

Minicourse contents:

Lecture 1:

Lecture 1 (pdf) available at <http://www.fmt.if.usp.br/~luisdias>

See link on Twitter: [@ProfLuisDias](#)

- Intro: “More is Different”.
- The Kondo effect: a true “More is Different” phenomenon.
- Wilson’s numerical renormalization group method.

Lecture 2:

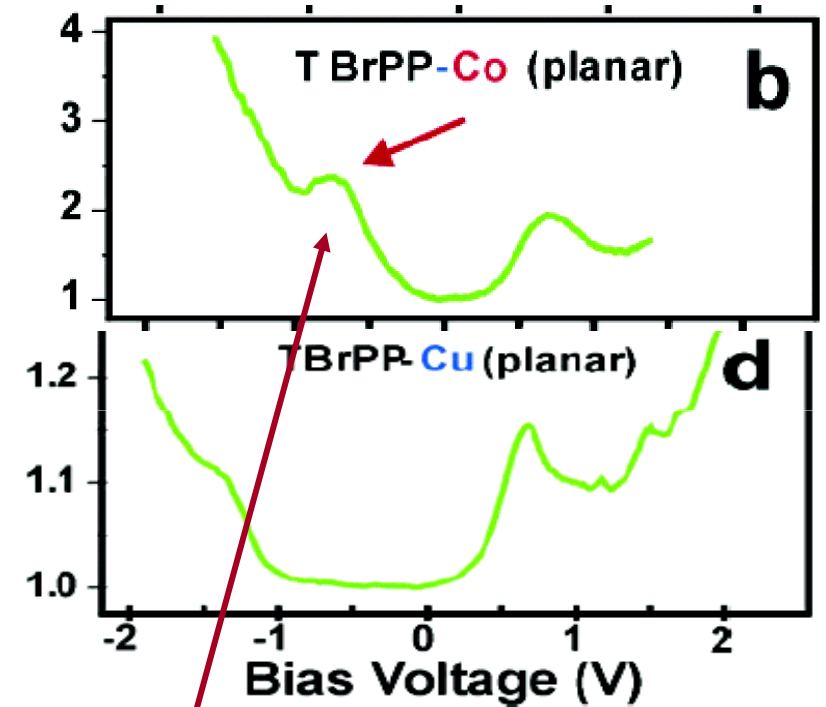
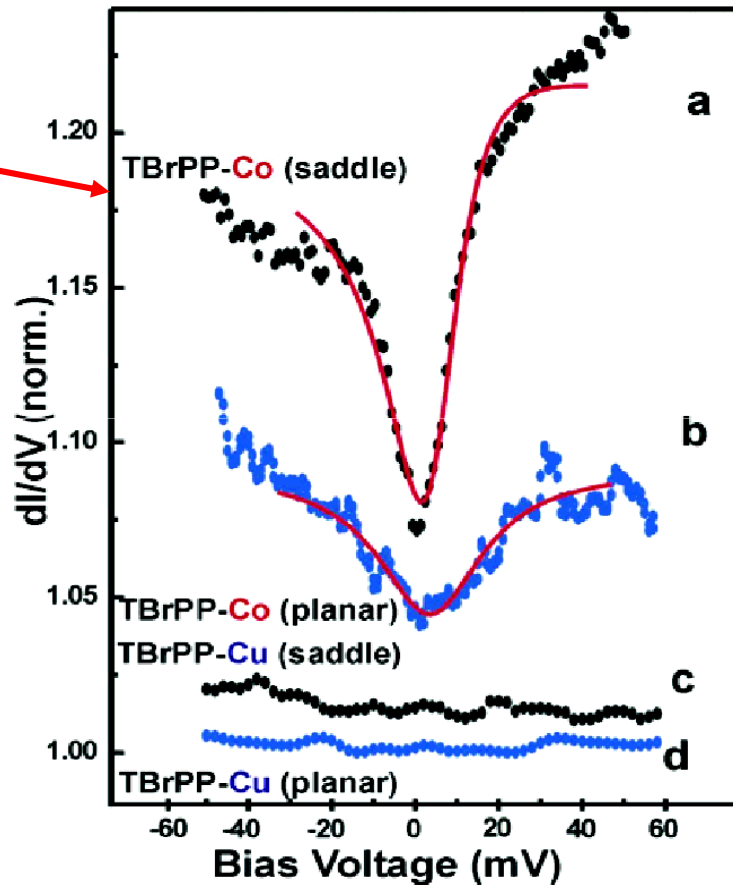
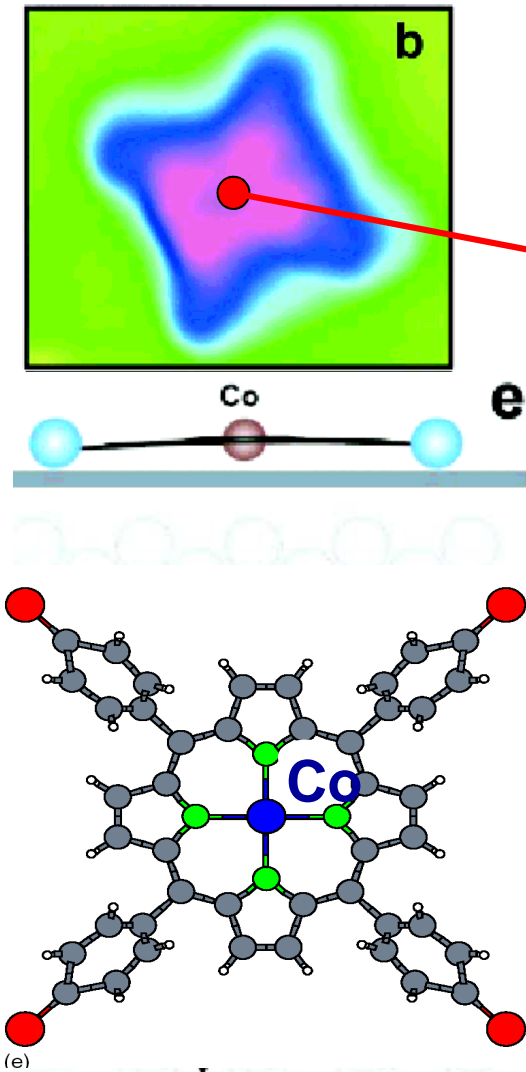
- Applications I: Magnetic molecules on surfaces.
- Applications II: Vacancies in graphene.

Application 1:
Magnetic molecules on surfaces
Combining NRG with Ab-initio methods

Kondo: Magnetic molecules on surfaces

STM measurements

V. Iancu, A. Deshpande, Saw W. Hla
Nano Lett. **6** 820 (2006)



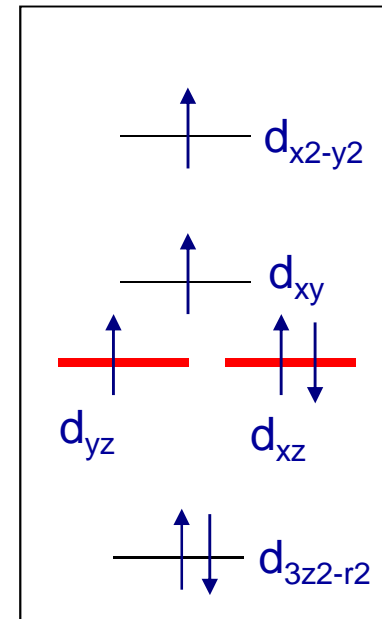
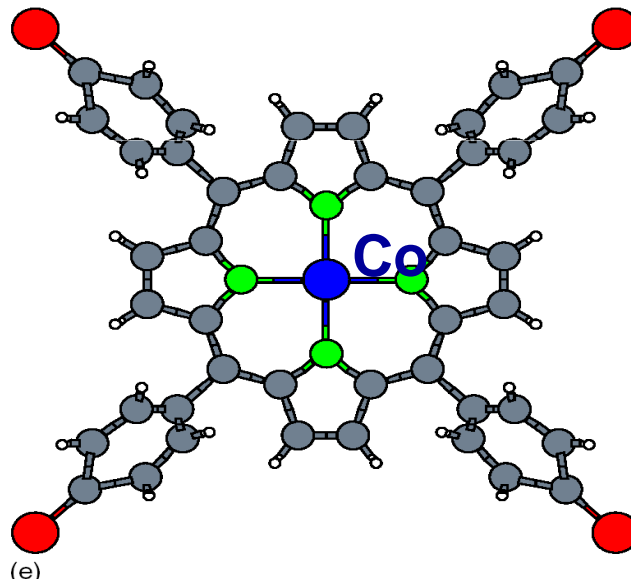
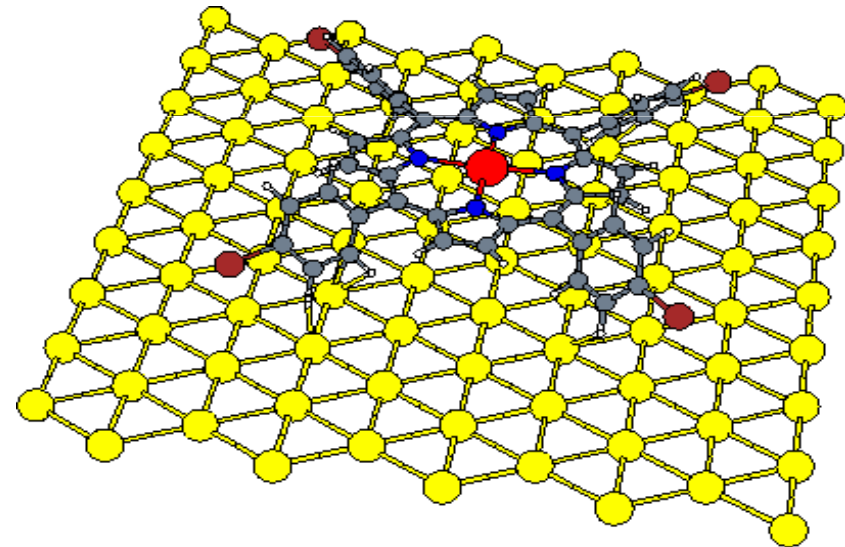
Co d_{3z^2-1} level (at ~ -0.7 eV)

Zero-bias dip: Kondo effect. $T_K \sim 130-170$ K

Cobalt Porphyrin molecule on a surface.

- Molecule: TBrPP-Co ($\text{CoBr}_4\text{N}_4\text{C}_{44}\text{H}_{24}$)
- Which microscopic model describes it?
- Naively one might expect $S=3/2$.

Co atomic configuration: $3d^7$
+ D_{4h} crystalline field:

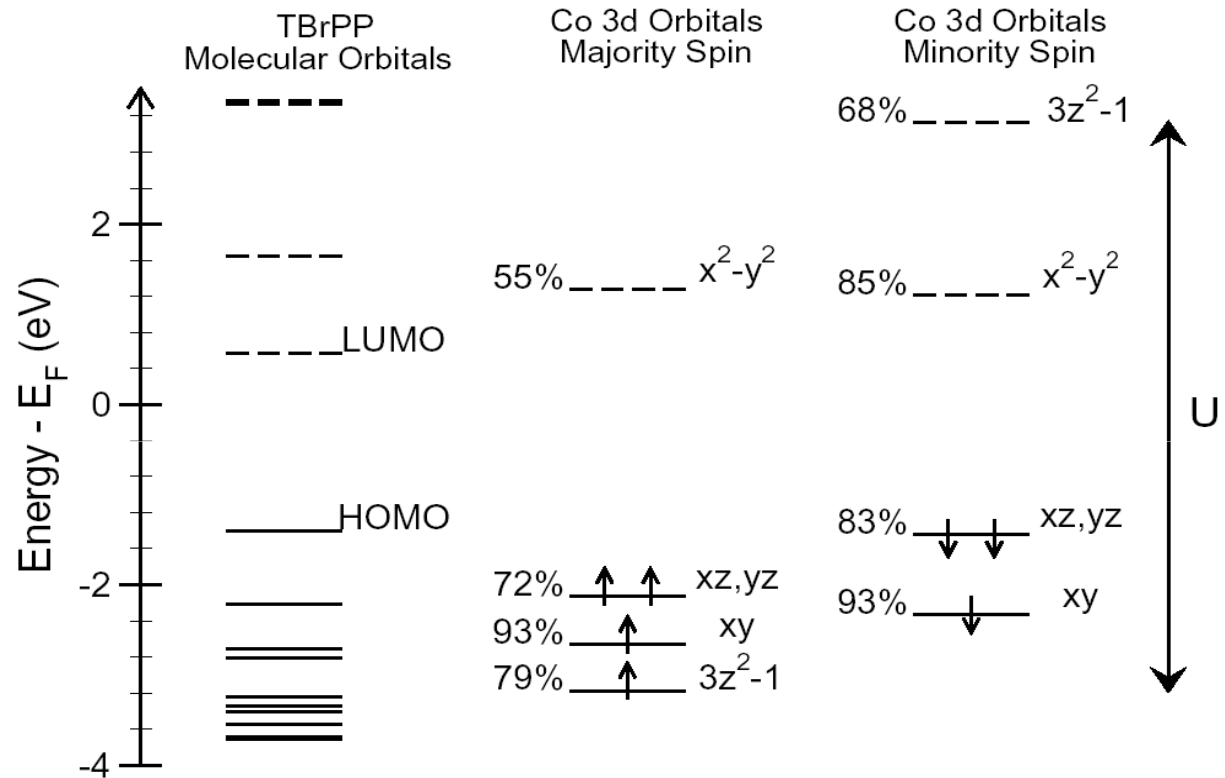
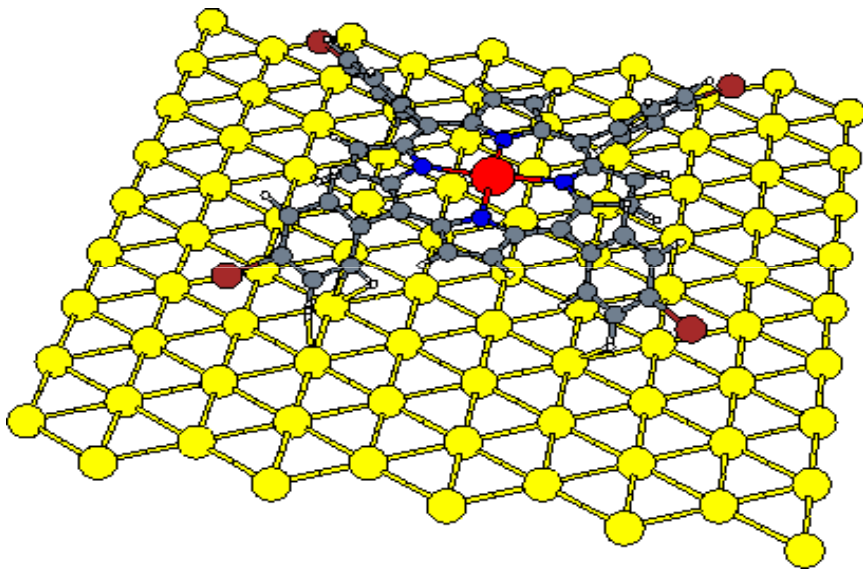


?

Is the simple “Hund’s rule filling” picture correct?

First-principles calculations (GW): hints for a microscopic model.

The answer: not quite!

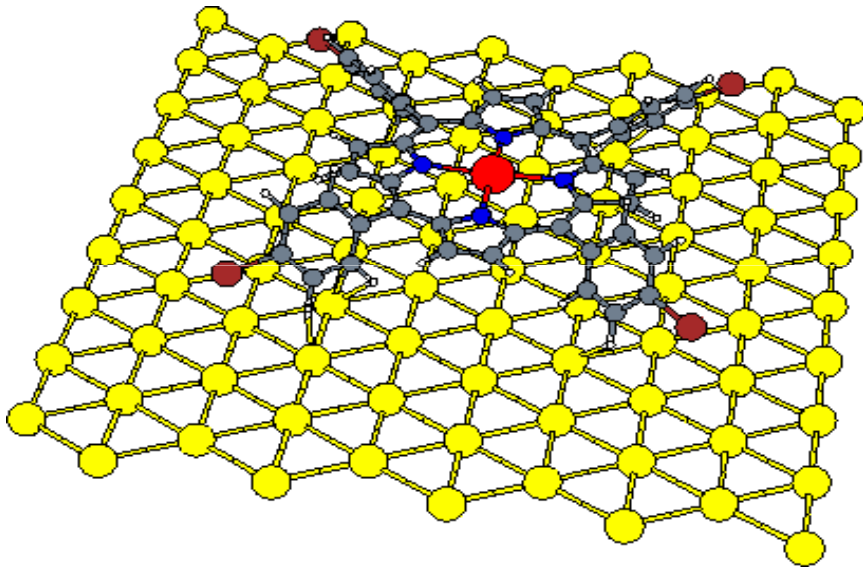


GW: Molecular levels "Co-like" levels

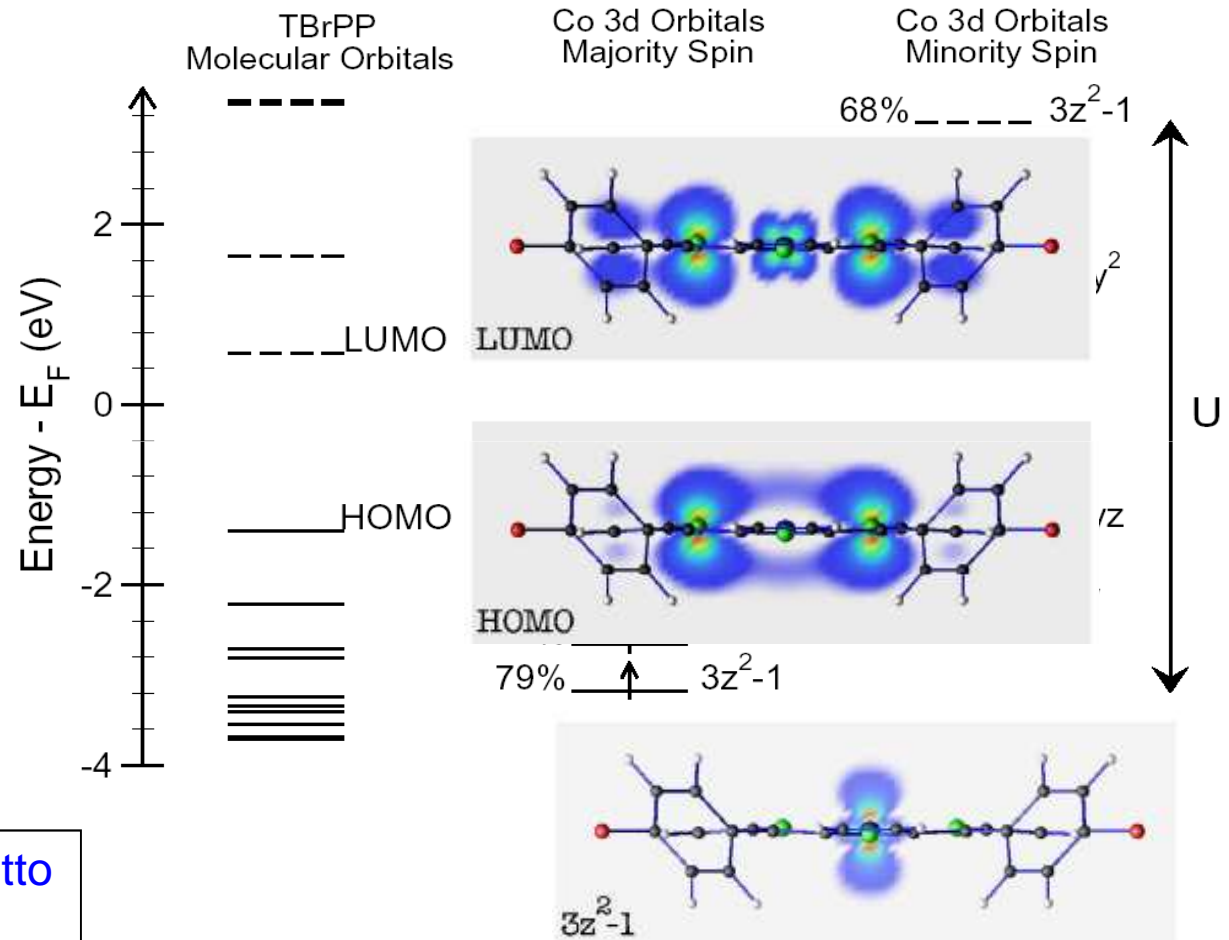
LDS, M Tiago, S Ulloa, F Reboredo E. Dagotto
PRB **80** 155443 (2009).

