Emergence of Majorana zero-modes in nanostructures..

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Seminário GRHAFITE
25 de junho de 2019

Outline

- Basics: Finding Majorana bound states in condensed matter systems.
- Why Majorana? The road to topological quantum computation.
- From our group:
- *Detecting* Majorana states with quantum dots.
- *Manipulating* Majorana states with (double) quantum dots.
- Quantum circuits with quantum dots and topological quantum wires.

What are Majorana fermions?

Dirac's Equation

- Dirac's equation: $\Psi(\mathbf{r})$ **: 4-spinor**

$$\boxed{i\hbar c\gamma^\mu\partial_\mu\Psi = mc^2\Psi}$$

http://en.wikipedia.org/wiki/Paul_Dirac /

Dirac matrices: obey Clifford algebra

4x4 matrices satisfying

$$\boxed{\begin{aligned}(\gamma^0)^\dagger &= \gamma^0 \\ (\gamma^i)^\dagger &= -\gamma^i\end{aligned}}$$

Solutions: **complex fields**

"particle" and "anti-particle"

"Standard" Rep.

(complex 4x4 matrices)

$$\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\gamma^i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix}$$

	Mass	Energy
$\Psi(\mathbf{r})$	m	E
$\Psi^\dagger(\mathbf{r})$	m	$-E$

Majorana Fermions

Majorana solution: Representarions of Dirac matrices with ***only imaginary non-zero elements*** while still satisfying

http://www.giornalettismo.com/archives/255332/il-ritorno-di-ettore-majorana/

$$\boxed{\begin{aligned}\tilde{\gamma}_0^\dagger &= \tilde{\gamma}_0 \\ \tilde{\gamma}_i^\dagger &= -\tilde{\gamma}_i\end{aligned}} \quad \Rightarrow \quad [i\tilde{\gamma}^\mu \partial_\mu - m]\,\Psi = 0$$

Real solutions: $$[i\tilde{\gamma}^\mu \partial_\mu - m]\,\gamma = 0 \qquad \boxed{\gamma = \gamma^\dagger}$$

- A Dirac fermion can be "written" in terms of two Majorana fermions

$$\begin{cases} \Psi = \frac{1}{2}\,(\gamma_1 + i\gamma_2) \\ \Psi^\dagger = \frac{1}{2}\,(\gamma_1 - i\gamma_2) \end{cases}$$

or

$$\boxed{\gamma_1 = \left(\Psi^\dagger + \Psi\right)}$$

E. Majorana, *Nuovo Cimento* **5**, 171 (1937)

Ettore Majorana: “the mistery”

- One of Enrico Fermi’s “Panisperna Boys”.

- In Fermi’s words:

“There are several categories of scientists in the world; those of second or third rank do their best but never get very far. Then there is the first rank, those who make important discoveries, fundamental to scientific progress. But then there are the geniuses, like Galilei and Newton. ***Majorana was one of these****.”*

- “Disappear” in 1938 during a boat trip between Palermo and Naples.

Ettore Majorana: "the mistery"

Source: Wikipedia

- Several explanations and "conspiracy theories":

Disappearance at sea and suggested explanations [edit]

Majorana disappeared in unknown circumstances during a boat trip from Palermo to Naples on 25 March 1938. Despite several investigations, his body was not found and his fate is still uncertain. He had apparently withdrawn all of his money from his bank account prior to making his trip to Palermo.[6] He may have travelled to Palermo hoping to visit his friend Emilio Segrè, a professor at the university there, but Segrè was in California at that time. On the day of his disappearance, Majorana sent the following note to Antonio Carrelli, Director of the Naples Physics Institute:

> Dear Carrelli,
>
> I made a decision that has become unavoidable. There isn't a bit of selfishness in it, but I realize what trouble my sudden disappearance will cause you and the students. For this as well, I beg your forgiveness, but especially for betraying the trust, the sincere friendship and the sympathy you gave me over the past months.
>
> I ask you to remember me to all those I learned to know and appreciate in your Institute, especially Sciuti: I will keep a fond memory of them all at least until 11 pm tonight, possibly later too.
>
> E. Majorana

