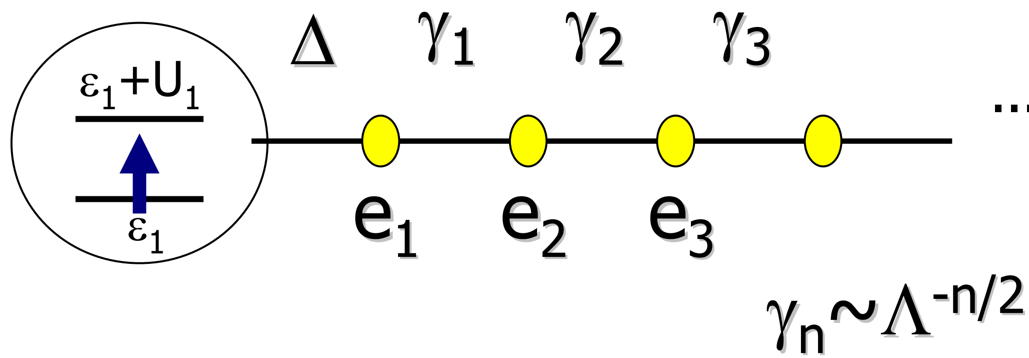


$$\bar{\rho}(\epsilon) = \frac{\Delta_2/\pi}{(\epsilon^+ - \epsilon_2)^2 + \Delta_2^2}$$

Numerical Renormalization Group







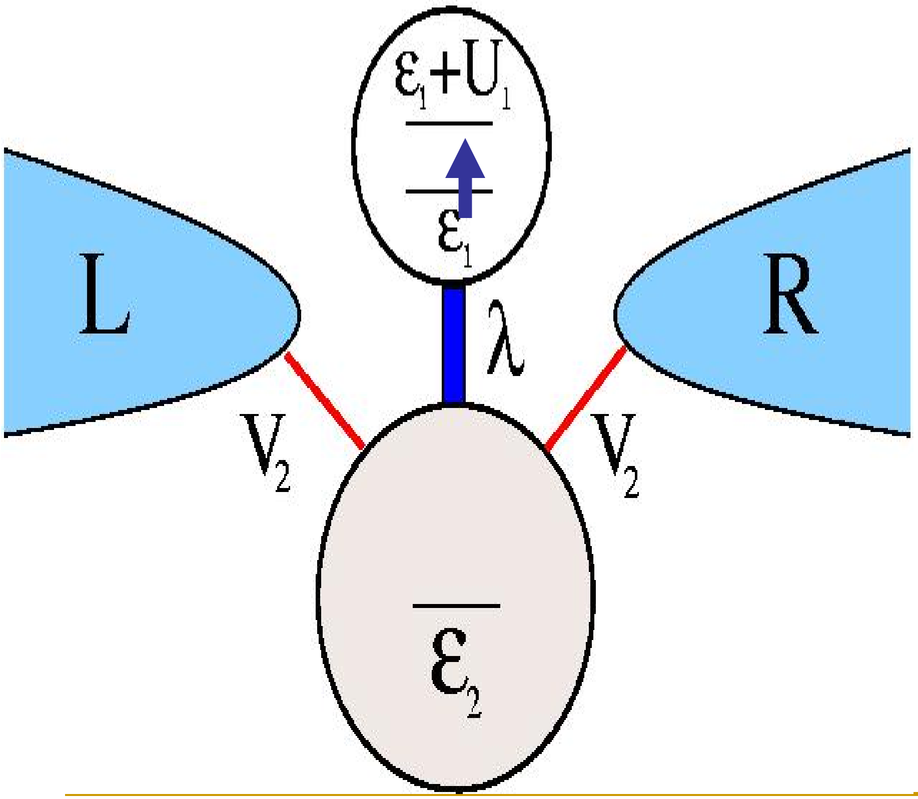
Wilson's NRG method:

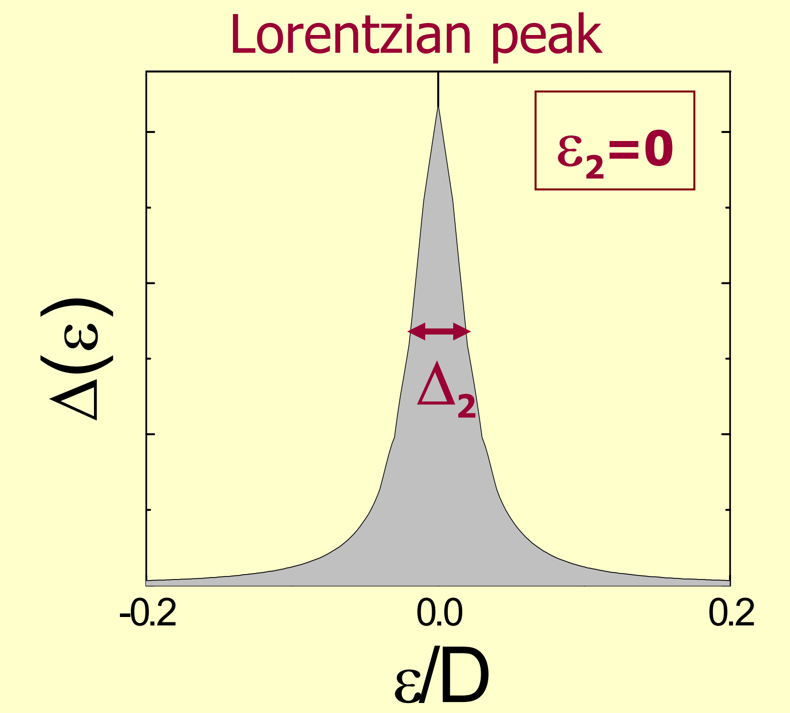
- Logarithmic discretization of the hybridization function: Lanczos method.
- Iterative numerical solution of the many-body Hamiltonian.
- Calculation of the spectral function: $A_{11}=-(1/\pi)\mathrm{Im}[G_{11}]$.