First-principles calculations (GW): hints for a microscopic model.

The answer: not quite!



LDS, M Tiago, S Ulloa, F Reboredo E. Dagotto
 PRB 80 155443 (2009).



First-principles calculations (GW): hints for a microscopic model.

Model: Anderson-like Hamiltonian

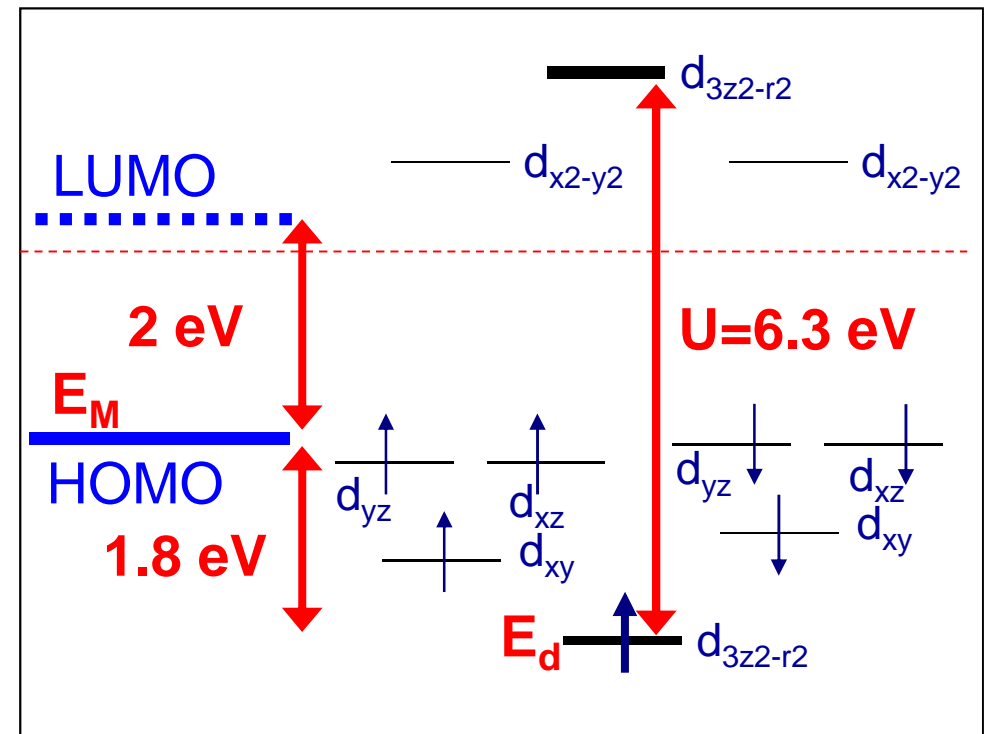
$$H = H_{\text{Molecule}} + H_{\text{Mol-Surface}}$$

$$H_{\text{Molecule}} = \sum_{\sigma} E_d \hat{n}_{d\sigma} + U \hat{n}_{d\uparrow} \hat{n}_{d\downarrow} + \sum_{\sigma} E_M \hat{n}_{M\sigma}$$

$$H_{\text{Mol-Surf}} = \sum_{\mathbf{k}, \sigma} V_{d\mathbf{k}} c_{d\sigma}^{\dagger} c_{\mathbf{k}\sigma} + \sum_{M\mathbf{k}, \sigma} V_{M\mathbf{k}} c_{M\sigma}^{\dagger} c_{\mathbf{k}\sigma} + \text{h.c.}$$

$$V_{d(M)\mathbf{k}} = \langle \phi_{d(M)} | \hat{H} | \psi_{\mathbf{k}} \rangle$$

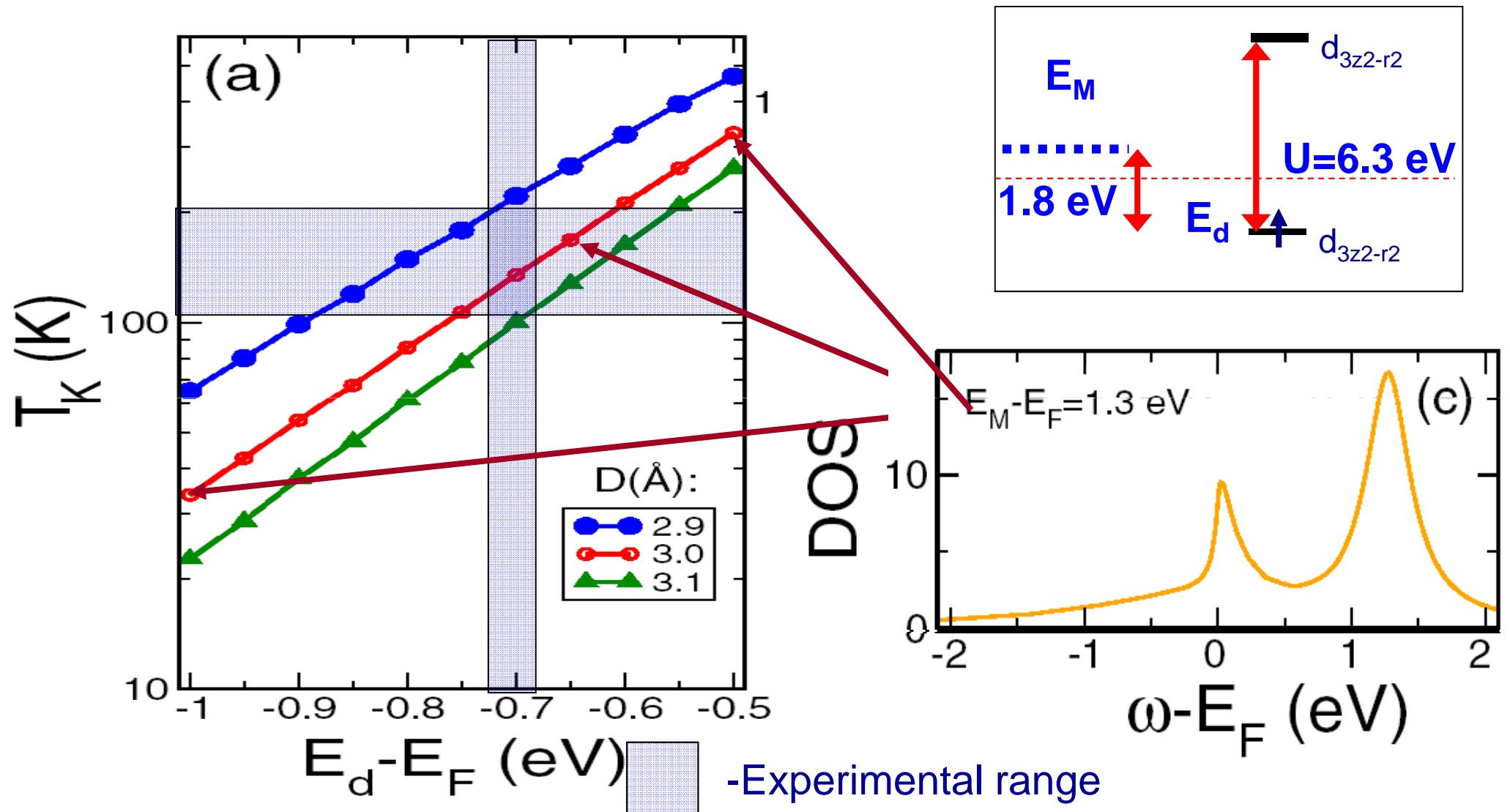
S=1/2 model



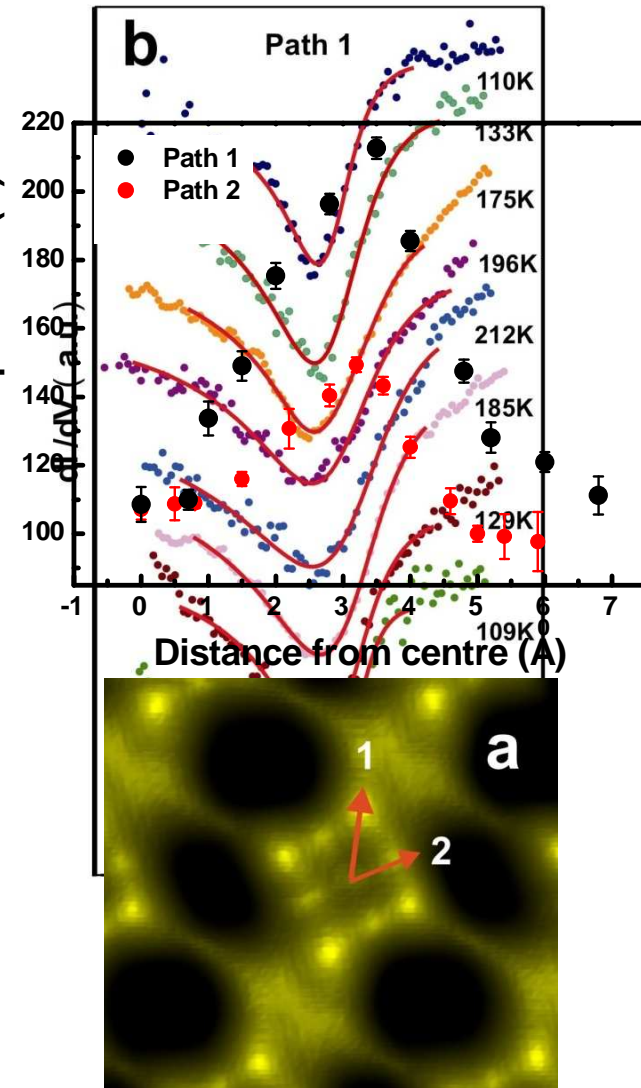
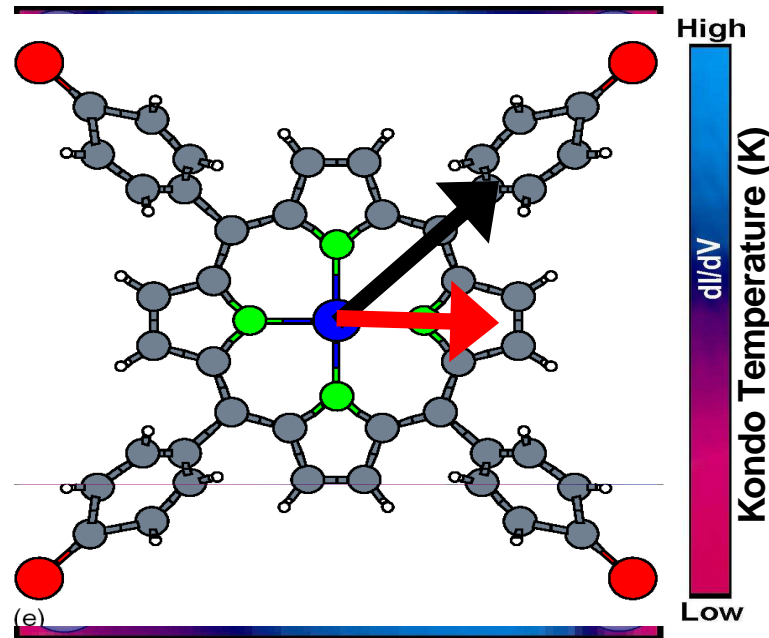
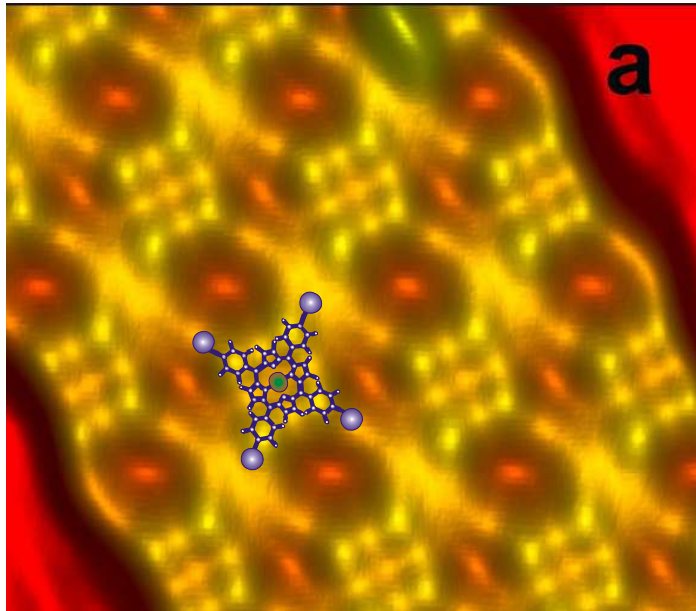
GW: Molecular levels "Co-like" levels

← Not easy to calculate with GW!

NRG calculations (Kondo temperature).

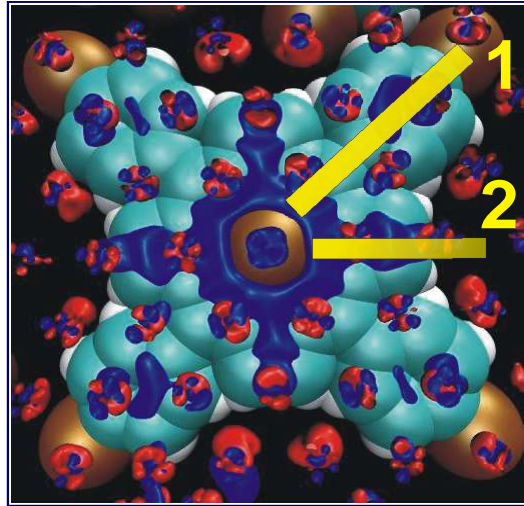


Spatially extended Kondo effect

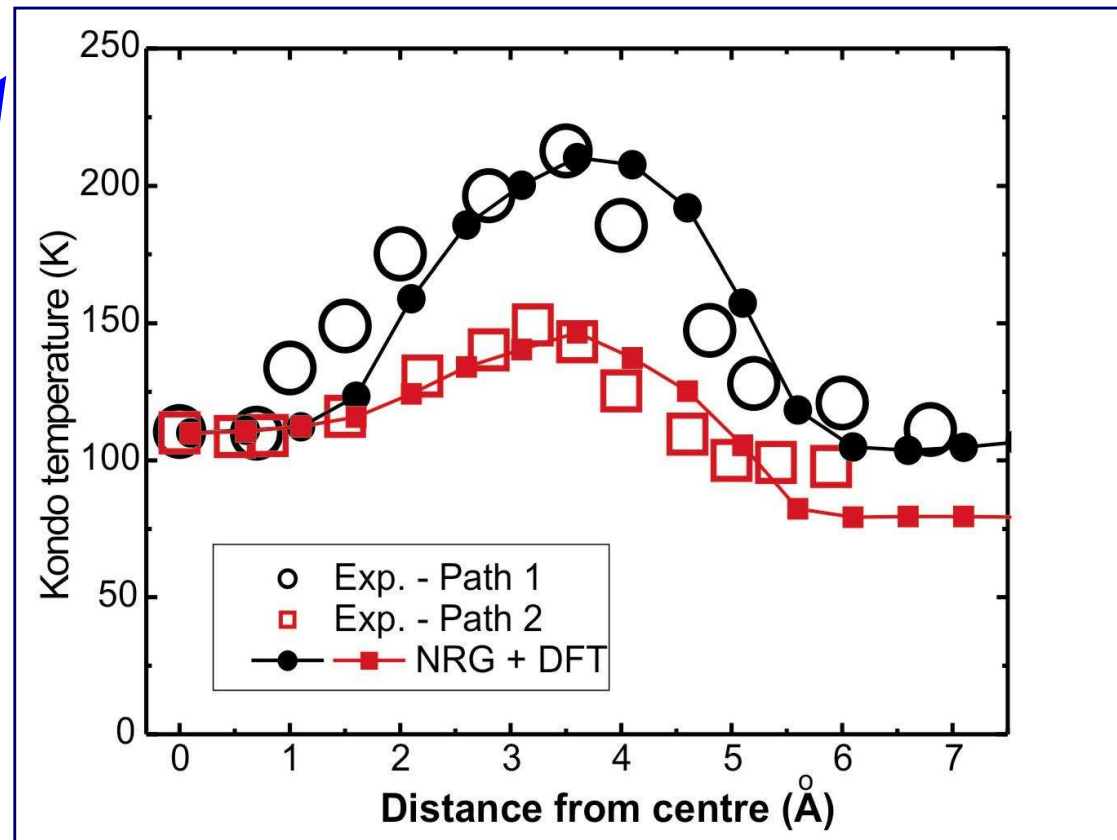
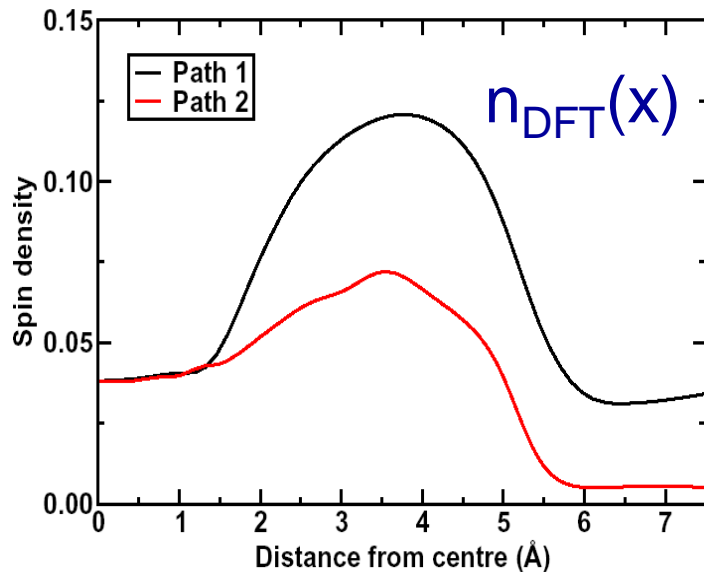


- Co-porphyrin structures: low-temperature STM dI/dV .
- Kondo resonance found *away* from central Co atom!
- Spatially-dependent Kondo temperature.