This was followed rapidly by a telegram cancelling his earlier plans. He apparently bought a ticket from Palermo to Naples and was never seen again.[6]

Several possible explanations for his disappearance have been proposed, including:

Hypothesis of suicide
proposed by his colleagues Amaldi, Segrè and others[*citation needed*]

Hypothesis of escape to Argentina
proposed by Erasmo Recami and Carlo Artemi (who has developed a detailed hypothetical reconstruction of Majorana's possible escape and life in Argentina)[*citation needed*]

Hypothesis of escape to Venezuela
proposed the Rai 3 talk show "*Chi l'ha Visto?*" published a statement stating that Majorana was alive between 1955 and 1959, living in Valencia, Venezuela.[*citation needed*]

Hypothesis of escape to a monastery
proposed by Sciascia (putatively the Charterhouse of Serra San Bruno)[*citation needed*]

Hypothesis of kidnapping or murder
by Bella, Bartocci, and others, to avoid his participation in the construction of an atomic weapon[*citation needed*]

Hypothesis of escape to become a beggar
by Bascone and Venturini (called the "*omu cani*" or "dog man" hypothesis)[10]

Where do we find Majorana fermions?

Majorana quasiparticles in condensed matter systems?

- Fractional Quantum Hall liquids (ν=5/2): “non-Abelian anyons”.

  Moore and Read, *Nucl. Phys. B* (1991).

- Two-channel Kondo non-Fermi-liquid state.

  Emery, Kivelson, *PRB* (1992).
  Coleman, Ioffe, Tsvelik *PRB* (1995).
  Maldacena, Ludwig, *Nucl. Phys. B.* (1997).
  Zhang, Hewson, Bulla, *Solid State Comm.* (1999).

- Interface of topological insulators with BCS superconductors

  Fu and Kane, *Phys. Rev. Lett.* (2008).

- Spin-polarized (“spinless”) p-wave superconductors.

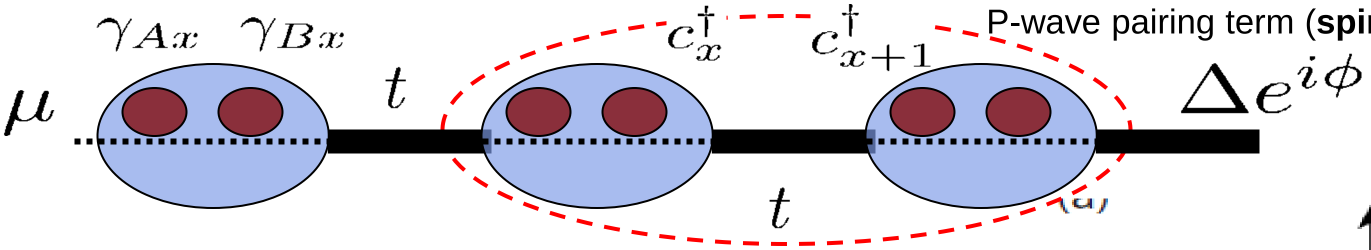
  Read and Green, *Phys. Rev. B* (2000).
  Kitaev, *Phys. Usp.* (2001).

Motivation: entanglement of particles with non-abelian statistics (“Ising anyons”); topologically protected quantum computation.