Side Dot: "Filtering" of the effective DoS.

$$\left[G_{11}(\epsilon^+)\right]^{-1} = \epsilon^+ - \epsilon_1 - \Sigma^*(\epsilon) - \Lambda(\epsilon) + i\Delta(\epsilon)$$

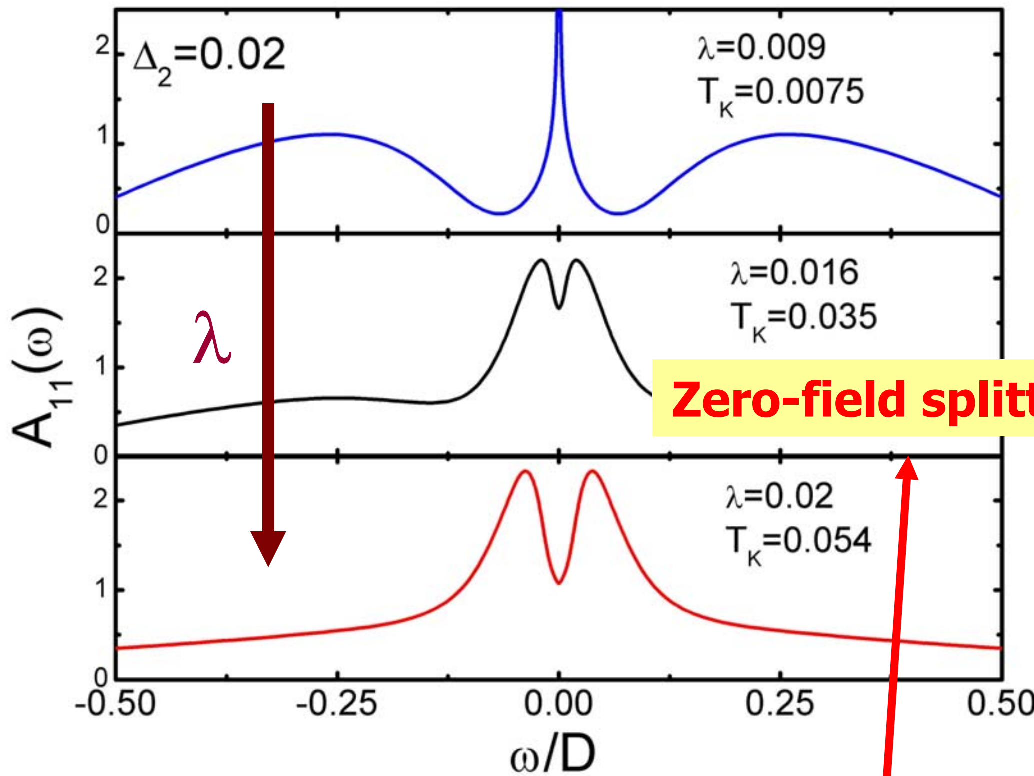
$$\Delta(\epsilon) = \pi\bar{\rho}(\epsilon)\lambda^2$$

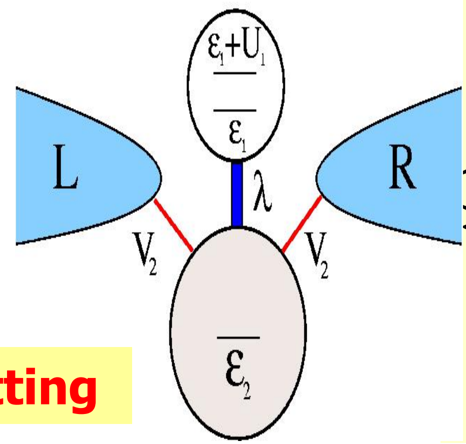


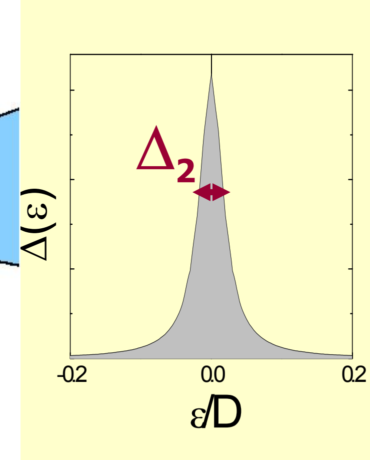


$$\bar{\rho}(\epsilon) = \frac{\Delta_2/\pi}{(\epsilon^+ - \epsilon_2)^2 + \Delta_2^2}$$