Spatially extended Kondo effect

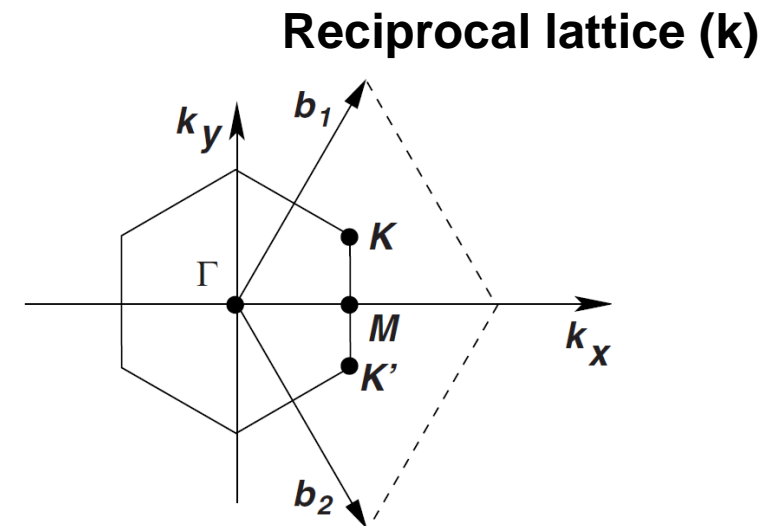
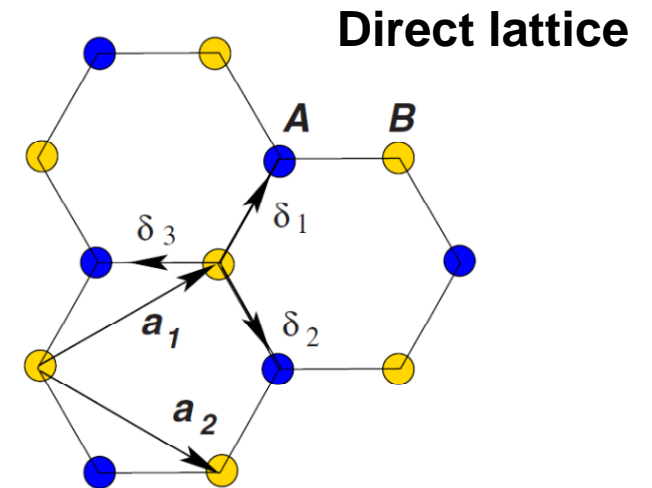
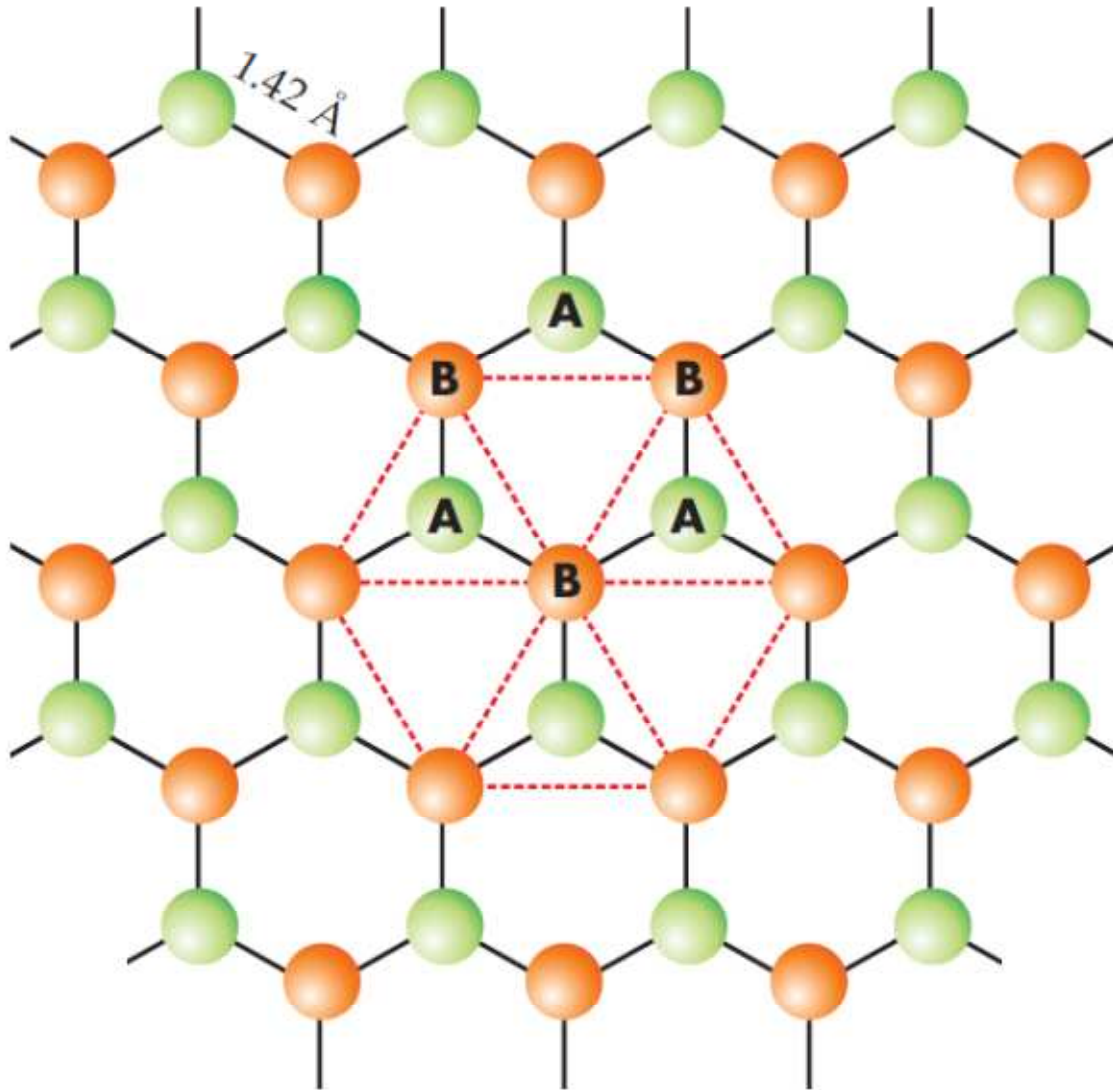


- DFT calculations: molecular spin density $n_{\text{DFT}}(x)$.
- NRG calculations (Kondo model): $J(x) = J_0 + An_{\text{DFT}}(x)$
- Good agreement with experimental data.

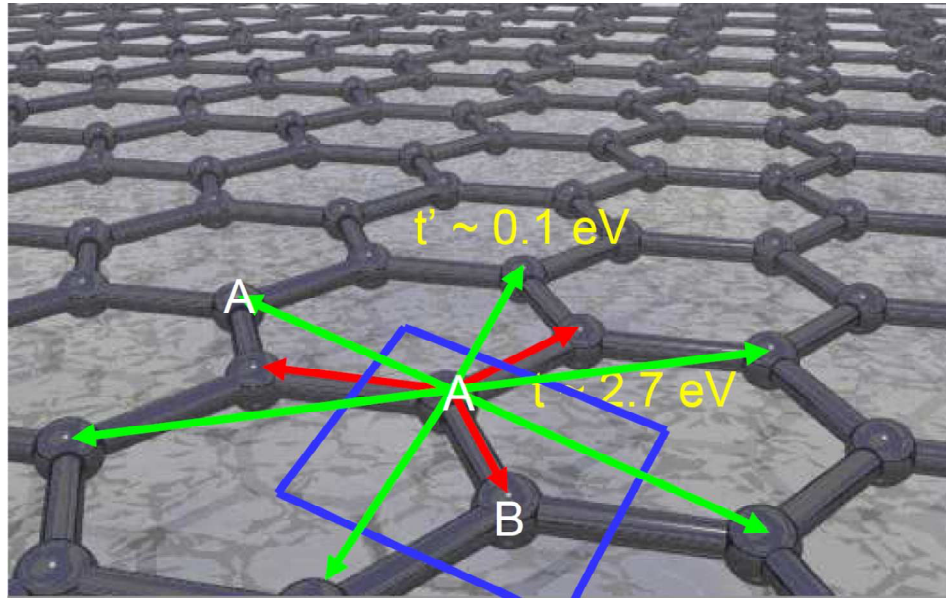


Application 2: Kondo effect in graphene.

Graphene basics: triangular lattices.



Electronic structure of graphene.



Tight-binding model

$$H = -t \sum_{\langle i,j \rangle, \sigma} \left(a_{\sigma,i}^\dagger b_{\sigma,j} + \text{H.c.} \right) - t' \sum_{\langle\langle i,j \rangle\rangle, \sigma} \left(a_{\sigma,i}^\dagger a_{\sigma,j} + b_{\sigma,i}^\dagger b_{\sigma,j} + \text{H.c.} \right)$$

...1st and 2nd neighbours

$$0.02t \leq t' \leq 0.2t$$

Phil R. Wallace

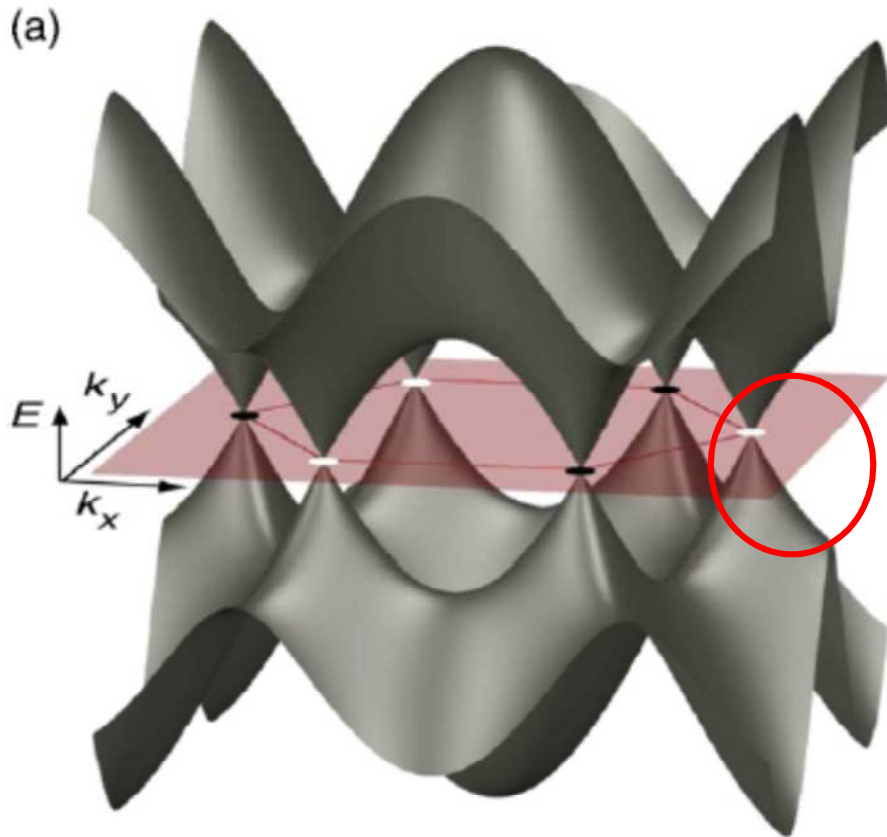
Energy dispersion for graphene (derived in the 40's!)

$$E_{\pm}(\mathbf{k}) = \pm t \sqrt{3 + f(\mathbf{k})} - t' f(\mathbf{k})$$

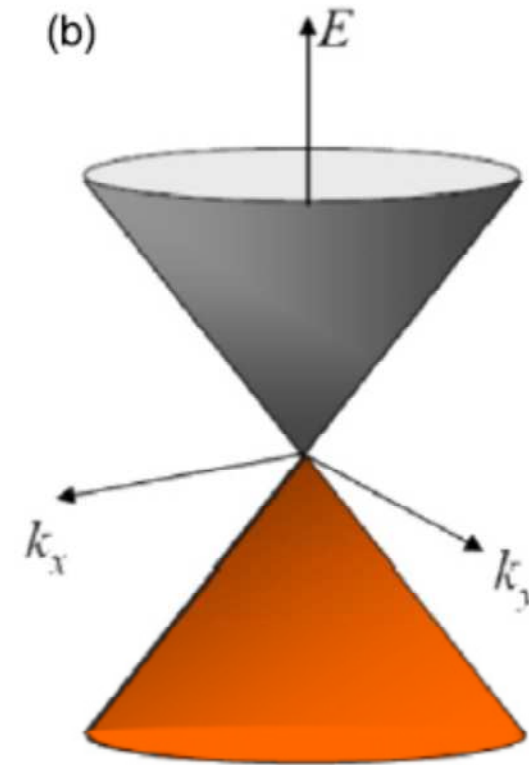
$$f(\mathbf{k}) = 2 \cos(\sqrt{3}k_y a) + 4 \cos\left(\frac{\sqrt{3}}{2}k_y a\right) \cos\left(\frac{3}{2}k_x a\right)$$

Wallace, P.R., *Phys. Rev.* **71** 622 (1947)

Dirac cones: massless fermions



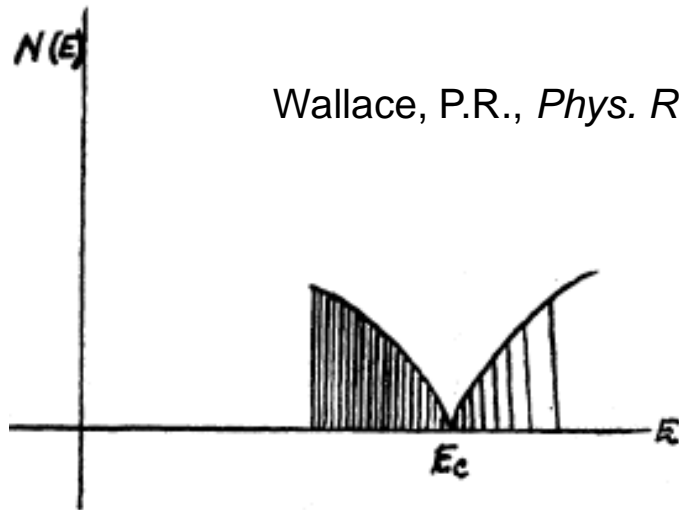
$$E_{\pm}(\mathbf{k}) = \pm |t| \sqrt{3 + f(\mathbf{k})}$$



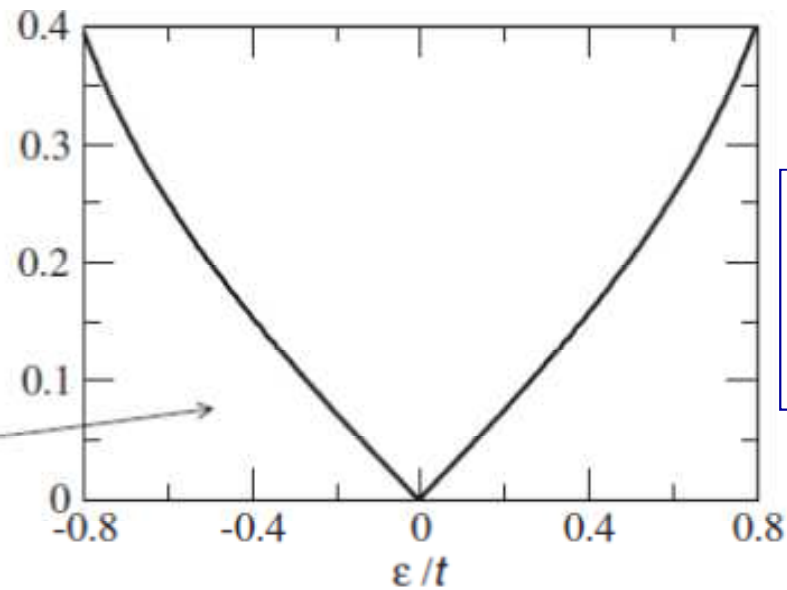
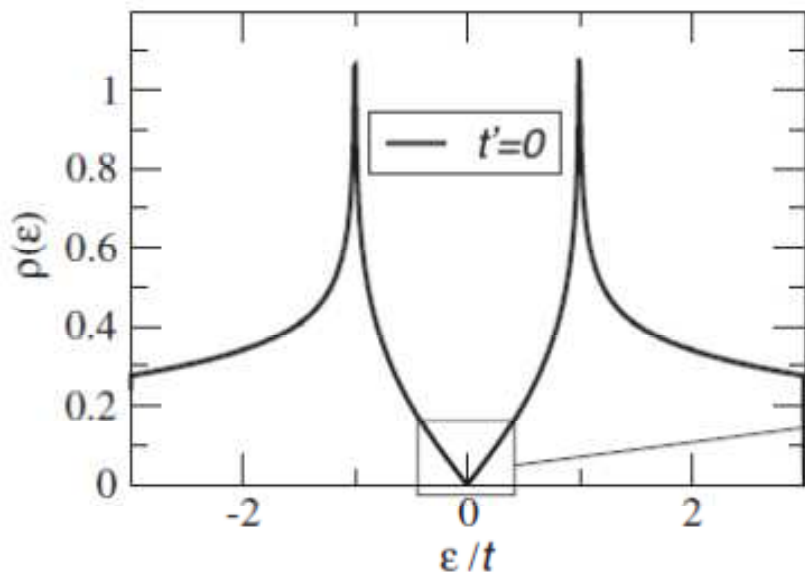
$$E_{\pm}(\mathbf{q}) = \pm v_F |\mathbf{q}|$$

Analogy: linear dispersion of massless “relativistic” particles (“E=pc”)

Graphene: density of states



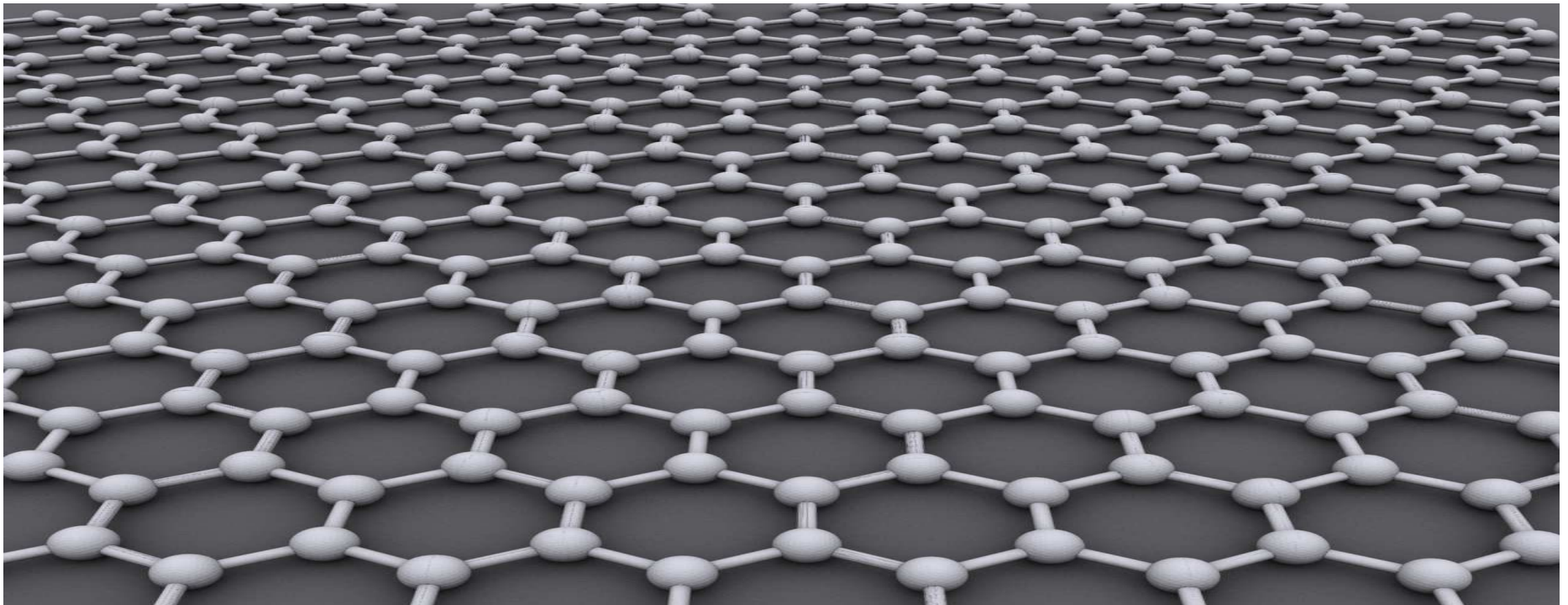
$\rho(E) \propto |E|$ (linear)
Close to the charge neutrality point.



$$\rho(E) = \frac{2A_c}{\pi} \frac{|E|}{v_F^2}$$

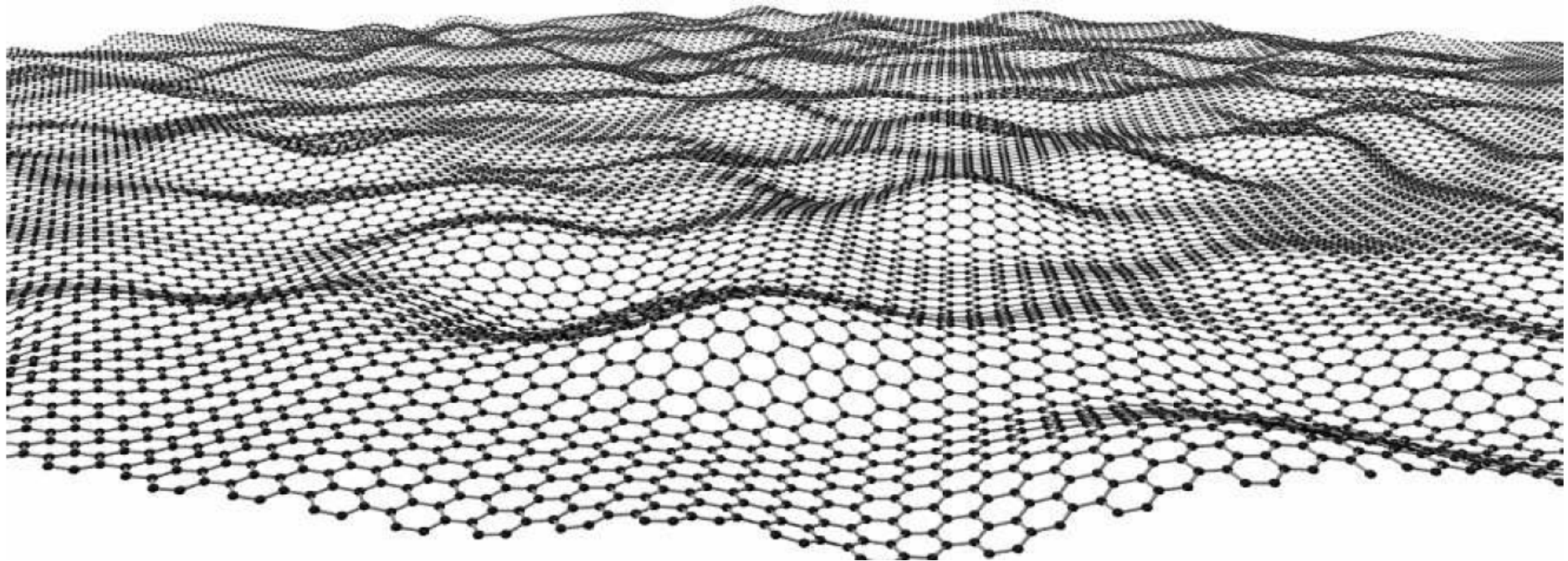
Graphene: How we think it is...