1D p-wave superconductor (Kitaev model)

$$\boxed{H = -\mu \sum_x c_x^\dagger c_x - \frac{1}{2} \sum_x \left(t c_x^\dagger c_{x+1} + \Delta e^{i\phi} c_x c_{x+1} + h.c. \right)}$$

P-wave pairing term (**spinless** fermions)



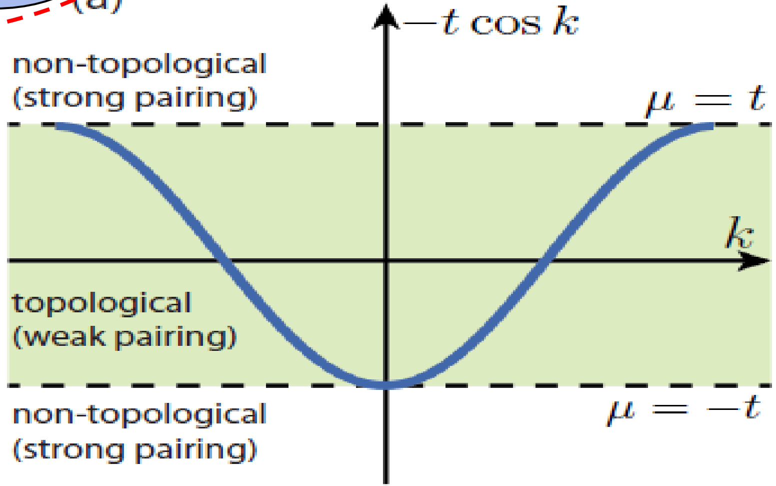
Energy spectrum:

$$\boxed{E(k) = \pm\sqrt{(t\cos k + \mu)^2 + (\Delta \sin k)^2}}$$

$\boxed{|\mu| > t}$ Gapped (E_+-E_->0): **trivial**

$\boxed{\mu = \pm t}$ <u>Gapless modes (E=0) :</u>
$k = \pm\pi$ or $k = 0$

$\boxed{|\mu| < t}$ Gapped: **topological** (Δ≠0)



J. Alicea, *Rep. Prog. Phys.* **75**, 076501 (2012)

Majorana states in the Kitaev model.

Map into a “chain of Majorana modes” using:

$$\begin{cases} c_x = \frac{e^{-i\phi/2}}{2}\left(\gamma_{B,x} + i\gamma_{A,x}\right) \\ c_x^\dagger = \frac{e^{+i\phi/2}}{2}\left(\gamma_{B,x} - i\gamma_{A,x}\right) \end{cases}$$

$$\boxed{H = -\mu \sum_x c_x^\dagger c_x - \frac{1}{2}\sum_x \left(t c_x^\dagger c_{x+1} + \Delta e^{i\phi} c_x c_{x+1} + h.c.\right)}$$

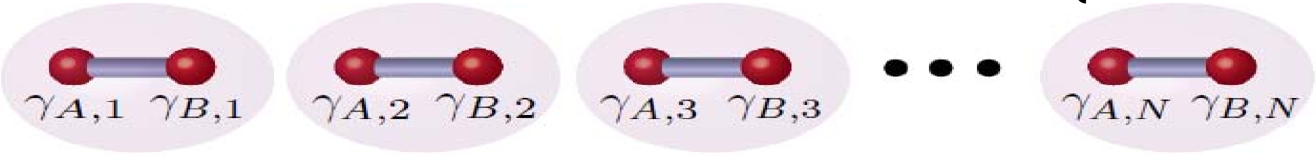
$$\Downarrow$$

$$\boxed{H = -\frac{\mu}{2}\sum_x^N \left(1 + i\gamma_{B,x}\gamma_{A,x}\right) - \frac{i}{4}\sum_x^{N-1} \left(\Delta + t\right)\gamma_{B,x}\gamma_{A,x+1} + \left(\Delta - t\right)\gamma_{A,x}\gamma_{B,x+1}}$$

Majorana states in the Kitaev model.

$$H = -\frac{\mu}{2}\sum_{x}^{N}\left(1 + i\gamma_{B,x}\gamma_{A,x}\right) - \frac{i}{4}\sum_{x}^{N-1}\left(\Delta + t\right)\gamma_{B,x}\gamma_{A,x+1} + \left(\Delta - t\right)\gamma_{A,x}\gamma_{B,x+1}$$