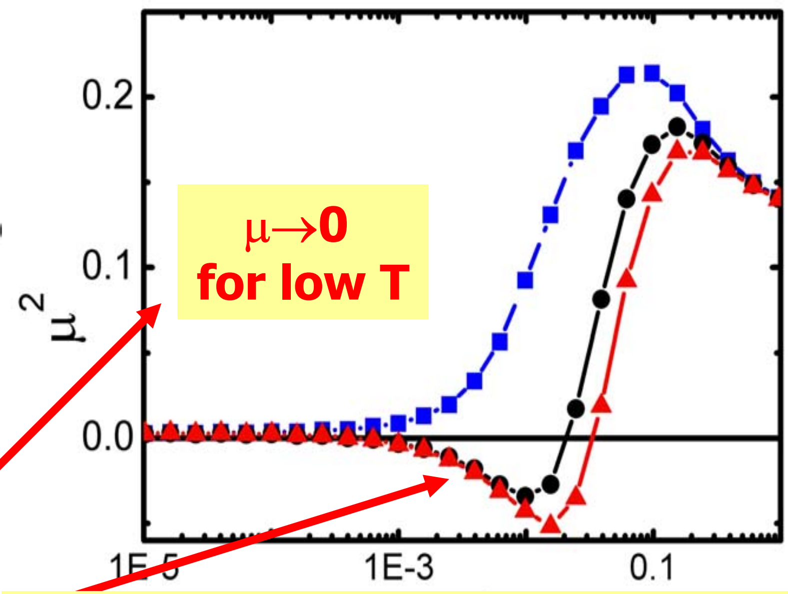
Splitting in the Kondo peak







Zero-field splitting



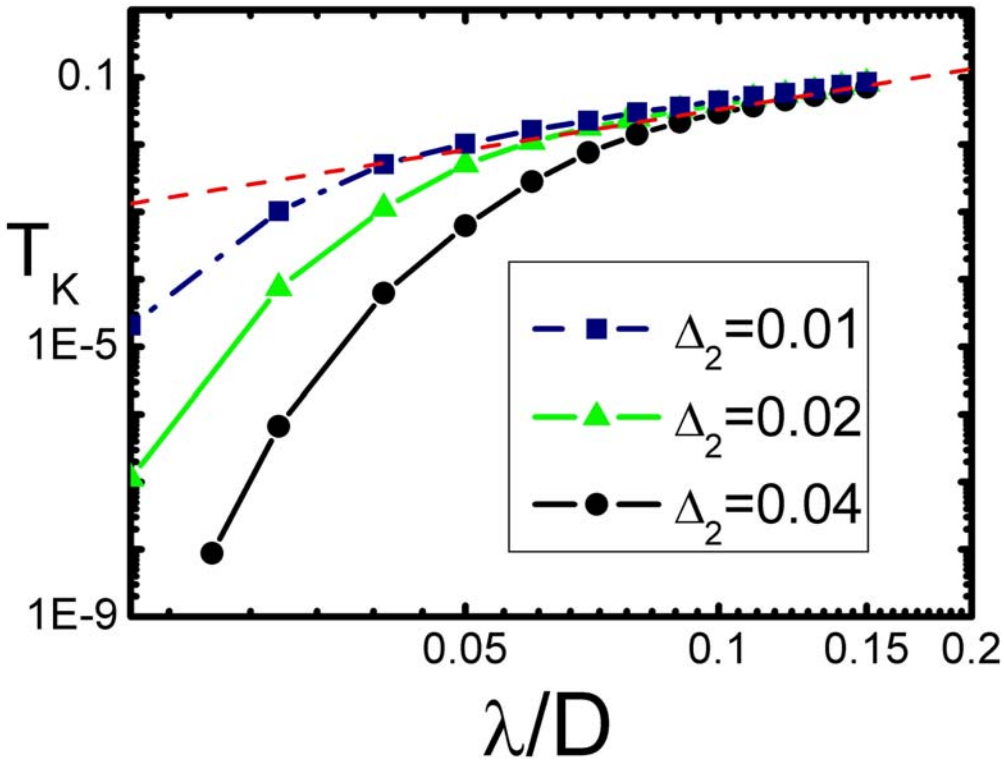
μ→0 for low T

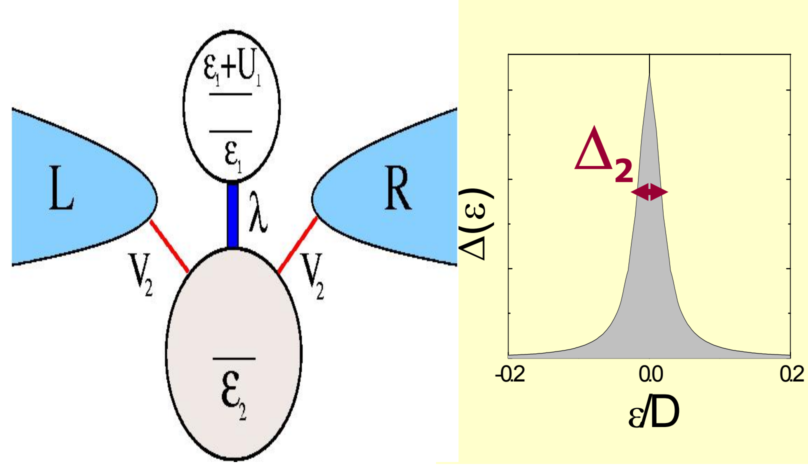
- Resonance in the DoS at $\varepsilon_2=0$:
  - Kondo peak splits for $T_K>\Delta_2$
  - Kondo screening is *preserved.*
  - Diamagnetic phase for $\Delta_2<T<T_K$

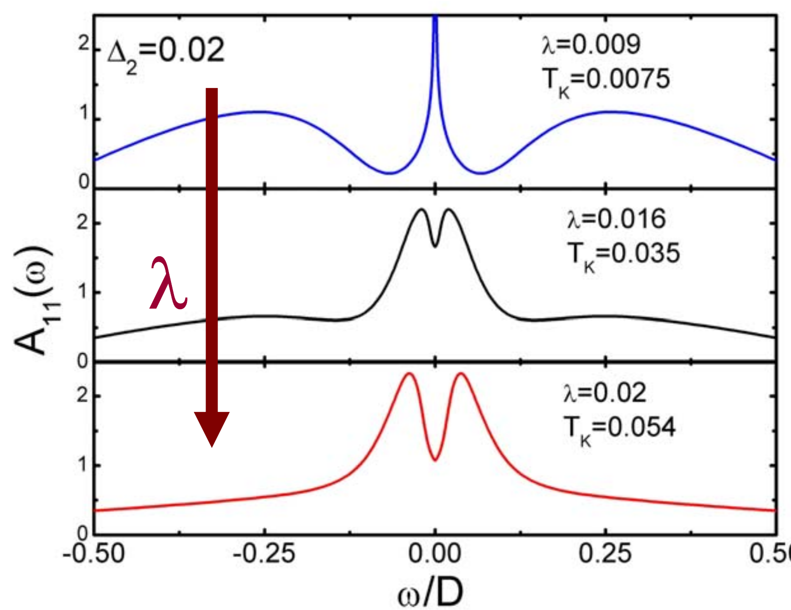
Consistent with narrow-band results:
Hofstetter, Kehrein, *PRB* **59 R**112732 (1999)

LDS *et al. Phys. Rev. Lett.* **97** 096603 (2006)

Side-dot: Splitting the Kondo peak







- Large λ: $T_K \sim \lambda$.
- T_K decreases with Δ_2.
- Splitting ~ T_K .