“Ideal” graphene: planar, 2D, ordered, nice...



... and how it really is.

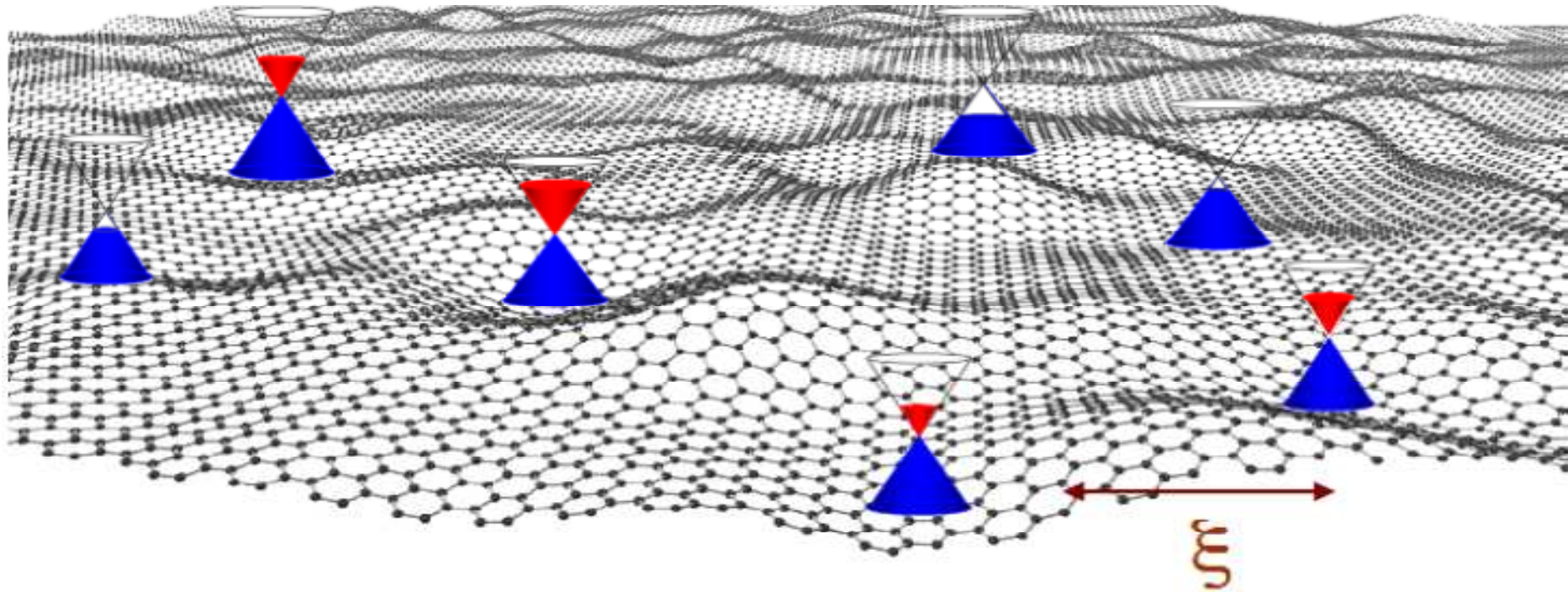
... versus “real” graphene: wavy, corrugated, membrane-like.



Graphene: “long-range disorder”.

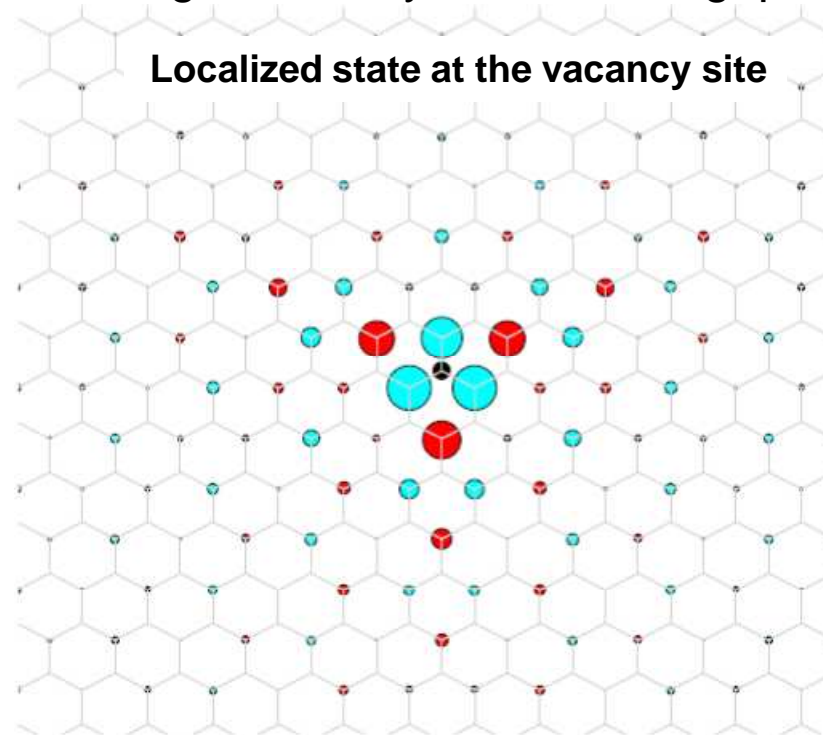
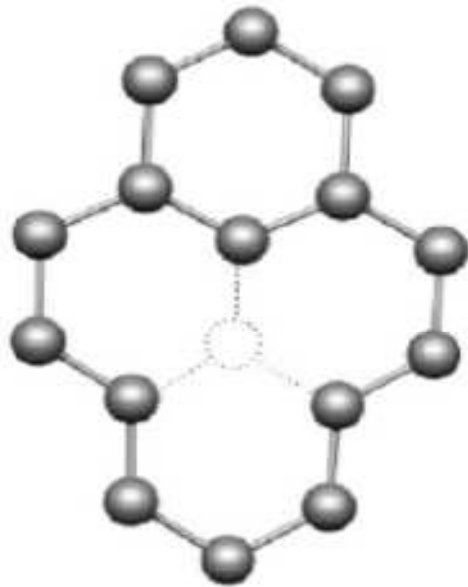
$$U_{\text{dis}}(\mathbf{r}_i) = \sum_{j=1}^{N_{\text{imp}}} W_j e^{-\frac{(\mathbf{r}_i - \mathbf{R}_j)^2}{2\xi^2}}$$

Disorder potential

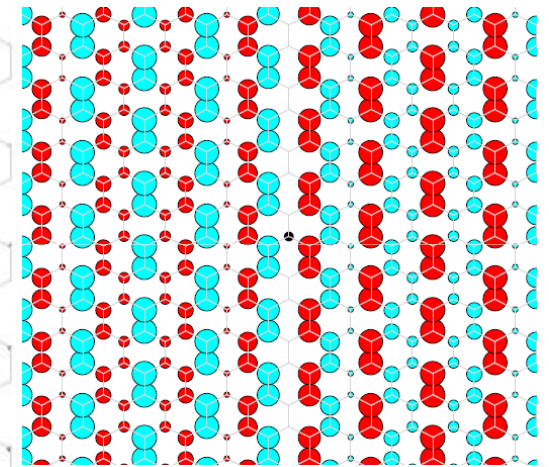


Short-range disorder (vacancies)

Tight-binding calculations: single vacancy leads to midgap state



Typical delocalized state

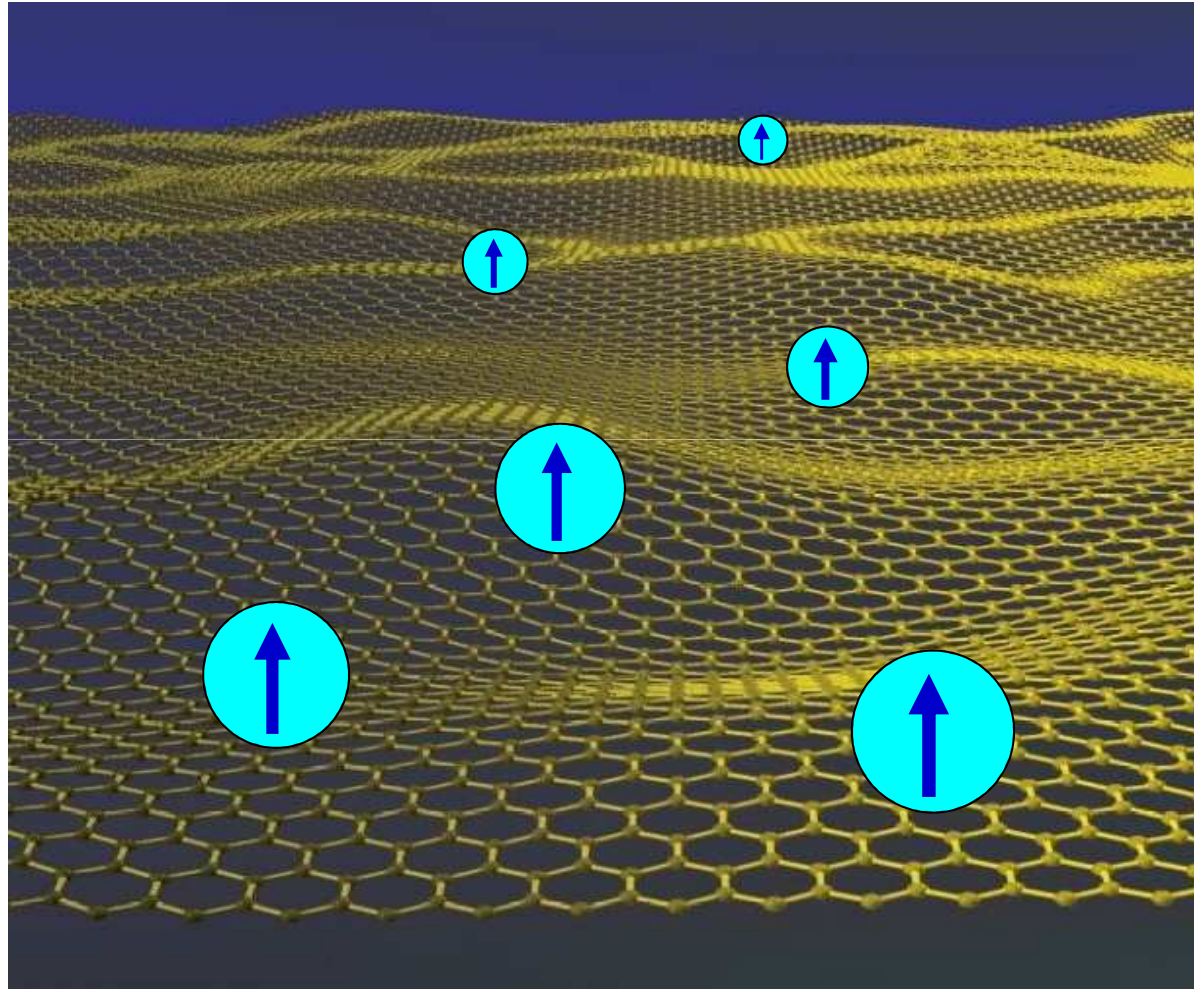


Vacancy tight-binding model:

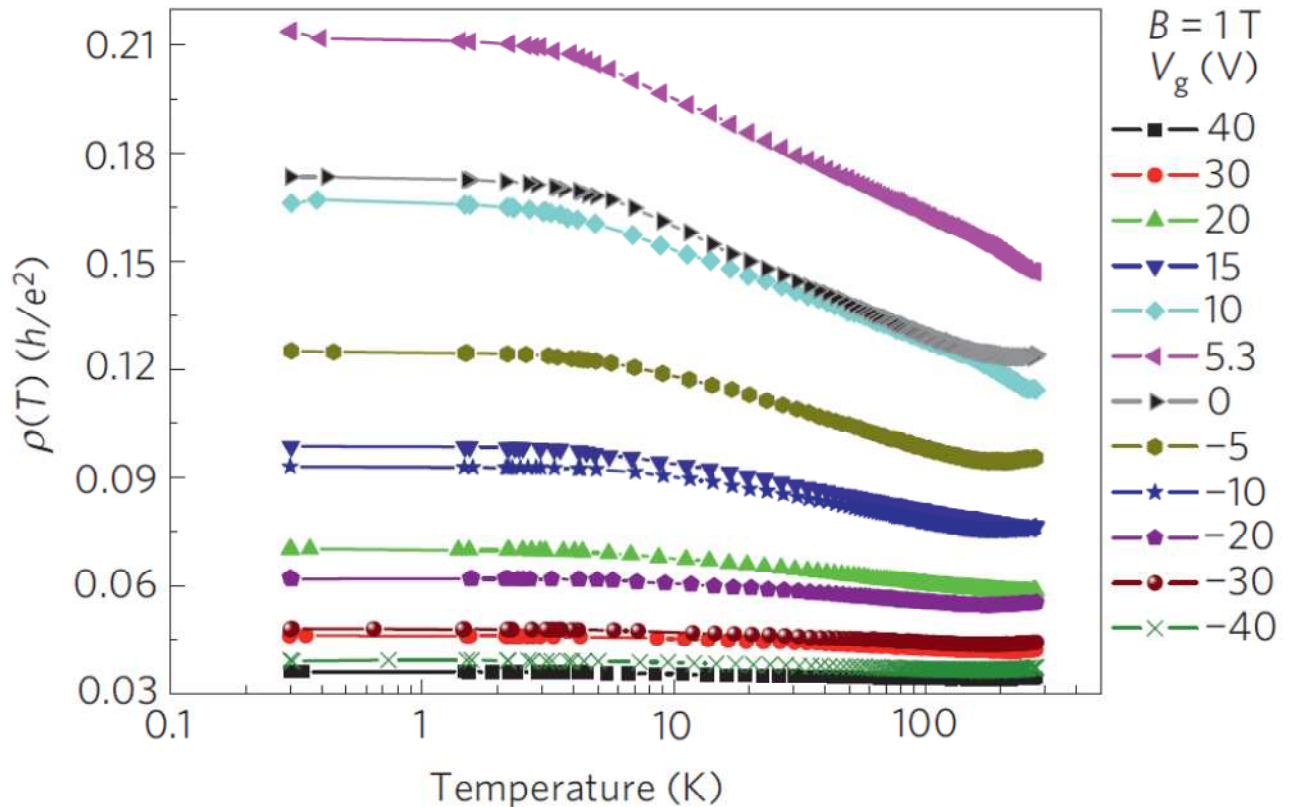
$$H_v = -t \sum_{\langle i,j \rangle} c_i^\dagger c_j + t \sum_{\langle v,j \rangle} c_v^\dagger c_j + \text{H.c.}$$

$$H_v |\nu\rangle = \begin{cases} \varepsilon_\nu |\nu\rangle & \text{for } \varepsilon \neq 0, \quad |\nu\rangle \text{ is extended,} \\ 0 |\nu\rangle & \text{for } \nu = v, \quad |v\rangle \text{ is localized.} \end{cases}$$

Vacancies = localized spins?