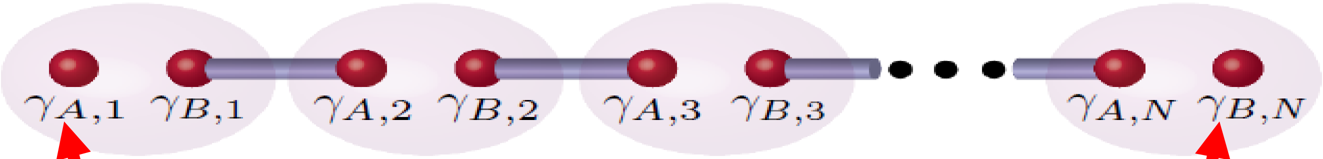
$\boxed{|\mu| > t}$ Gapped: **trivial.** Special case: $\begin{cases} \mu \neq 0 \\ t = \Delta = 0 \end{cases}$



$$E_+ - E_- = 2\mu$$

$\boxed{|\mu| < t}$ Gapped: **topological.** Special case: $\begin{cases} \mu = 0 \\ t = \Delta \neq 0 \end{cases}$

$$E_+ - E_- = 2\Delta$$



Topological regime: Majorana modes (e=μ=0!!!) at the *edges* of the chain!

Can the Kitaev model be realized experimentally?

How to realize a p-wave SC: Quantum wires.

Theory: Lutchyn et al. PRL, **105**, 077001 (2010); Oreg et al. PRL,**105**, 077002 (2010);

- Step 1:

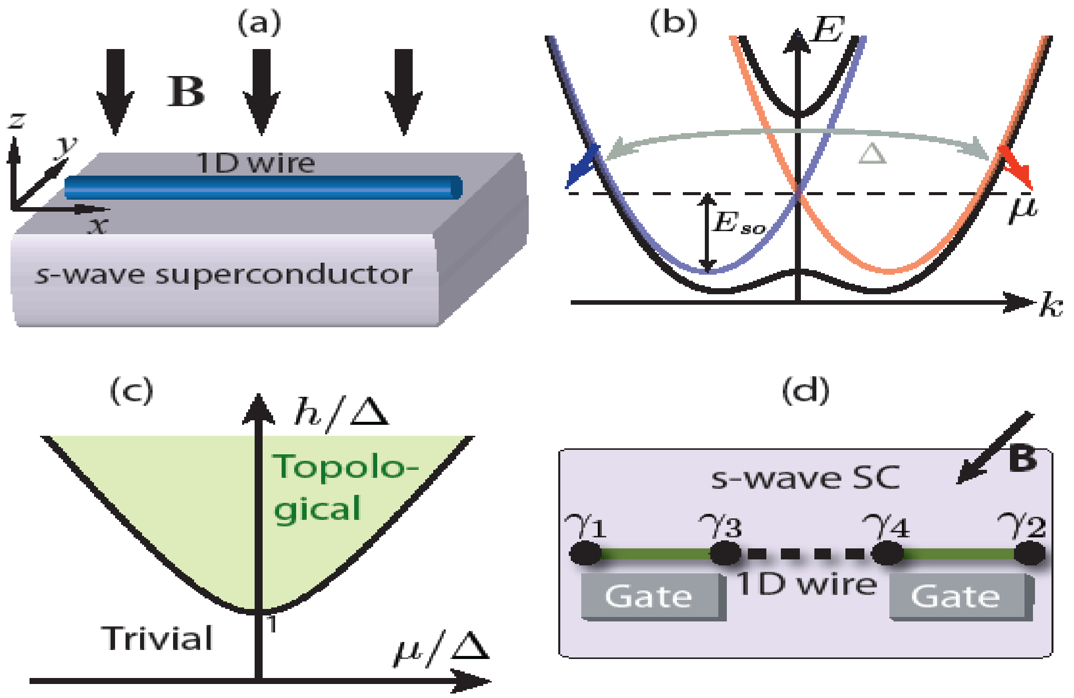
create spinless 1D fermions.