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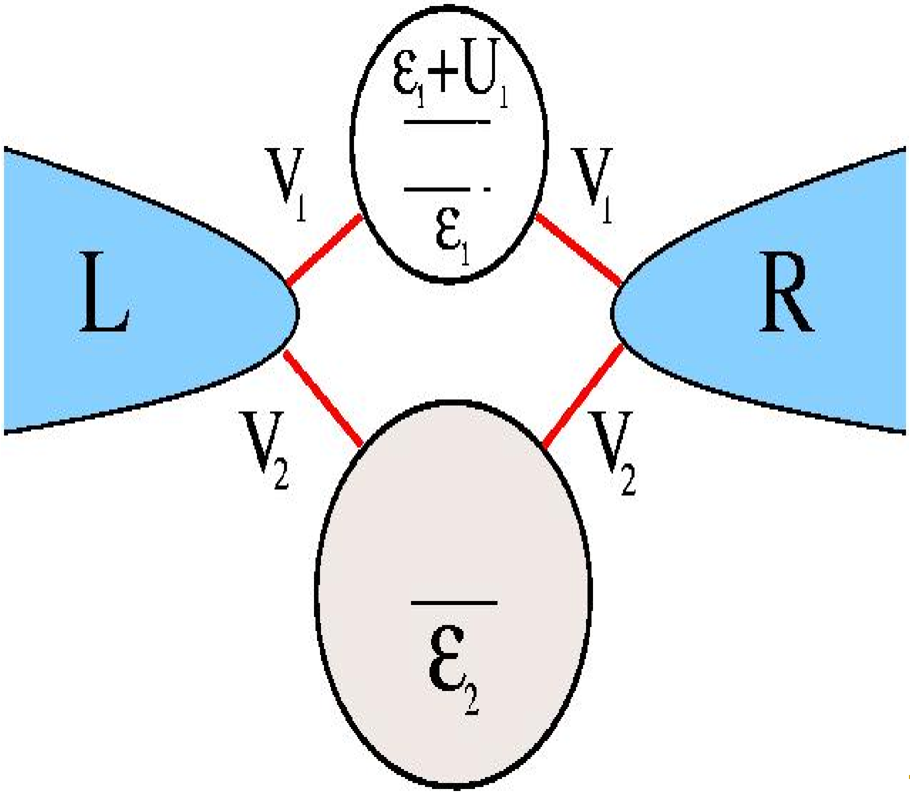
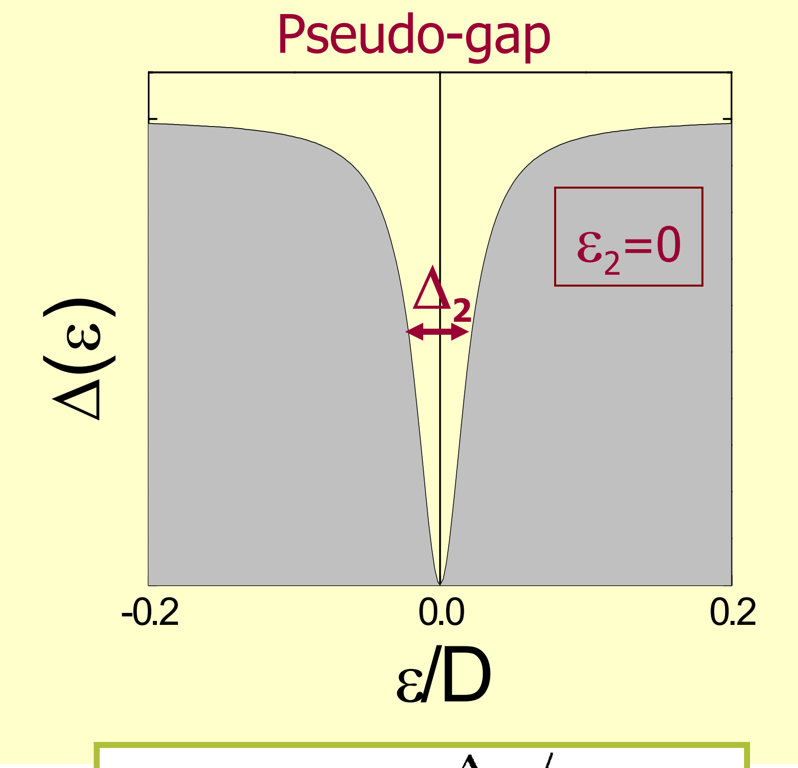
Parallel Dot: Pseudo-Gapped effective DoS

$$[G_{11}(\epsilon^+)]^{-1} = \epsilon^+ - \epsilon_1 - \Sigma^*(\epsilon) - \Lambda(\epsilon) + i\Delta(\epsilon)$$

