Dias da Silva *et al.* Reply: The preceding Comment [1] raises two points of interpretation concerning results of our study of an asymmetric double-quantum-dot device [2]: the zero-field splitting of the Kondo peak cannot be explained using the Friedel sum rule (FSR) formula for the spectral density at the Fermi energy, and this splitting can be reproduced using weakly interacting quasiparticles. We agree with the second point, which reinforces a physical picture put forward in [2]. However, we dispute the first point and stand by our FSR explanation for the Kondo peak splitting.

The zero-field splitting of the Kondo resonance is seen most readily for an interacting ($U_1 > 0$) quantum dot 1 (the "side dot") connected to leads only through a larger, noninteracting dot 2. We note in [2] that evolution of the spectral density $A_{11}(\omega)$ for the side dot from a single peak (for small interdot coupling λ) to a double peak (for larger λ) is closely mimicked by the spectral density $A_{11}^{(0)}(\omega)$ for a *noninteracting* side dot ($U_1 = 0$), provided that the width of the Lorentzian density of states is adjusted to match the Kondo temperature of the interacting problem. Figure 1(a) of [1] provides a nice confirmation of this correspondence.

The two-peak structure in $A_{11}^{(0)}(\omega)$ can be thought of as arising from interference between single-particle resonances on the two dots. This leads to a picture [2] of the peak splitting for $U_1 > 0$ as resulting from interference between a single-particle resonance in the dot-2 density of states and the many-body Kondo resonance centered on dot 1. The latter, of course, can be described in terms of heavy quasiparticles, as shown explicitly in [1].

One can also view the zero-field splitting of the Kondo peak as a nontrivial consequence of the introduction of an additional energy scale into the effective hybdridization $\Delta(\omega)$ of the single-impurity Anderson model. In the side-dot example, this scale is the width Δ_2 of the resonance on dot 2. At frequencies $|\omega| \gg \Delta_2$ the impurity "sees" a low effective hybridization and A_{11} rises towards a correspondingly high Fermi-level value, while at frequencies $|\omega| \ll \Delta_2$ the impurity sees a higher effective hybridization and the spectral density must dip to fulfill the FSR requirement, $A_{11}(0) = \cos^2\varphi/[\pi\Delta(0)]$.

The principal point made in [1] is that for the parameter set shown in Fig. 2 of [2], particle-hole symmetry ensures that the phase factor entering the FSR is $\varphi = 0$. This observation in no way invalidates our explanation for the Kondo peak splitting, which—contrary to the claim in [1]—is not based on $\cos^2\varphi$ being less than unity. Rather, the key is that $1/[\pi\Delta(0)]$ provides an *upper bound* on $A_{11}(0)$ [2]. In any situation where the low effective hybridization at high energies causes $A_{11}(\omega)$ on either side of the Fermi level to lie above the bound set by $\Delta(0)$, the Kondo peak must split to satisfy the FSR.

The FSR explanation also applies away from particle-hole symmetry. Figure 1 shows φ vs the dot-2 level energy ϵ_2 with all other parameters as in Fig. 2(b) of [2]. $A_{11} = -\pi^{-1}\mathrm{Im}G_{11}$, computed as in [2], was used to obtain $\mathrm{Re}G_{11}$ and hence φ. In all cases, $A_{11}(0)$ obeys the FSR to within numerical error. The Kondo resonance (insets to Fig. 1) still shows two clear peaks for $\epsilon_2 = 0.01D$, but the smaller peak disappears for $|\epsilon_2| \gtrsim 0.03D$ (not shown), in which range $A_{11}(\omega)$ rises above the FSR value on only one side of the Fermi level.

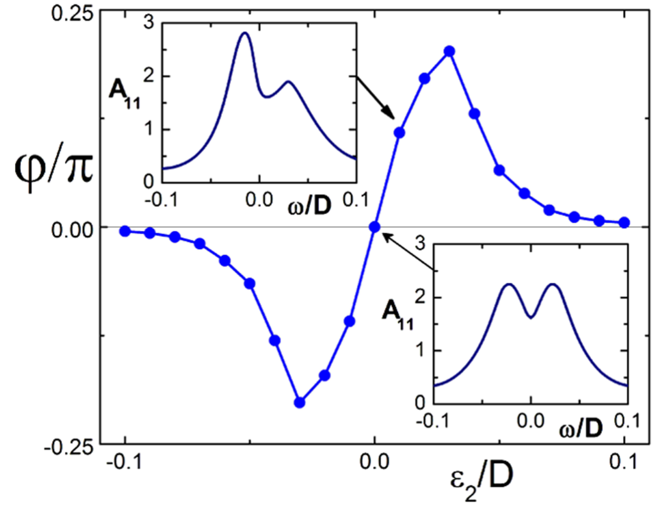

FIG. 1 (color online). FSR phase φ vs ϵ_2 for a side-dot device. See text for details. Insets: $A_{11}(\omega)$ for $\epsilon_2 = 0$ and $0.01D$.

Finally, we note that for $\Delta(\omega)$ dipping sharply at low frequencies (instead of rising), the FSR predicts a narrow *upturn* superimposed on a broader Kondo resonance. This prediction is borne out in numerical results [3].

This work was partially supported by NSF DMR-0312939 (K. I.) and NSF-IMC/NIRT (L. D. S., N. S., S. U.).

Luis G. G. V. Dias da Silva,[1] Nancy P. Sandler,[1]
Kevin Ingersent,[2] and Sergio E. Ulloa[1]
[1]Department of Physics and Astronomy
Nanoscale and Quantum Phenomena Institute
Ohio University
Athens, Ohio 45701-2979, USA
[2]Department of Physics
University of Florida
P.O. Box 118440
Gainesville, Florida, 32611-8440, USA

Received 17 May 2007; published 14 November 2007
DOI: 10.1103/PhysRevLett.99.209702
PACS numbers: 72.15.Qm, 73.23.−b, 73.63.Kv

[1] L. Vaugier, A. A. Aligia, and A. M. Lobos, preceding Comment, Phys. Rev. Lett. **99**, 209701 (2007).
[2] L. G. G. V. Dias da Silva, N. P. Sandler, K. Ingersent, and S. E. Ulloa, Phys. Rev. Lett. **97**, 096603 (2006).
[3] L. G. G. V. Dias da Silva, N. P. Sandler, K. Ingersent, and S. E. Ulloa (to be published).