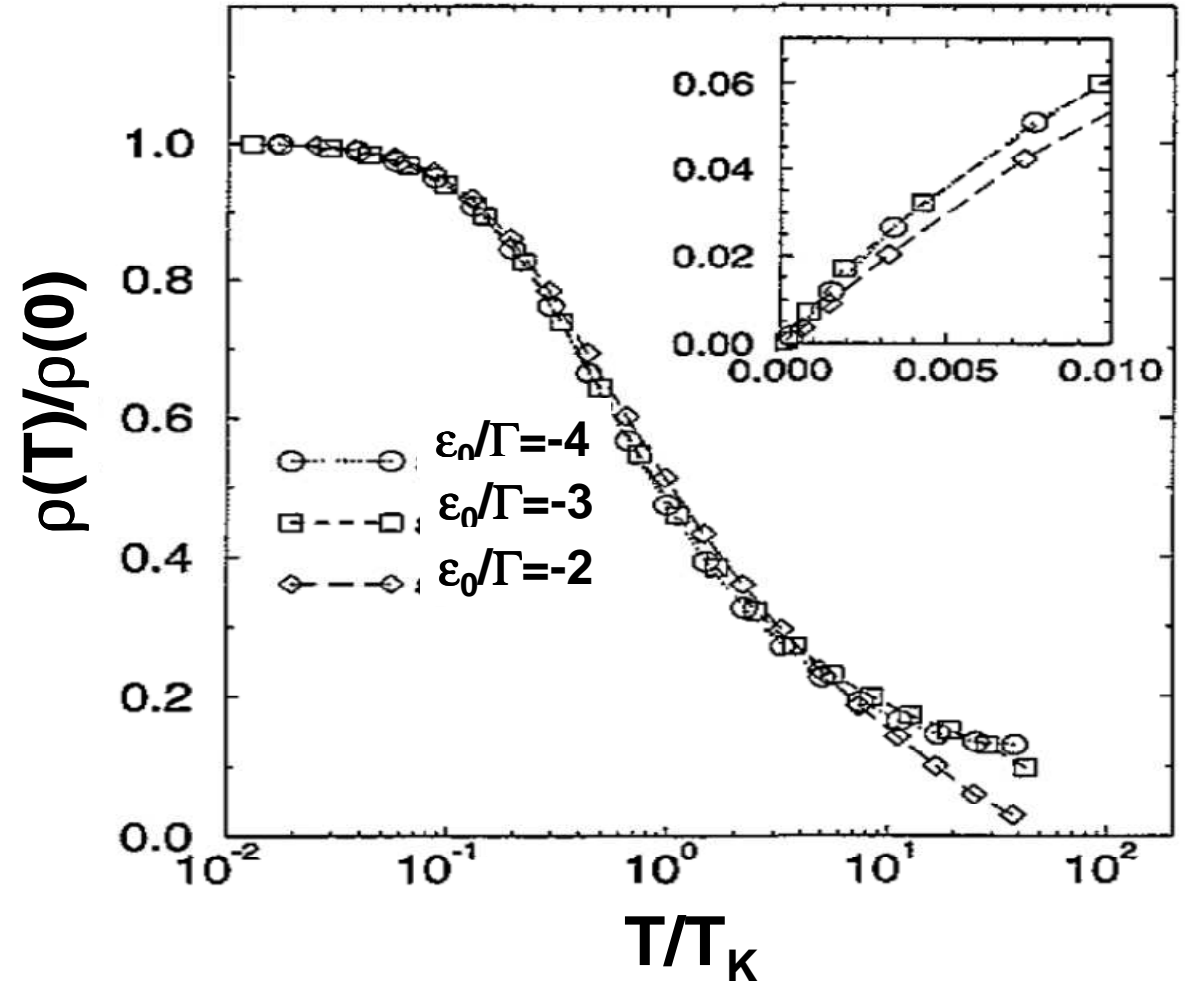
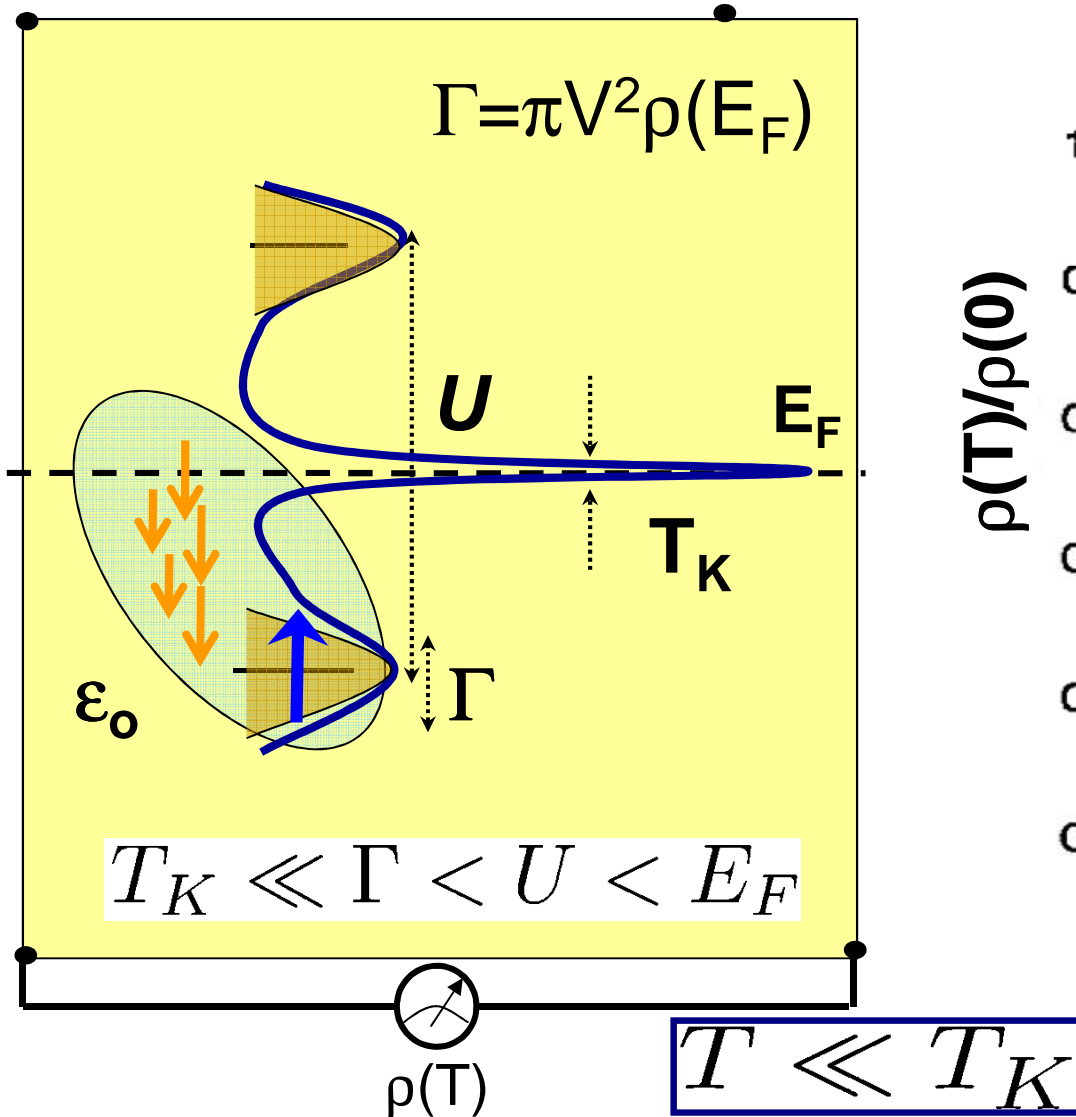
Resistivity $\rho(T)$ in irradiated graphene.



Jian-Hao Chen et al., *Nature Phys.* **7** 535 (2011)

Vacancies (short-range disorder) intentionally caused by irradiation.
Resistivity *increases* at low temperatures !! We have seen that before...

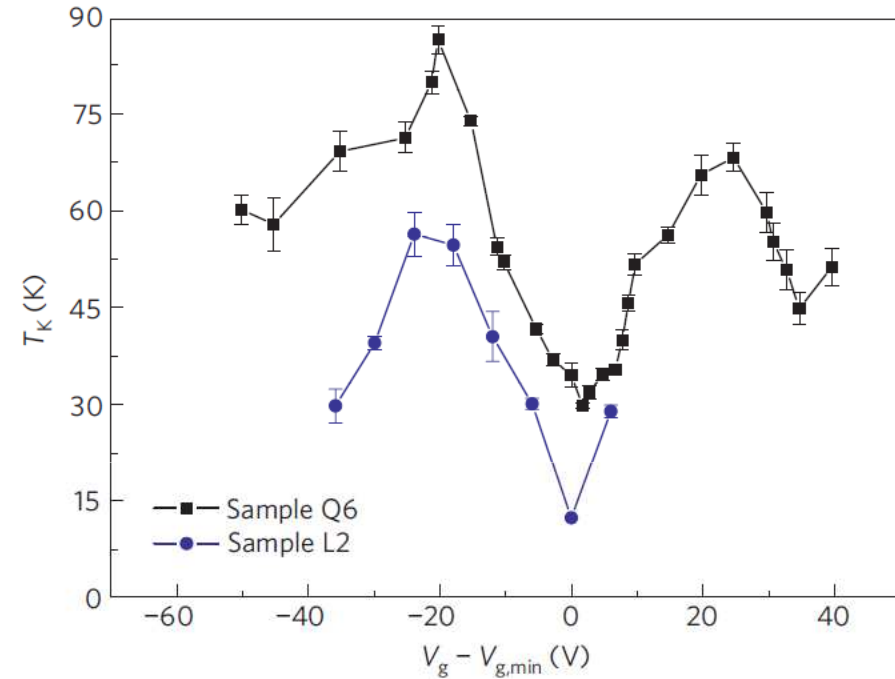
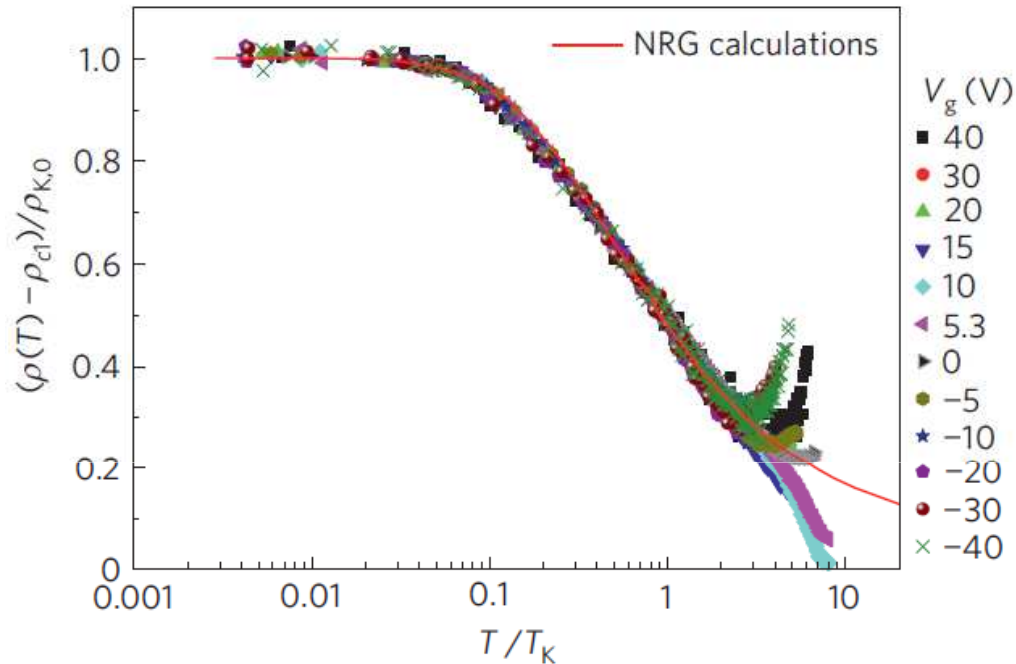
Kondo effect: NRG calculations



NRG calculations: scaling with T_K

T.A. Costi, et al., *J Phys Cond Mat.* **6** 2519 (1994).

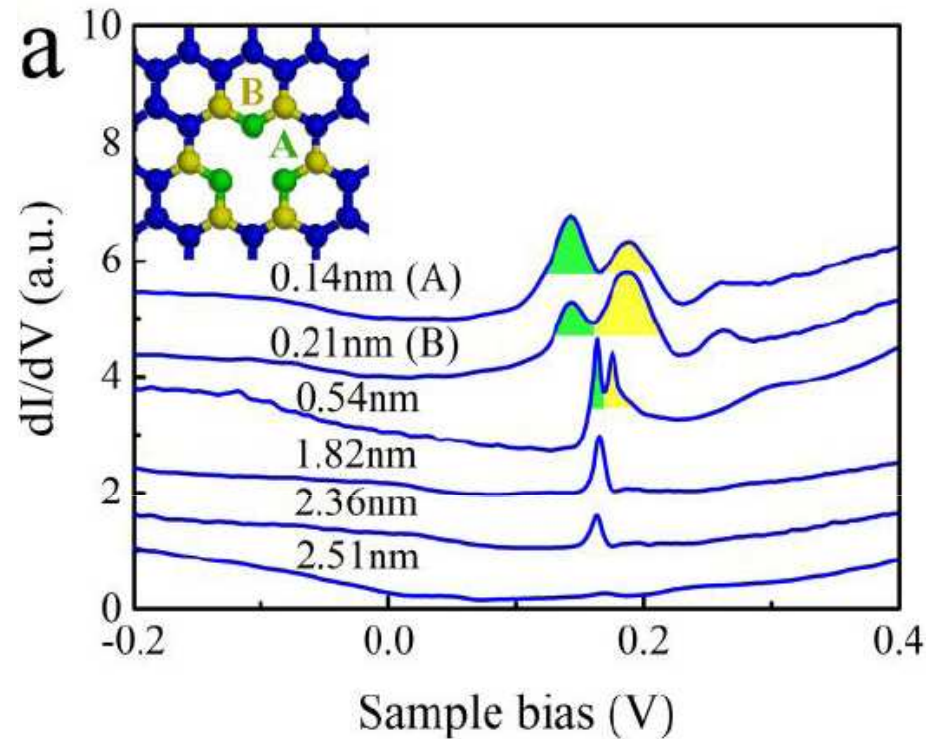
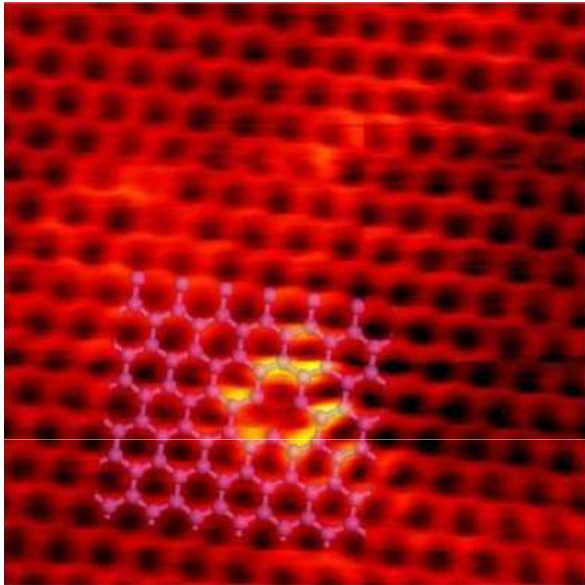
Kondo-like $\rho(T)$ features in irradiated graphene



Jian-Hao Chen et al., *Nature Phys.* **7** 535 (2011)

Resistivity vs Temperature measurements in disordered graphene. Vacancies (short-range disorder) intentionally caused by irradiation. Left: Kondo-like scaling in T/T_K . Right: T_K vs gate voltage.

Vacancy magnetism in graphene: STM



Yu Zhang et al., Phys. Rev. Lett. **117**, 166801 (2016)

STS spectra (right) showing Hubbard peaks with $U \sim 30$ meV.

Our calculations: $U \sim 640$ meV

V. Miranda, LDS, C.H. Lewenkopf
PRB **94** 075114 (2016) – Editor's Suggestion

Kondo effect in graphene: a few questions.

Where does the localized (magnetic) state come from?

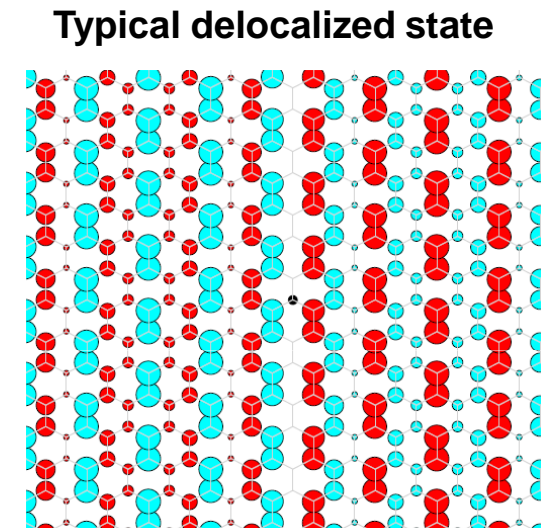
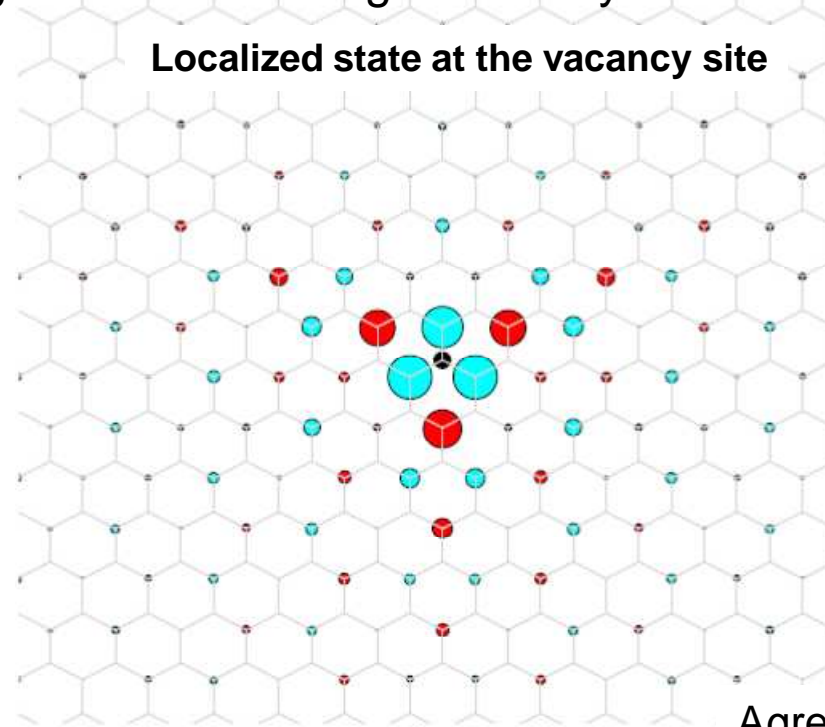
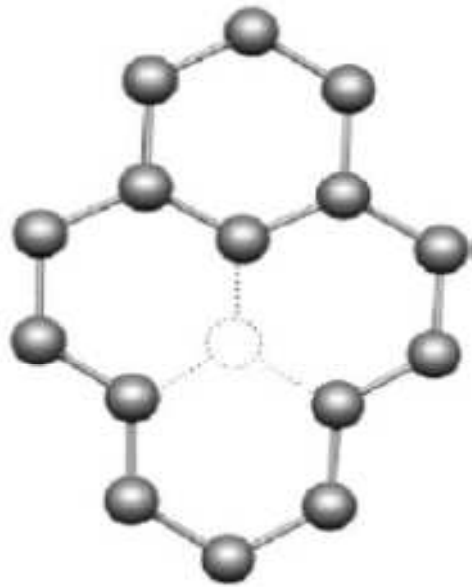
R: Vacancies (=mid-gap states) V. M. Pereira et al., *PRB* **77** 115109 (2008)

How does it couple to the continuous band?

Does this system retain features of an Anderson model coupled to Dirac fermions?

Mid-gap state in the presence of vacancies.

Tight-binding calculations: single vacancy leads to midgap state



Agreement with previous results

Vacancy tight-binding model:

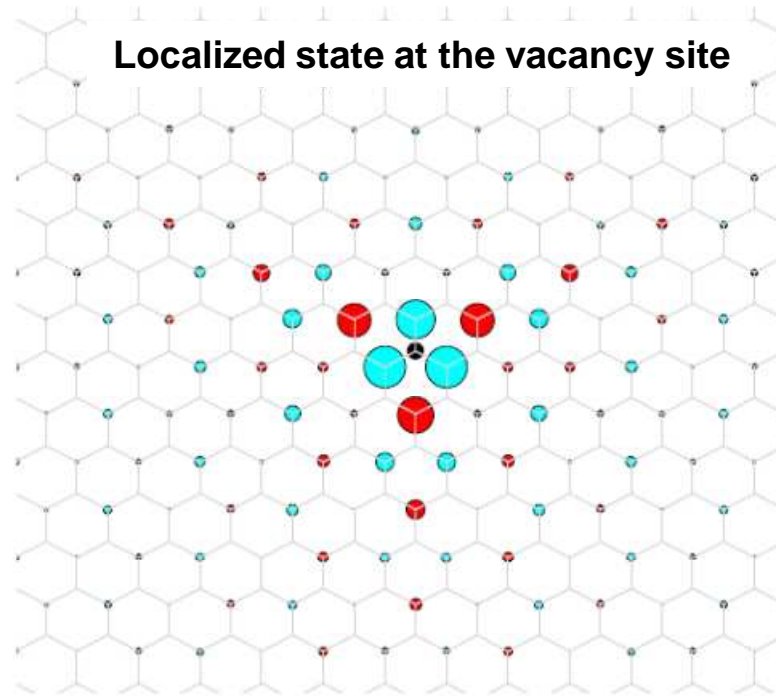
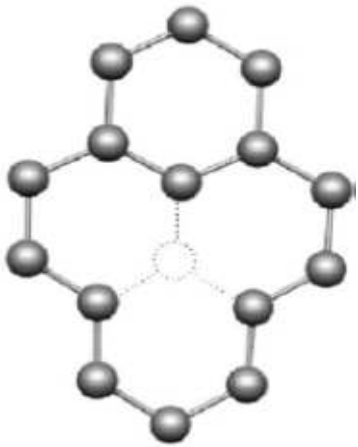
V. M. Pereira et al., *PRB* **77** 115109 (2008)

$$H_v = -t \sum_{\langle i,j \rangle} c_i^\dagger c_j + t \sum_{\langle v,j \rangle} c_v^\dagger c_j + \text{H.c.}$$

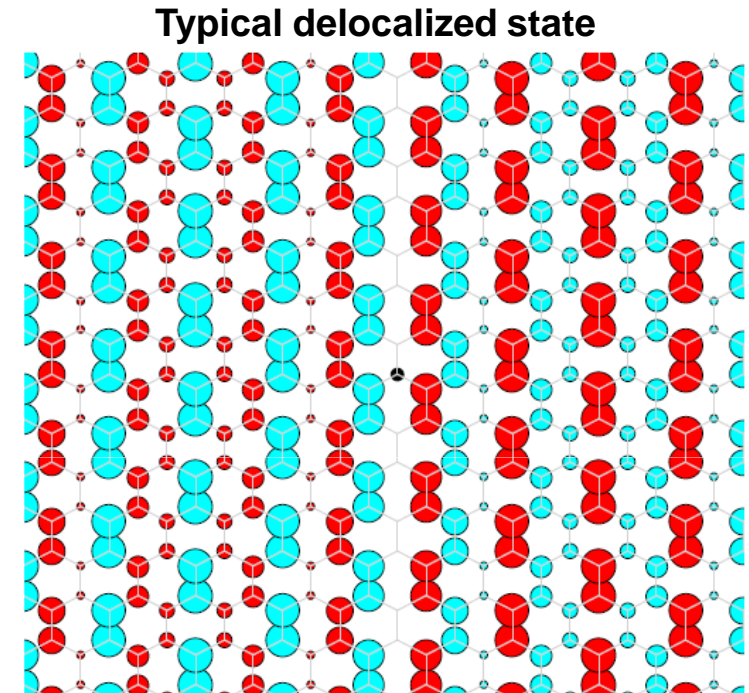
$$H_v |\nu\rangle = \begin{cases} \epsilon_\nu |\nu\rangle & \epsilon_\nu \neq 0, \quad |\nu\rangle \text{ is extended,} \\ 0 |v\rangle & \epsilon_\nu = 0, \quad |v\rangle \text{ is localized.} \end{cases}$$

Mid-gap state is not coupled!!

$$H_v |\nu\rangle = \begin{cases} \varepsilon_\nu |\nu\rangle \\ 0 |v\rangle \end{cases}$$



$$\varepsilon_v = 0, \quad |v\rangle \text{ is localized}$$



$$\varepsilon_\nu \neq 0, \quad |\nu\rangle \text{ is extended}$$

Localized and delocalized states are decoupled:
No Kondo possible.

$$\langle v | H_v | \nu \rangle = 0 \quad \text{if } v \neq \nu !$$

Kondo effect in graphene: a few questions.