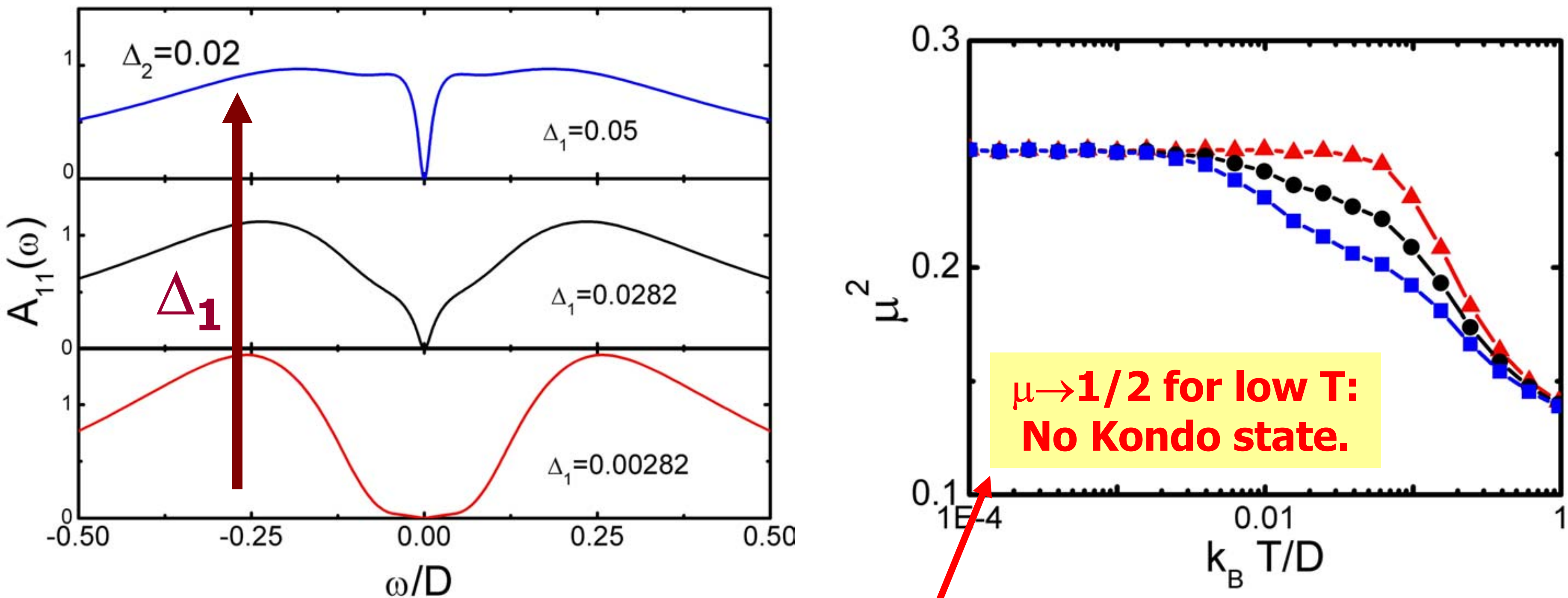


$$\Delta(\epsilon) = \Delta_1 \left[1 - \pi\bar{\rho}(\epsilon)\Delta_2\right]$$

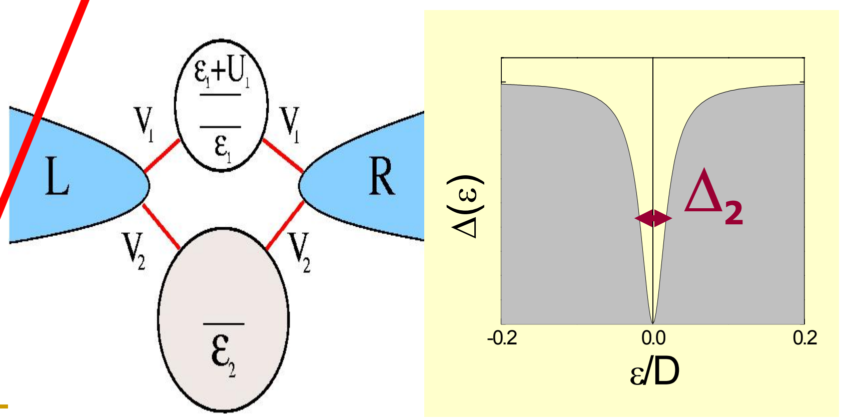


$$\bar{\rho}(\epsilon) = \frac{\Delta_2/\pi}{(\epsilon^+ - \epsilon_2)^2 + \Delta_2^2}$$

Parallel dots: p-h symmetric case



- $\varepsilon_2=0$; Pseudogap: $\Delta(\varepsilon)\sim|\varepsilon-E_F|^2$.
- Particle-hole symmetry ($\varepsilon_1=-U/2$)
  - No Kondo state for any Δ_1, Δ_2.
  - No local moment screening.



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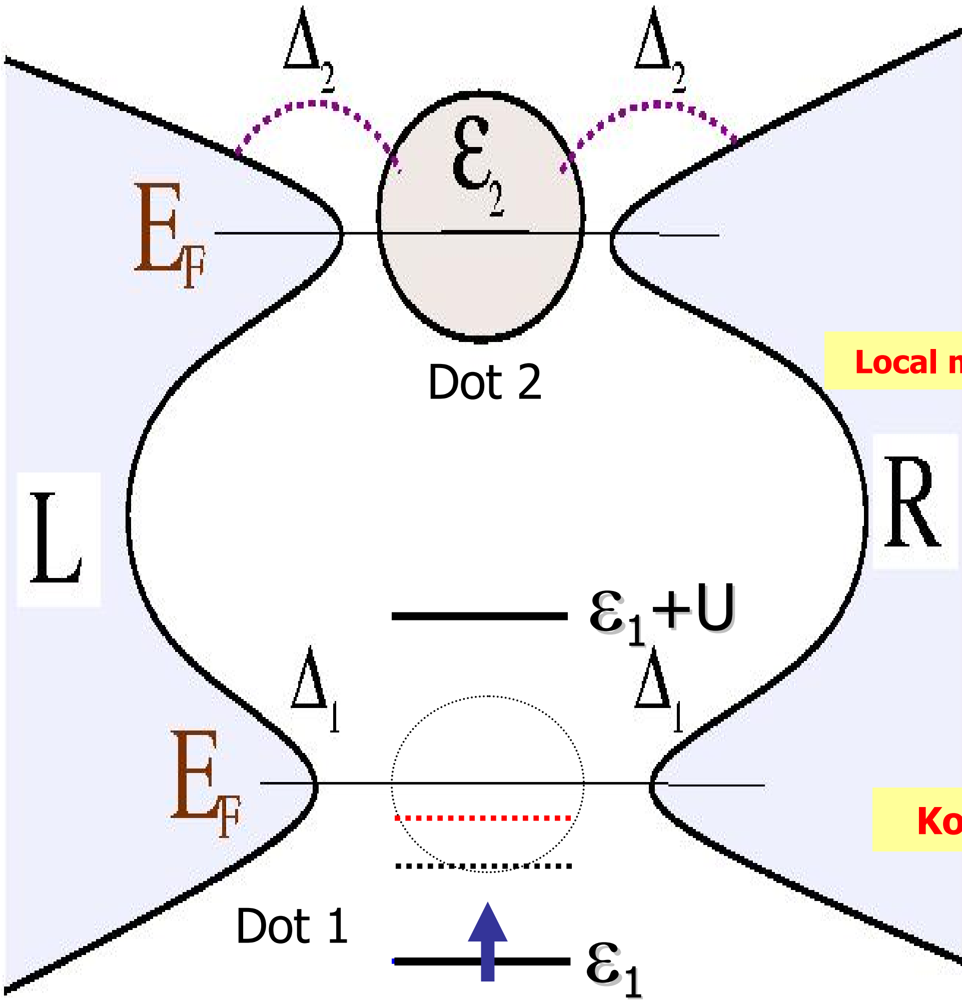
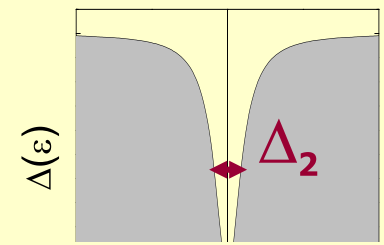
Parallel dots: quantum critical transition