Where does the localized (magnetic) state comes from?

R: Vacancies (=mid-gap states) V. M. Pereira et al., *PRB* **77** 115109 (2008)

How does it couple to the continuous band?

R1: Rippling, Jahn-Teller-like distortion M. A. Cazallila et al., arXiv 1207.3135 (2012)

R2: Long-range disorder (this work)

Does this system retain features of an Anderson model coupled to Dirac fermions?

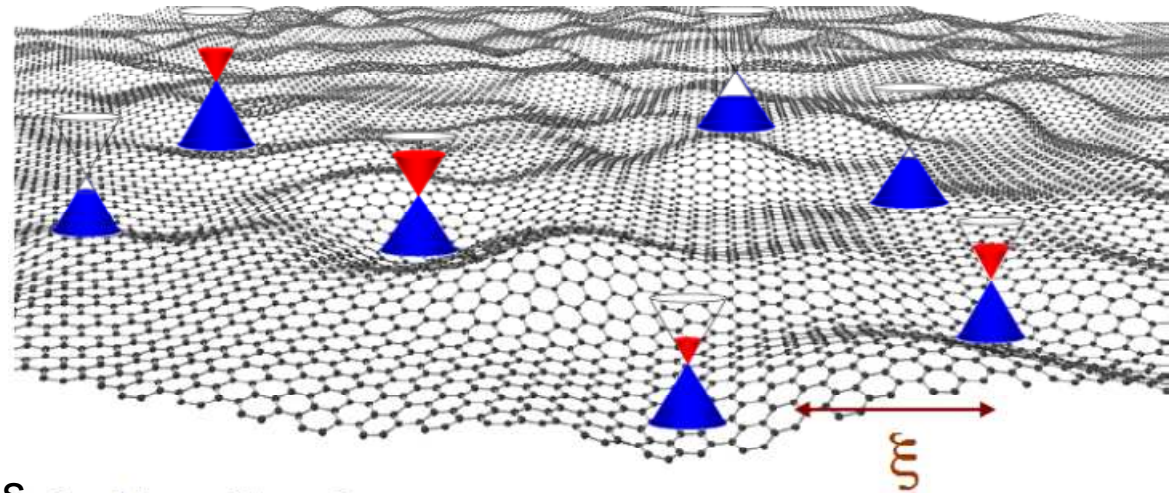
How to couple? Long-range disorder.

How to couple the localized state to the graphene band?

Disorder (weak)

$$U_{\text{dis}}(\mathbf{r}_i) = \sum_{j=1}^{N_{\text{imp}}} W_j e^{-\frac{(\mathbf{r}_i - \mathbf{R}_j)^2}{2\xi^2}}$$

$$H = H_{\text{v}} + U_{\text{dis}}$$



Our “basis”: localized and extended states

$$H_{\text{v}} = -t \sum_{\langle i,j \rangle} c_i^\dagger c_j + t \sum_{\langle \text{v},j \rangle} c_{\text{v}}^\dagger c_j + \text{H.c.}$$

$$H_{\text{v}}|\nu\rangle = \begin{cases} \varepsilon_\mu |\mu\rangle & \text{and } \varepsilon_\mu \neq 0, \quad |\mu\rangle \text{ is extended,} \\ \varepsilon_0 |0\rangle & \text{and } \varepsilon_0 = 0 \quad |0\rangle \text{ is localized.} \end{cases}$$

Projectors:

$$\mathbf{1} = \hat{P} + \hat{Q} \equiv \sum_{\mu} |\mu\rangle \langle \mu| + |0\rangle \langle 0|$$

$$H = \hat{P}H\hat{P} + \hat{Q}H\hat{P} + \hat{P}H\hat{Q} + \hat{Q}H\hat{Q}$$

The trick: projecting into different sectors!

How to couple the localized state to the graphene band?

Disorder (weak)

Extended

Coupling

Localized

$$H = \hat{P}H\hat{P} + \hat{Q}H\hat{P} + \hat{P}H\hat{Q} + \hat{Q}H\hat{Q}$$

$$\hat{P}H\hat{P} = \sum_{\mu} |\mu\rangle \varepsilon_{\mu} \langle \mu| + \sum_{\mu\mu'} |\mu\rangle \langle \mu| U_{\text{dis}} |\mu'\rangle \langle \mu'|$$

Effective band density of states.

$$\rho_{\text{dis}}(\omega) = \sum \delta(\omega - \varepsilon_{\beta})$$

$$\hat{Q}H\hat{P} + \text{h.c.} = \sum_{\mu} |0\rangle \langle 0| U_{\text{dis}} |\mu\rangle \langle \mu| + \text{h.c.}$$

Coupling to the localized state

$$t_{\beta 0} \equiv \langle \beta | U_{\text{dis}} | 0 \rangle$$

$$\hat{Q}H\hat{Q} = \sum_{\mu} |0\rangle \varepsilon_0 \langle 0| + |0\rangle \langle 0| U_{\text{dis}} |0\rangle \langle 0| = |0\rangle \varepsilon_0^{\text{dis}} \langle 0|$$

Renormalized state energy

$$\varepsilon_0^{\text{dis}} \equiv \langle 0 | U_{\text{dis}} | 0 \rangle$$

What about the Hubbard U ?

Tight-binding (single orbital) contributions to the charging energy of the localized state $|0\rangle$ give:

$$U = e^2 \int d^3r \int d^3r' \frac{\rho_0(r)\rho_0(r')}{|r - r'|}$$

with

$$\rho_0(r) \approx \sum_i |c_i^{(0)}|^2 |\chi_i(r)|^2$$

$$U \equiv U_{\text{diag}} + U_{\text{off}}$$

$$U_{\text{diag}} \sim \frac{U_C}{(\log N_i)^2}$$

“Diagonal”

$$U_{\text{off}} \sim e^2 \int d^2r \int d^2r' \frac{|\psi_0(r)|^2 |\psi_0(r')|^2}{|r - r'|}$$

“Off-diagonal”

In the NRG calculations, we use:

$$U = 0.5D \approx 1.5t = 4.2 \text{ eV}$$

consistent with recent DFT+cRPA calculations

Our estimates:

$$0.139 \text{ eV} \leq U_{\text{diag}} \leq 0.302 \text{ eV}$$

$$1.859 \text{ eV} \leq U_{\text{off}} \leq 3.723 \text{ eV}$$

$$U^*/t = 1.6 \pm 0.2 \quad \text{M. Schüler et al., PRL 111 036601 (2013)}$$

What about the Hubbard U ?

Tight-binding (single orbital) contributions to the charging energy of the localized state $|0\rangle$ give:

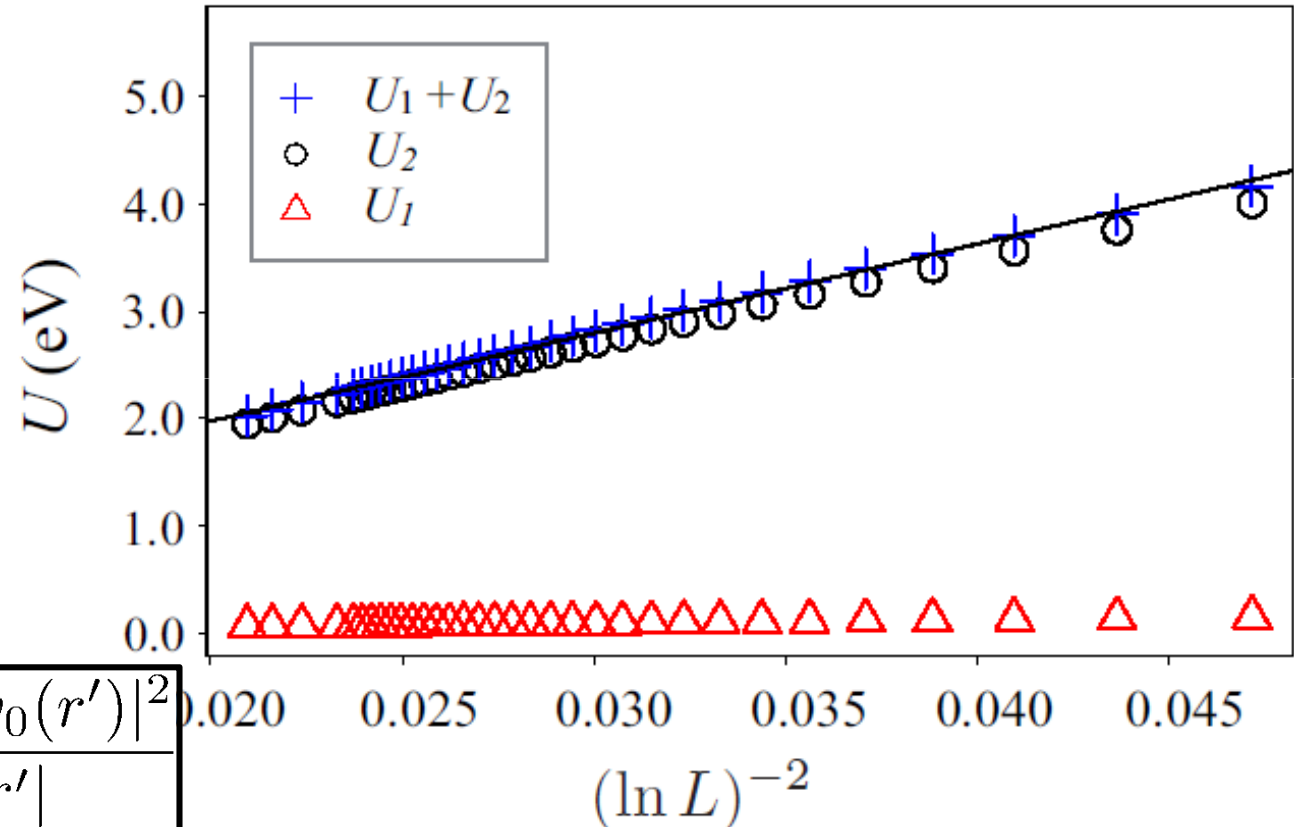
$$U \equiv U_1 + U_2$$

“Diagonal”

$$U_1 \sim \frac{U_C}{(\log N_i)^2}$$

“Off-diagonal”

$$U_2 \sim e^2 \int d^2 r \int d^2 r' \frac{|\psi_0(r)|^2 |\psi_0(r')|^2}{|r - r'|}$$



Kondo effect in graphene: a few questions.

Where does the localized (magnetic) state comes from?

R: Vacancies (=mid-gap states) V. M. Pereira et al., *PRB* **77** 115109 (2008)

How does it couple to the continuous band?

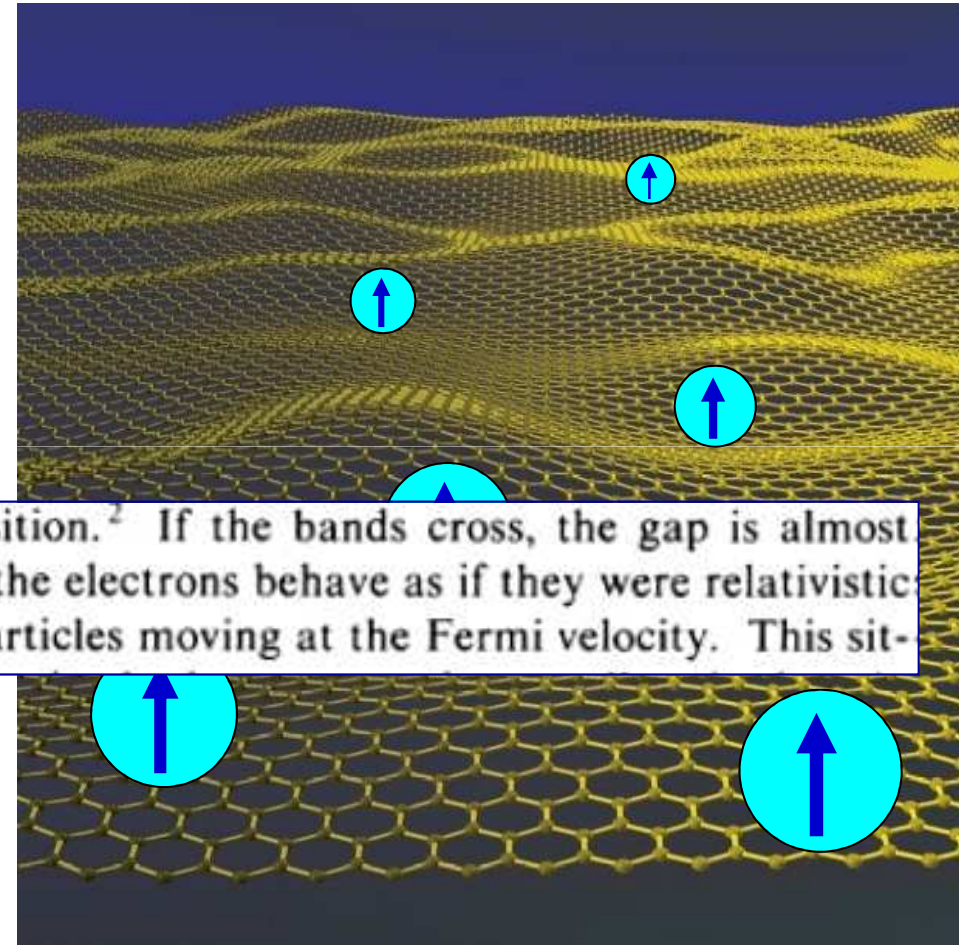
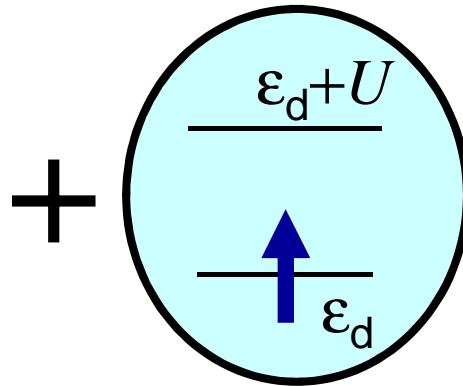
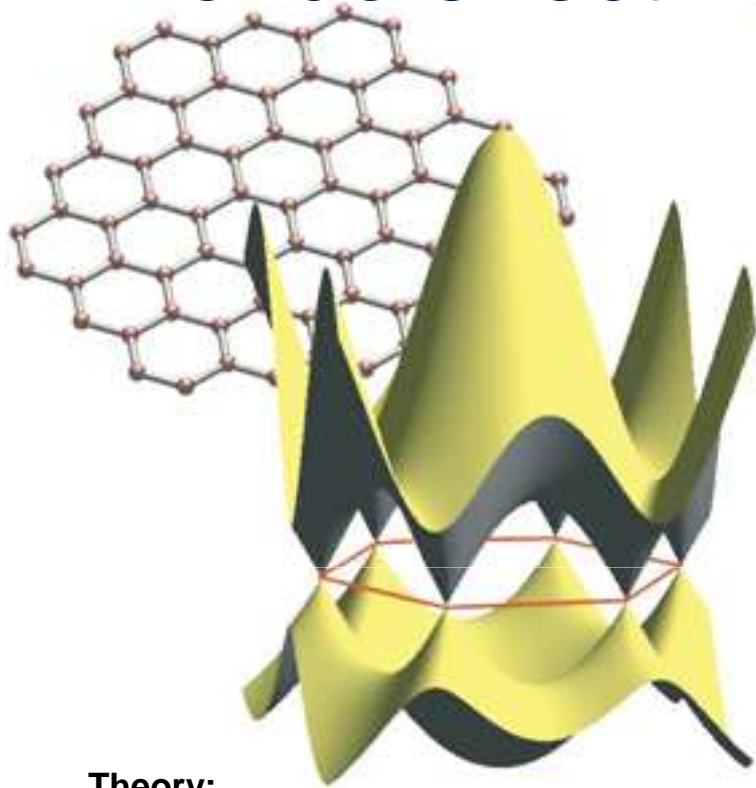
R1: Rippling, Jahn-Teller-like distortion M. A. Cazallila et al., arXiv 1207.3135 (2012)

R2: Long-range disorder (this work)

Does this system retain features of an Anderson model coupled to Dirac fermions?

NRG calculations: TB-derived Anderson model

Kondo effect with “massless Dirac Fermions”?



Theory:

- D. Withoff and E. Fradkin, PRL **64** 1835 (1990).
- C. Gonzalez-Buxton, K. Ingersent, PRB **57**, 14254 (1998)
- P.S. Cornaglia et al. PRL **102** 046801 (2009).
- M. Vojta, et al., Europhys. Lett. **90**, 27006 (2010).
- Review: L. Fritz and M. Vojta, arXiv:1208.3113 (2012)

=Quantum phase transition!

Q: Do we see this in our model?

Long-range disorder model: Hybridization.