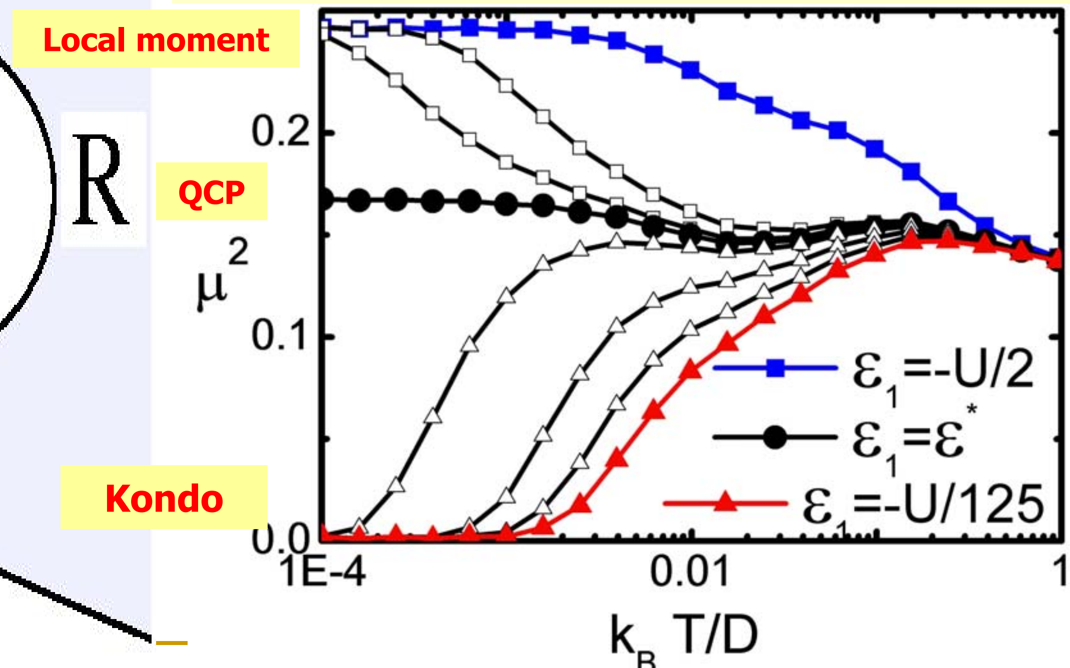


In between:
***Quantum critical transition* at $\varepsilon_1=\varepsilon^*$:**
$\mu^2 \rightarrow 1/6$

Pseudogap



Withoff, Fradkin, *PRL* **64** 1835 (1990).
Gonzales-Buxton, Ingersent, *PRB* **57** 14254 (1998).
Fritz, Vojta, *PRB* **70** 214427 (2004).



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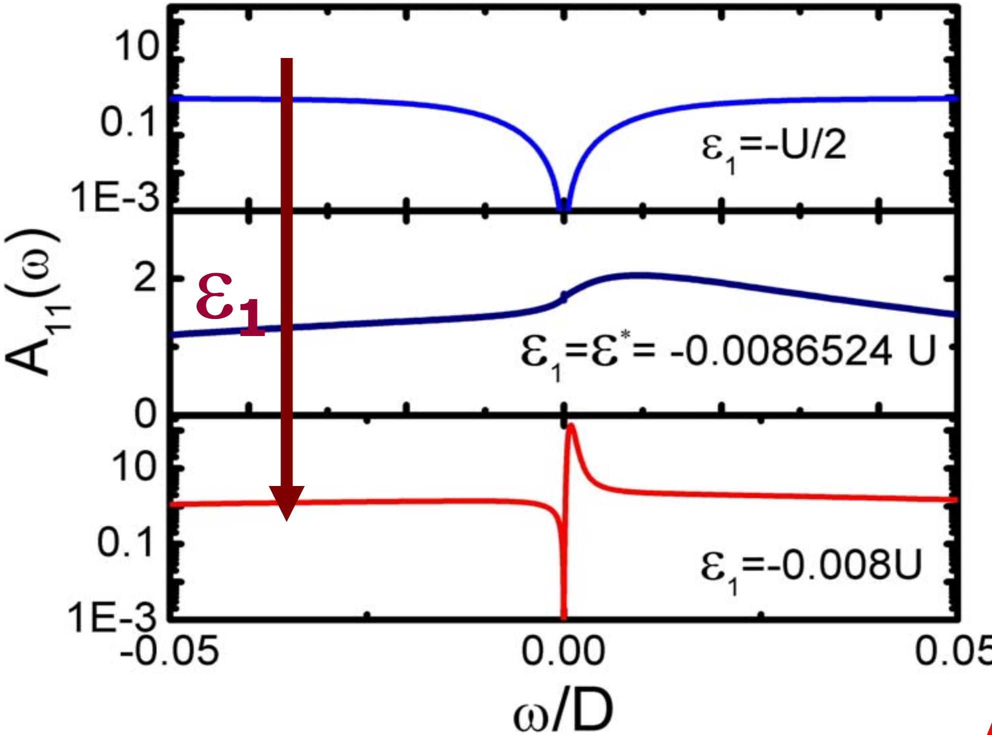
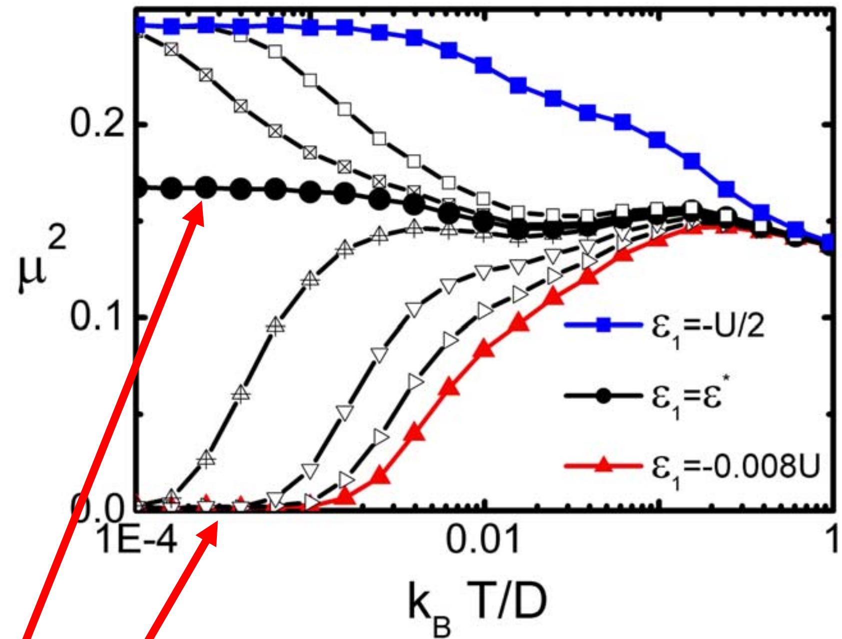
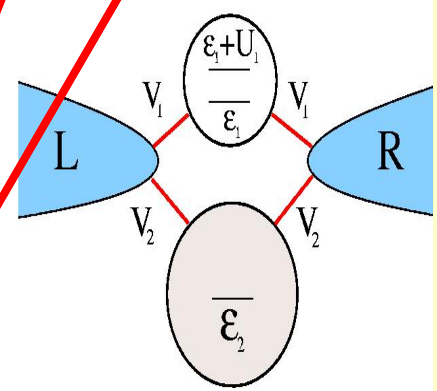
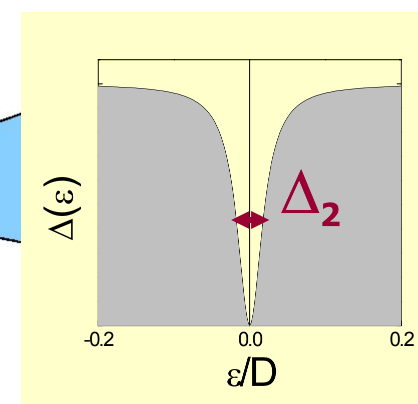
Parallel case: quantum critical transition