For each realization:

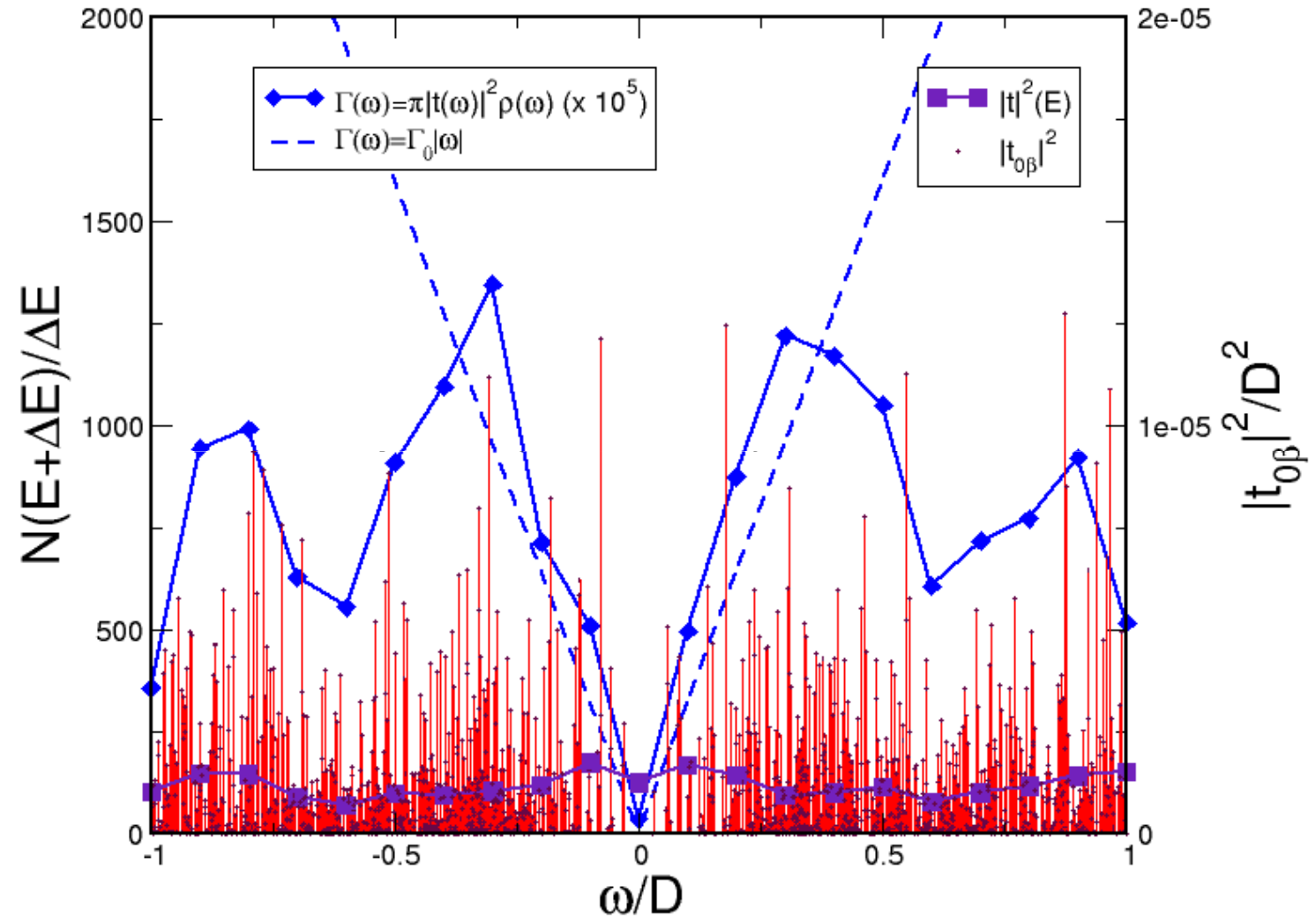
$$|t_{0\beta}|^2 \quad \varepsilon\beta$$



$$\Gamma_{\text{dis}}(\omega) = \pi |t_\omega|^2 \rho_{\text{dis}}(\omega)$$

And also:

$$\varepsilon_0^{\text{dis}} \equiv \langle 0 | U_{\text{dis}} | 0 \rangle$$



Long-range disorder model: Hybridization.

For each realization:

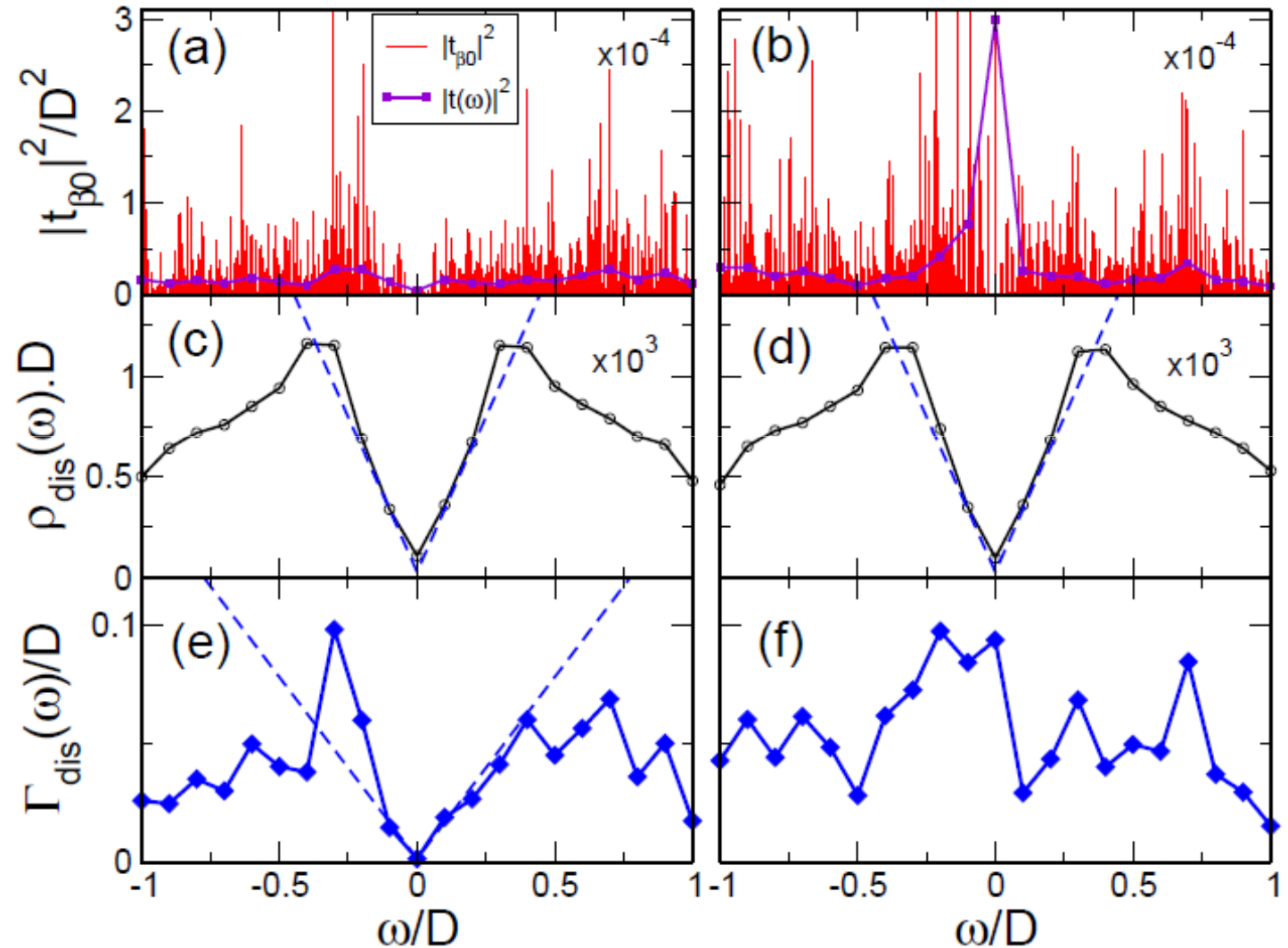
$$|t_{0\beta}|^2 \quad \varepsilon\beta$$

↓ ↓

$$\Gamma_{\text{dis}}(\omega) = \pi |t_\omega|^2 \rho_{\text{dis}}(\omega)$$

And also:

$$\varepsilon_0^{\text{dis}} \equiv \langle 0 | U_{\text{dis}} | 0 \rangle$$



Disorder: model parameters distributions

For each realization:

$$|t_{0\beta}|^2 \quad \varepsilon_\beta$$

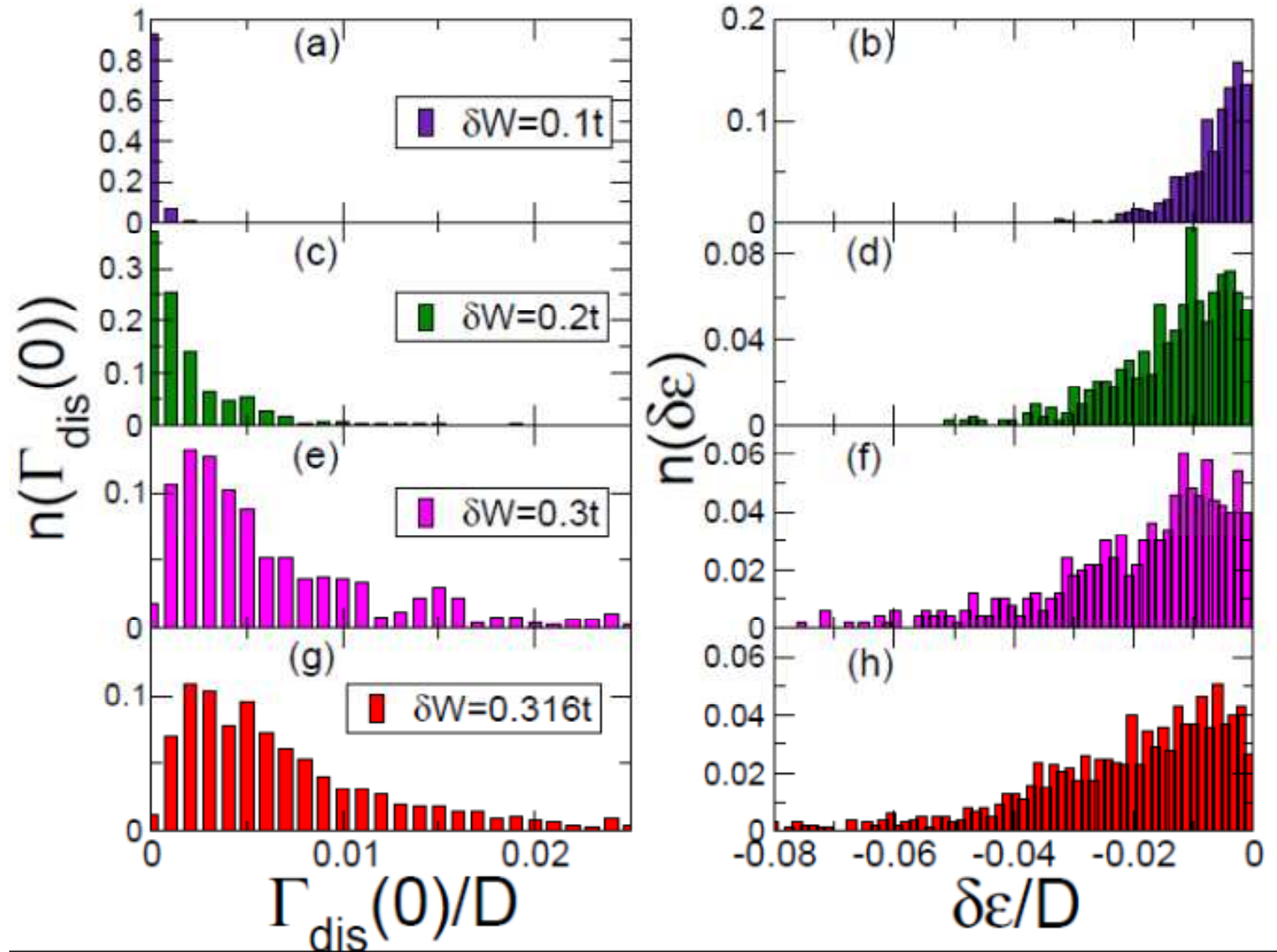


$$\Gamma_{\text{dis}}(\omega) = \pi |t_\omega|^2 \rho_{\text{dis}}(\omega)$$

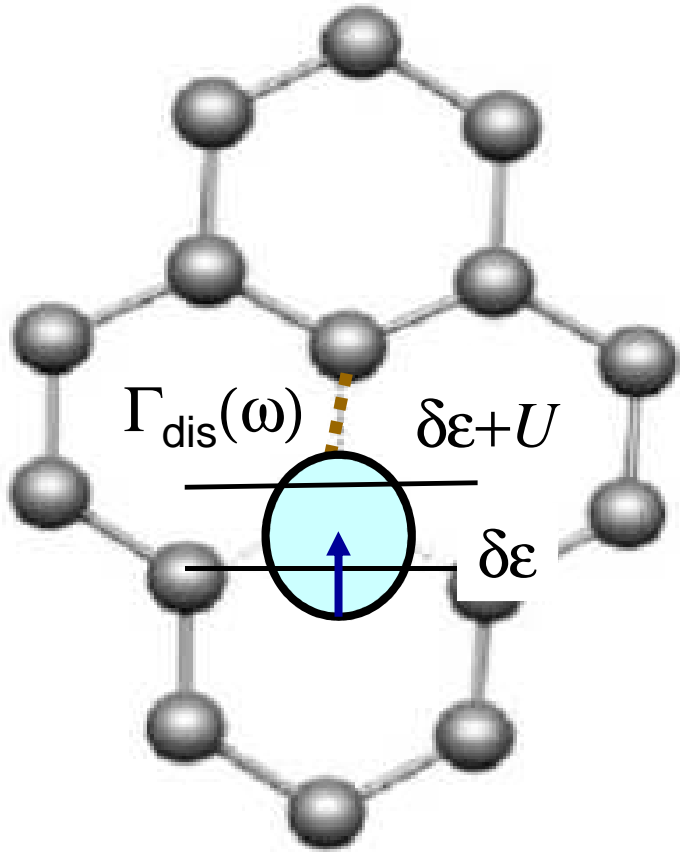
And also:

$$\varepsilon_0^{\text{dis}} \equiv \langle 0 | U_{\text{dis}} | 0 \rangle$$

$$\delta\varepsilon = \varepsilon_0^{\text{dis}} - \mu(V_g)$$



Effective Anderson model: disorder-mediated coupling.



$$H_A = H_{\text{state}} + H_{\text{band}} + H_{s-b} \text{ where:}$$

$$H_{\text{state}} = \delta\epsilon n_{0\sigma} + U n_{0\uparrow} n_{0\downarrow} \quad (\delta\epsilon = \epsilon_0^{\text{dis}} - \mu(V_g))$$

$$H_{\text{band}} = \int_{-D-\Delta\mu}^{D-\Delta\mu} d\omega \omega c_{\omega\sigma}^\dagger c_{\omega\sigma}$$

$$H_{s-b} = \int_{-D-\Delta\mu}^{D-\Delta\mu} d\omega \sqrt{\Gamma_{\text{dis}}(\omega)/\pi} \left(c_{0\sigma}^\dagger c_{\omega\sigma} + \text{H.c.} \right)$$

Anderson model with disorder-mediated coupling.

$\delta\epsilon$: impurity state energy; $\Gamma_{\text{dis}}(\omega)$: hybridization

$\mu(V_g)$: Fermi energy (gate-dependent)

$\mu(0)$: Fermi energy at charge neutrality. ($\Delta\mu = \mu(V_g) - \mu(0)$)

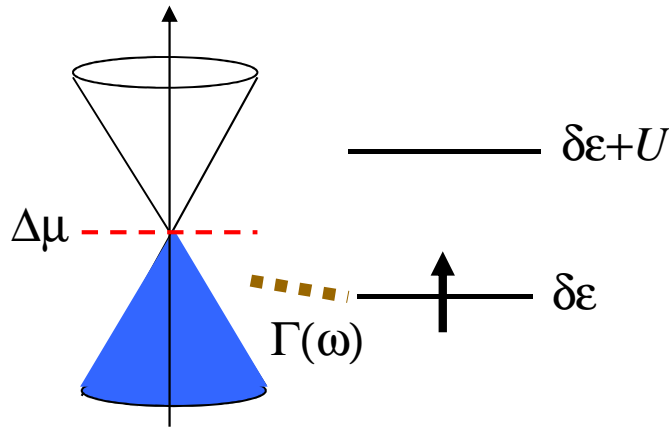
Anderson model + massless Dirac fermions

$\rho(\omega)$: density of states

$$\Gamma(\omega) = \pi V^2 \rho(\omega)$$

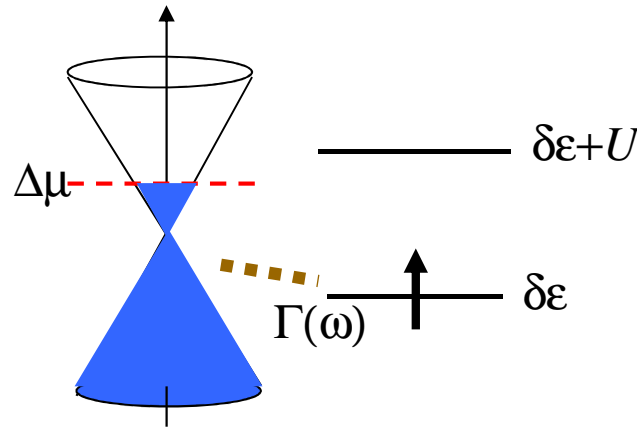
Anderson impurity coupled to a “Dirac band” with linear dispersion.

“Pseudogap” model



$$\Gamma(\omega \rightarrow 0) = 0 (\sim |\omega|)$$

“metallic” model



$$\Gamma(\omega \rightarrow 0) \neq 0$$

$\delta\varepsilon$: impurity state energy

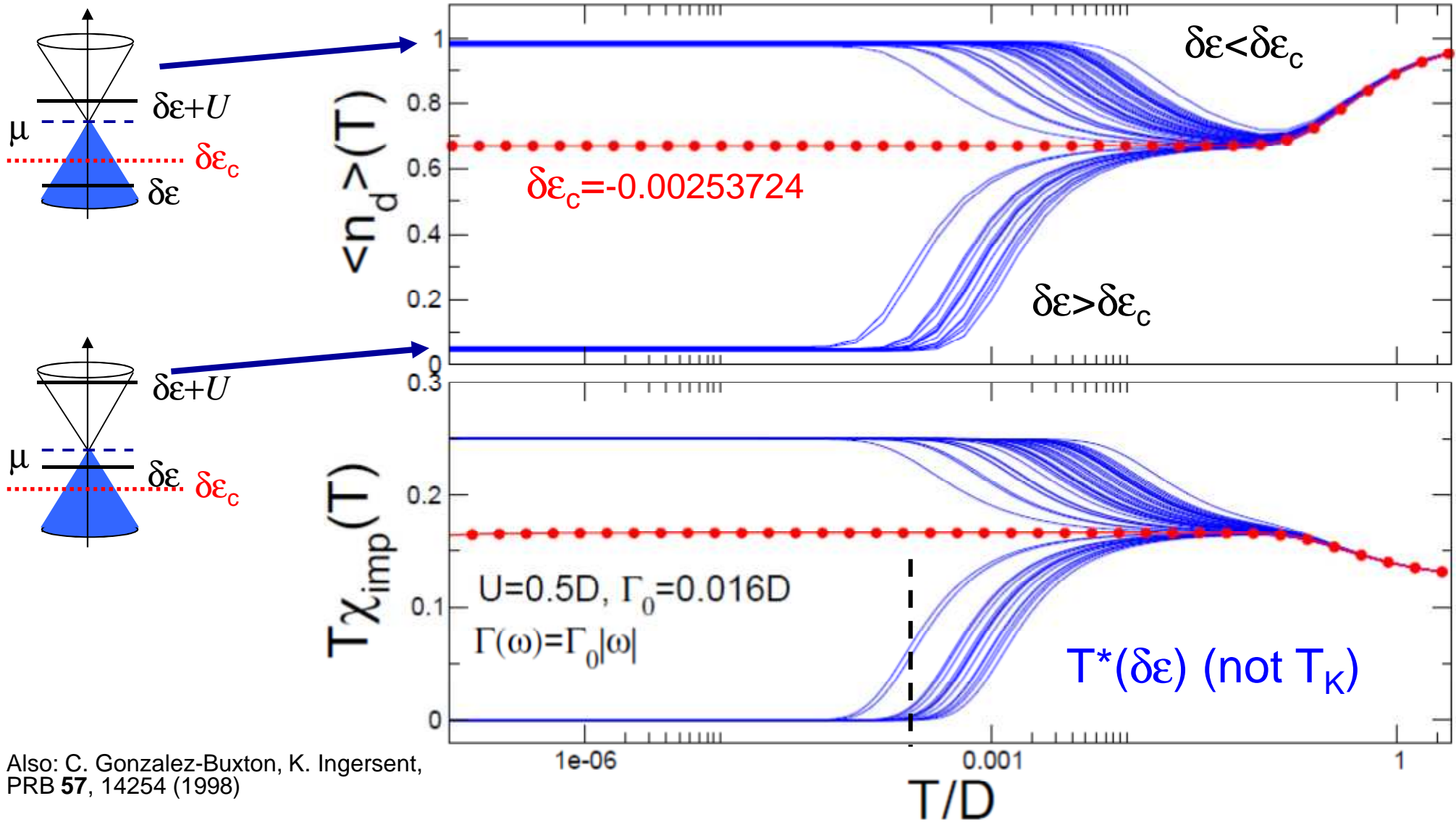
$$\Delta\mu = \mu(V_g) - \mu(0)$$

$\mu(V_g)$: Fermi energy (gate-dependent)