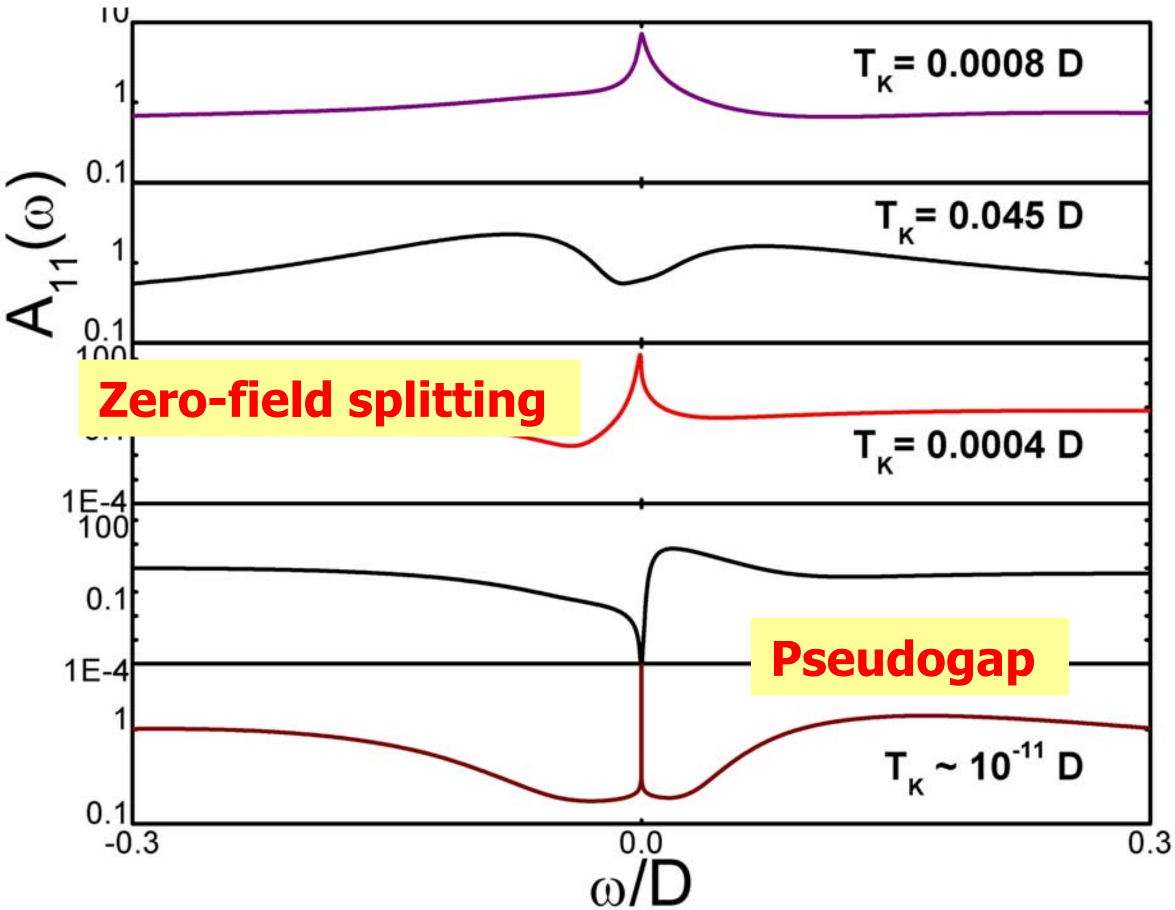


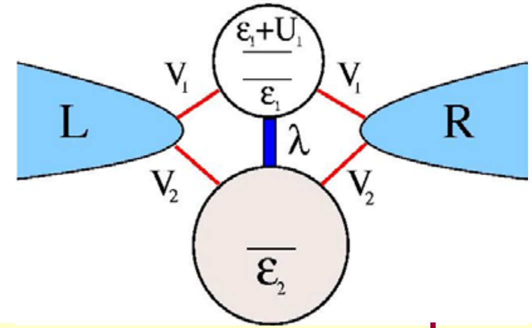
No p-h symmetry ($\varepsilon_1 \neq -U/2$)

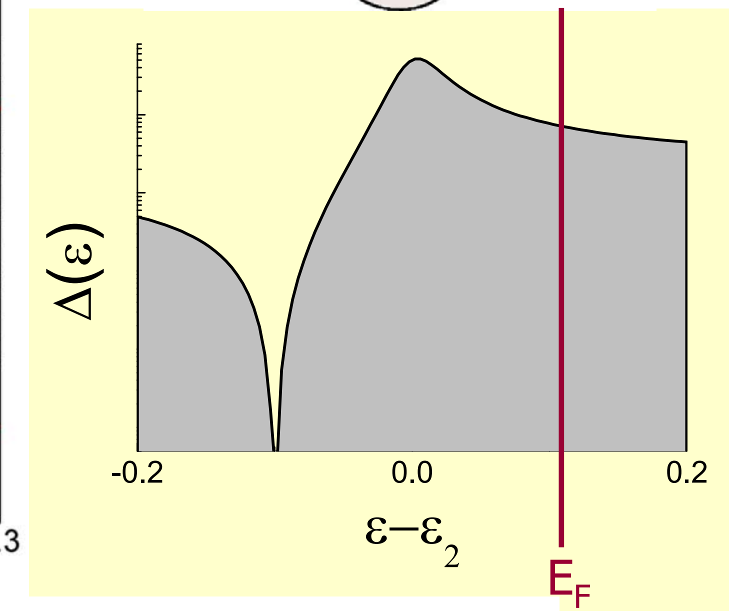
- Critical transition at $\varepsilon_1 = \varepsilon_1^*$.
- Kondo screening for $\varepsilon_1 > \varepsilon_1^*$.

LDS *et al.* *Phys. Rev. Lett.* **97** 096603 (2006)

General case: Kondo splitting+Pseudo-Gap







- By changing the gate voltage in Dot 2 (therefore ε_2), the three regimes can be reached.
- $\Delta(\varepsilon=0)$ determines the Kondo temperature.

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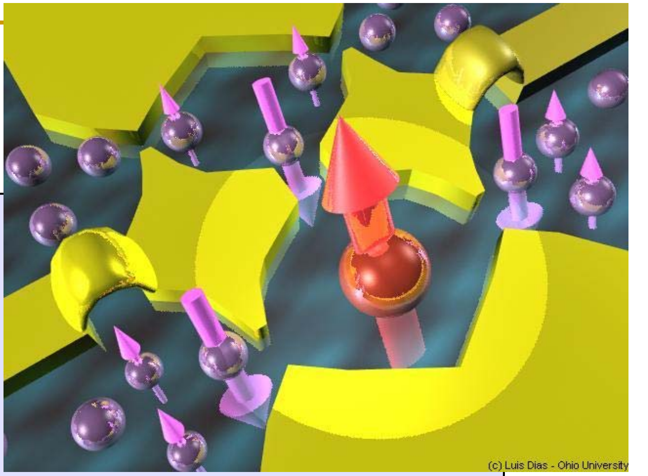
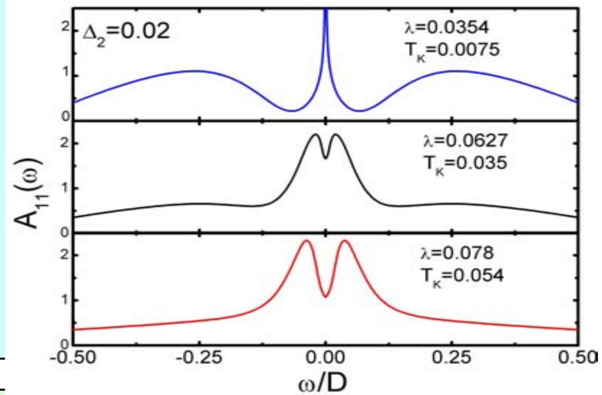
Conclusions

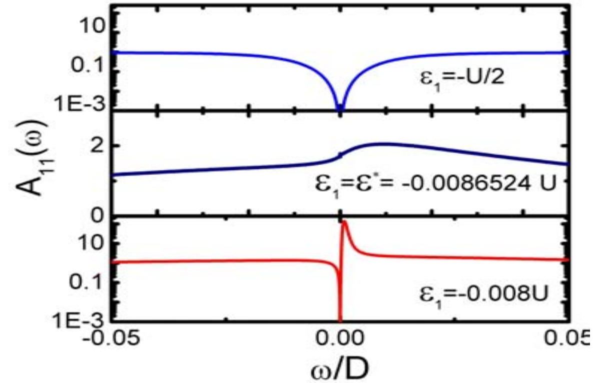


- **DQD systems: experimental realization of a Kondo impurity coupled to an effective DoS with resonances and pseudo-gaps.**

- ***Kondo filtering*: resonances in the effective DoS lead to splittings in the Kondo peak at E_F.**
- **Kondo singlet remains and T_K increases for larger interdot hybridization.**



- ***Quantum critical transition*: Coherent coupling through the leads induces a pseudo-gap in the effective DoS: quantum critical transition to non-Kondo regime.**
- **Different regimes can be achieved by tuning the gate voltage in Dot 2.**



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