$\mu(0)$: Fermi energy at charge neutrality

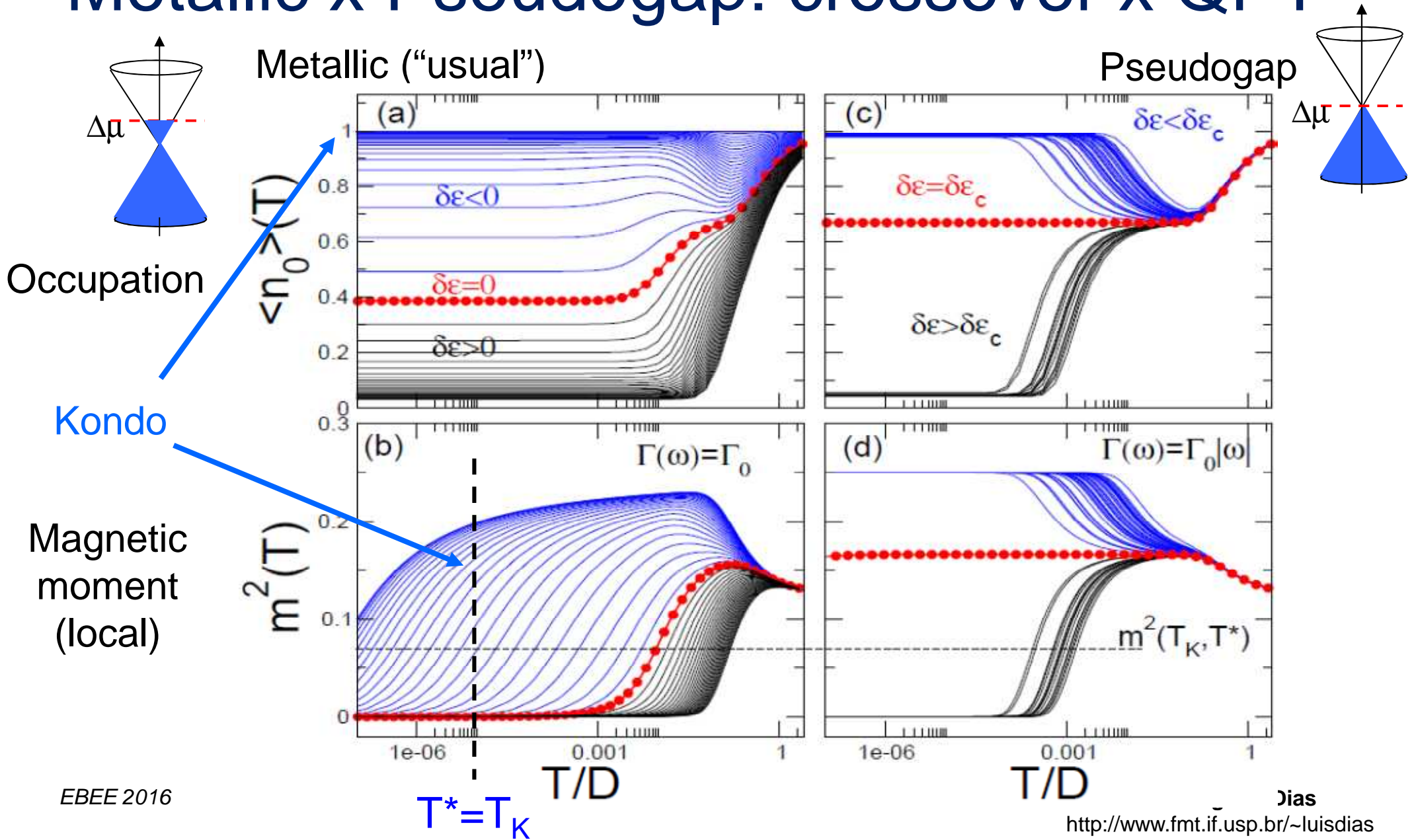
Realization of the “pseudogap Anderson model” for $V_g=0$.

Pseudogap model: quantum phase transition.

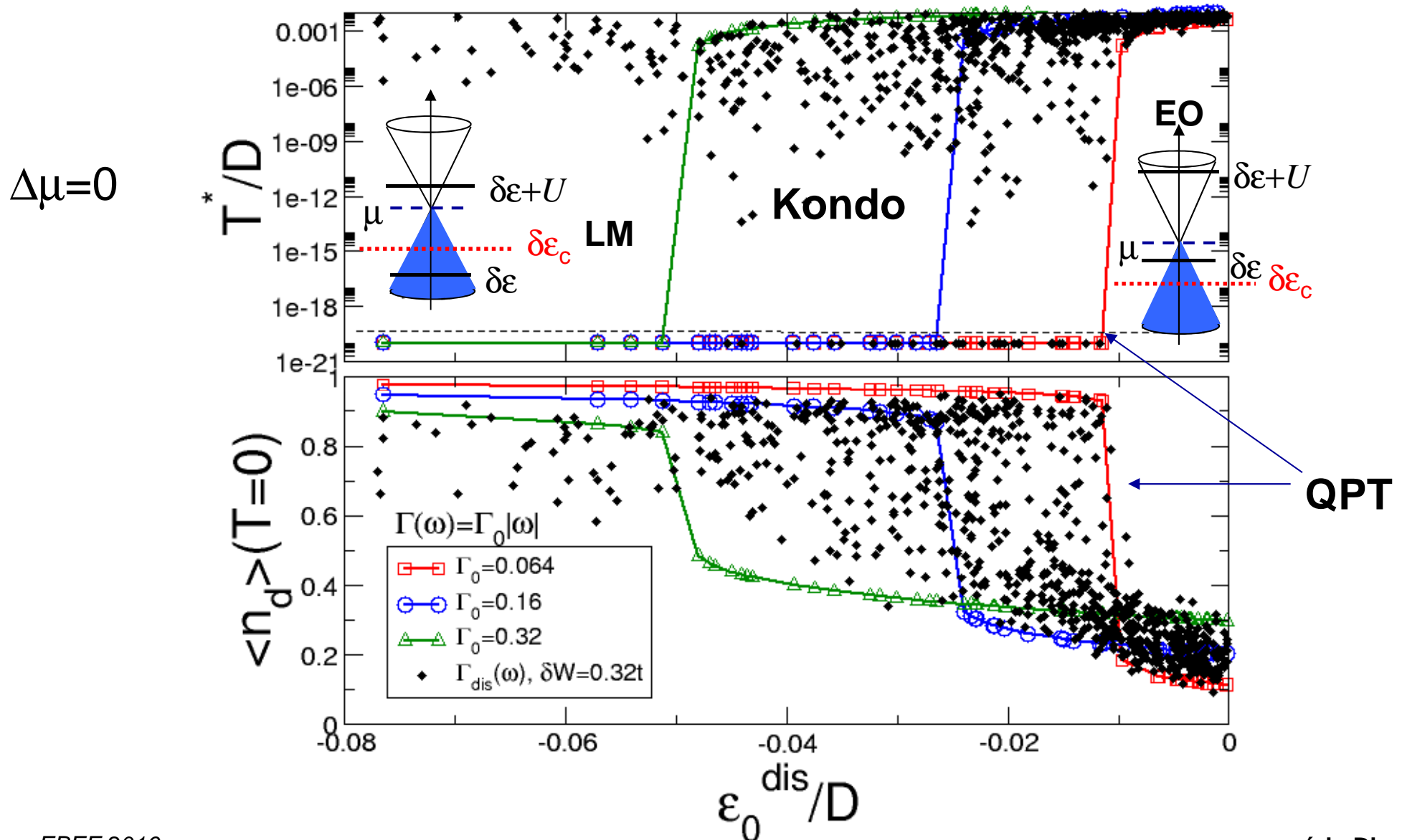


Also: C. Gonzalez-Buxton, K. Ingersent, PRB **57**, 14254 (1998)

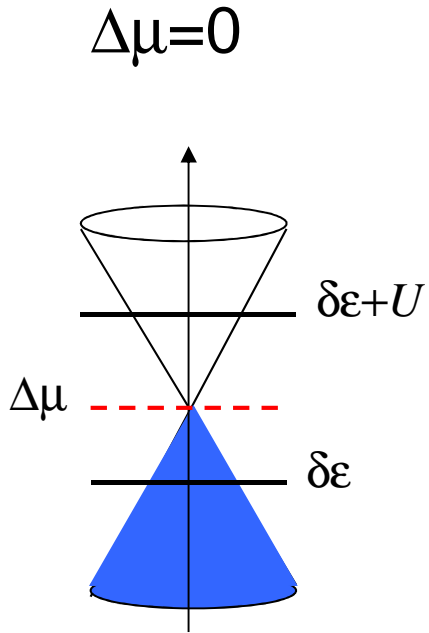
Metallic x Pseudogap: crossover x QPT



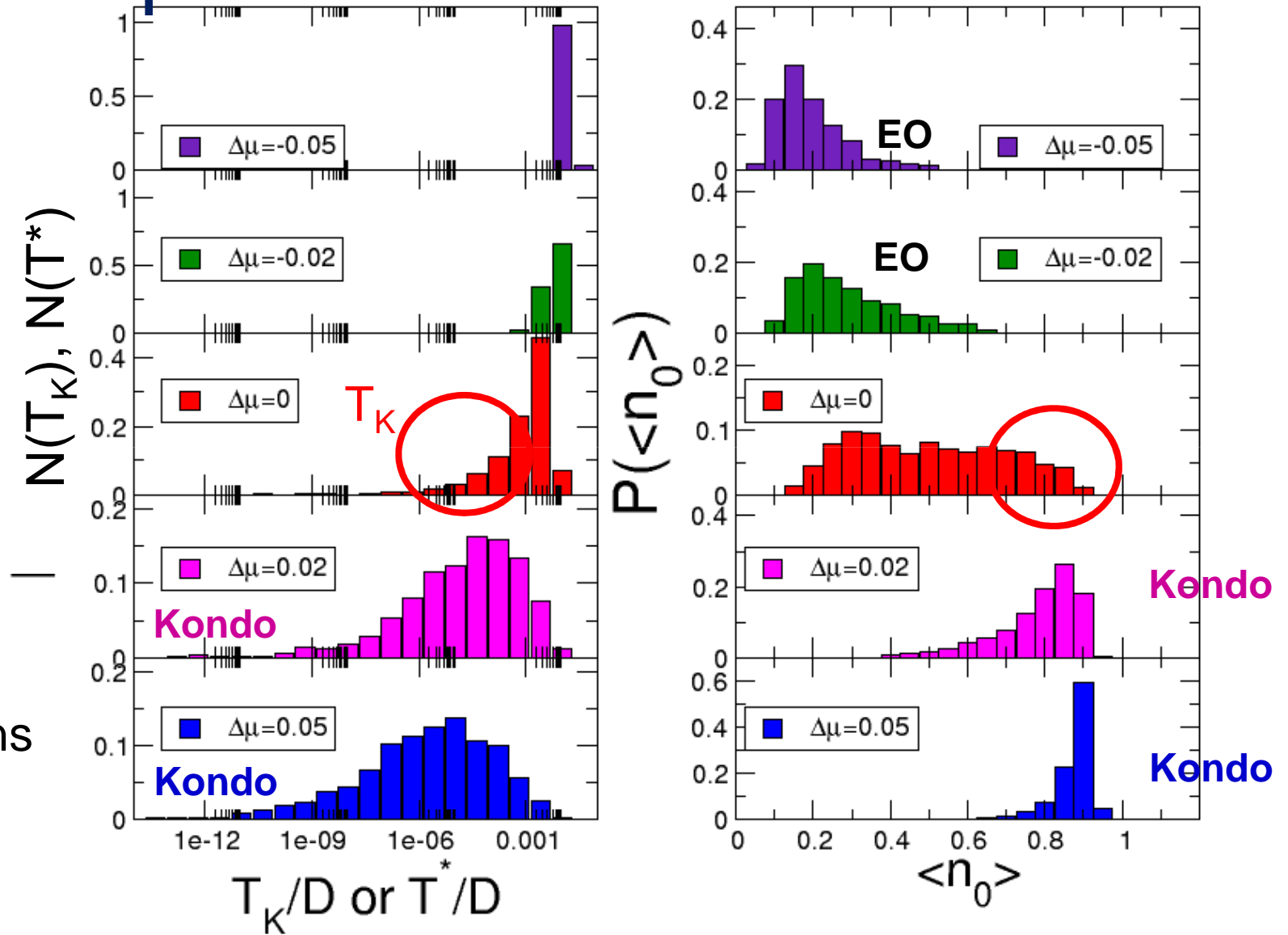
Pseudogap x Disorder: NRG calculations



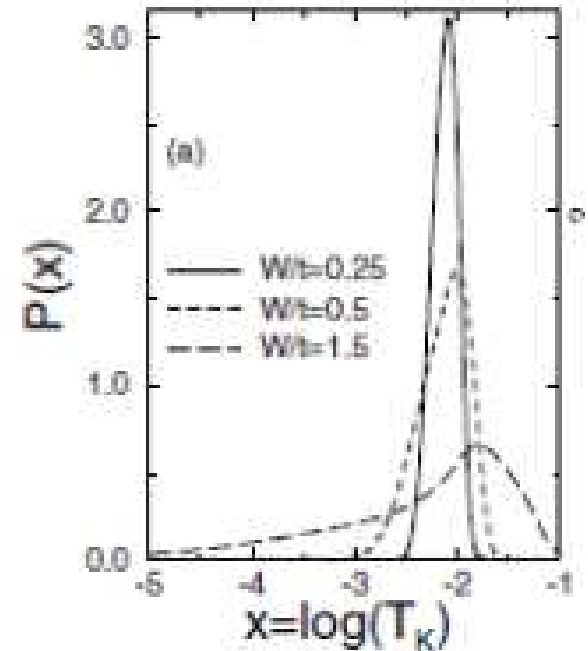
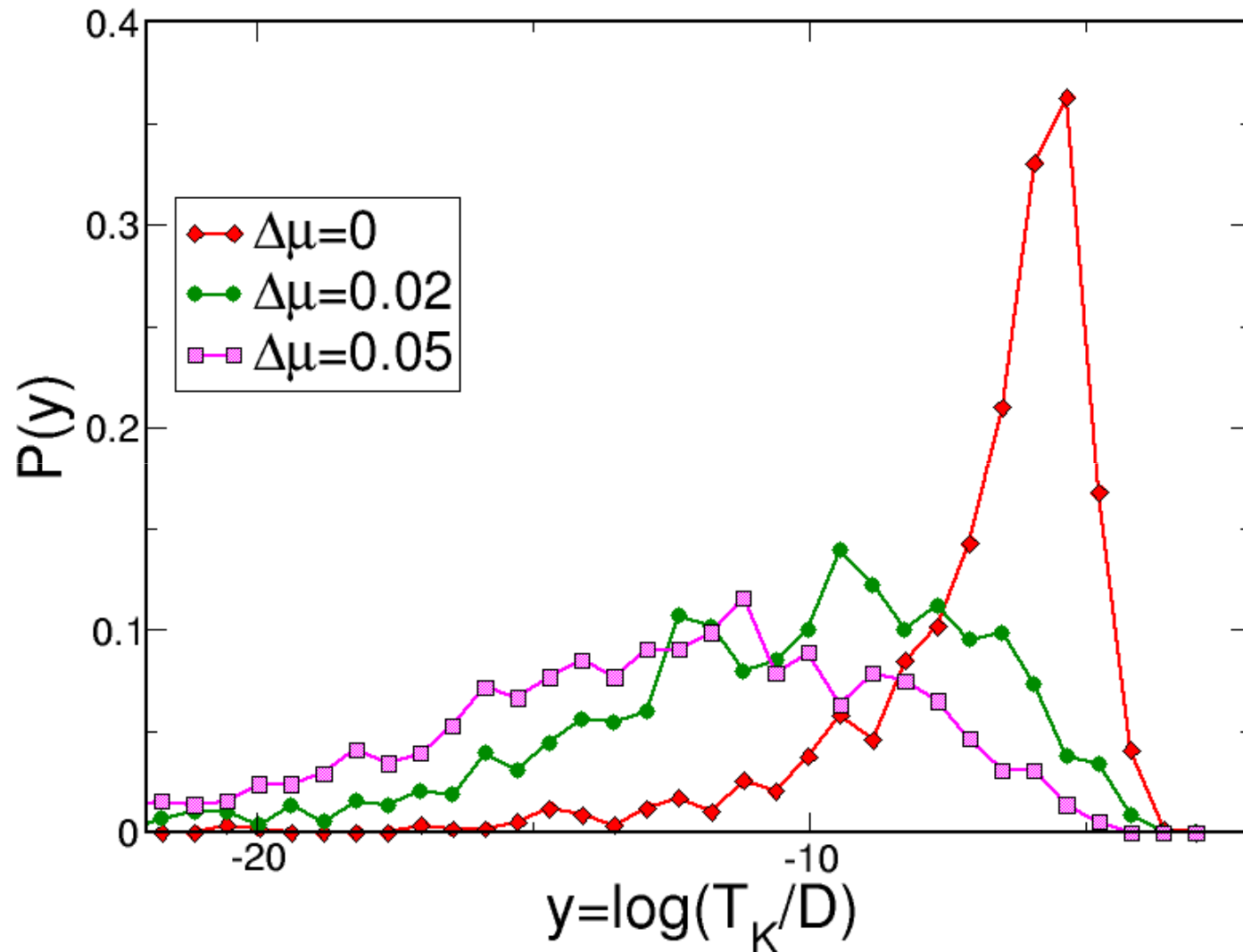
Kondo temperature distributions.



1000 realizations



$P(T_K)$ distributions: Griffiths phase.



E. Miranda and V. Dobrosavljevic,
PRL **86** 264 (2001)

$$P(T_K) \sim T_K^{(\alpha-1)}$$

Hallmark of a “Griffiths phase”

In a few words...“More is Different!”



“ The behavior of large and complex aggregates of elementary particles, it turns out, is not to be understood in terms of simple extrapolation of the properties of a few particles.

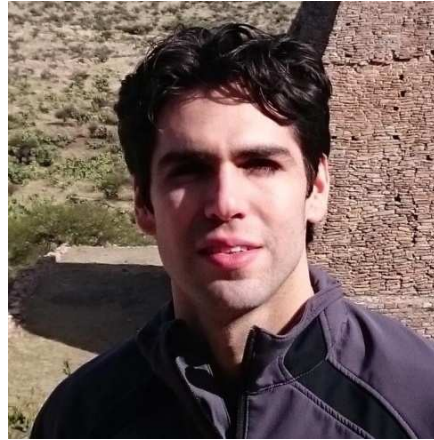
Instead, at each level of complexity entirely new properties appear and the understanding of the new behaviors requires research which I think is as fundamental in its nature as any other.“

Phillip W. Anderson, “More is different”,
Science **177** 393 (1972)

Current Group Members



Luis Dias da Silva
Professor



David Ruiz-Tijerina
Post-doc (now at
Manchester)



Dimy Nanclares
Graduate student



Marcos Medeiros
Graduate student



Raphael Levy
Graduate student

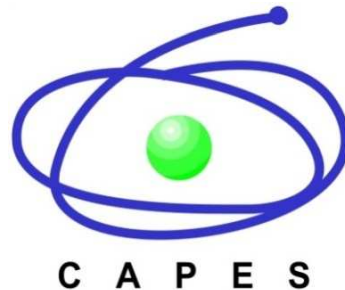


Acknowledgements

Collaborators on the works presented

Sergio Ulloa (Ohio U.)
Saw Hla (Ohio U. /Argonne)
Gayani Perera (Ohio U.)
Nichola Marzari (MIT)
Heather Kulik (MIT)

Elbio Dagotto (Oak Ridge)
Fernando Reboredo (Oak Ridge)
Murilo Tiago (Oak Ridge)
Caio Lewenkopf (UFF)
Vladimir Miranda (UFF)



Support: FAPESP (2016/18495-4); CAPES (PNPD program), CNPq (307107/2013-2 and 449148/2014-9); USP-PRP NAP Q-Nano.