Ingredients: spin-orbit, B field.

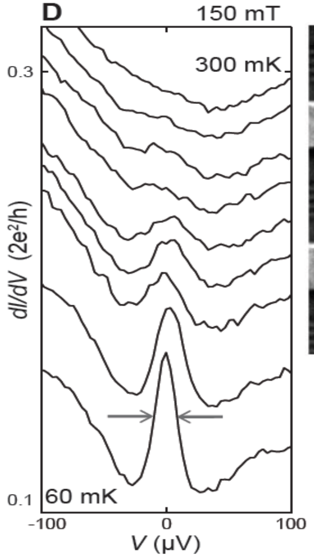
- Step 2:

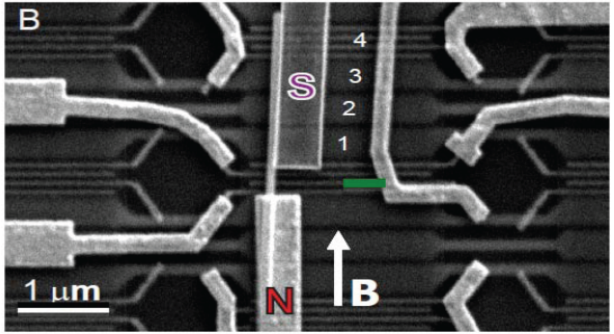
Introduce SC pairing.

Ingredients: proximity with a BCS SC



Experiment on InSb nanowires.

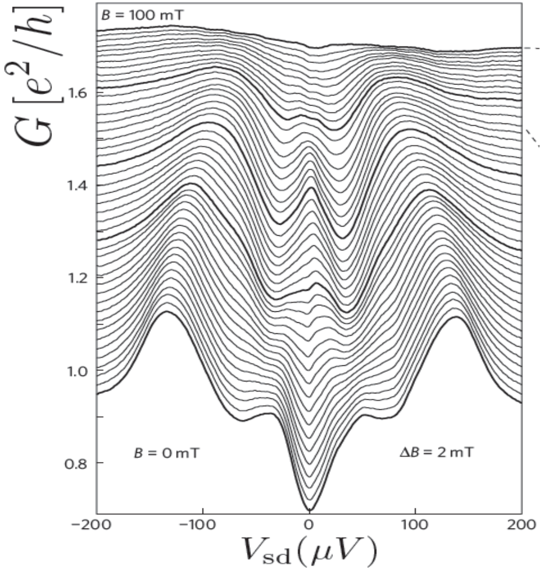


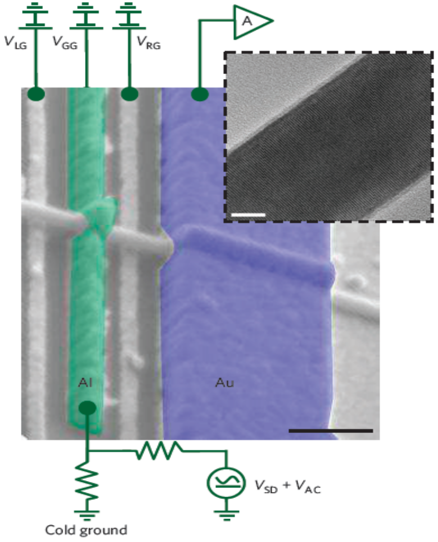


Leo Kouwenhoven (Delft).

Zero-bia peak in tunneling spectroscopy

Mourik *et al.*, Science **336**, 1003–1007 (2012)
Deng *et al.*, Nano Lett. **12**, 6414 (2012)
Das *et al.*, Nature Phys. **8**, 887 (2012)
Prada et al., Phys. Rev. B **86**, 180503 (2012)
Churchill et al., Phys. Rev. B **87**, 241401 (2013)





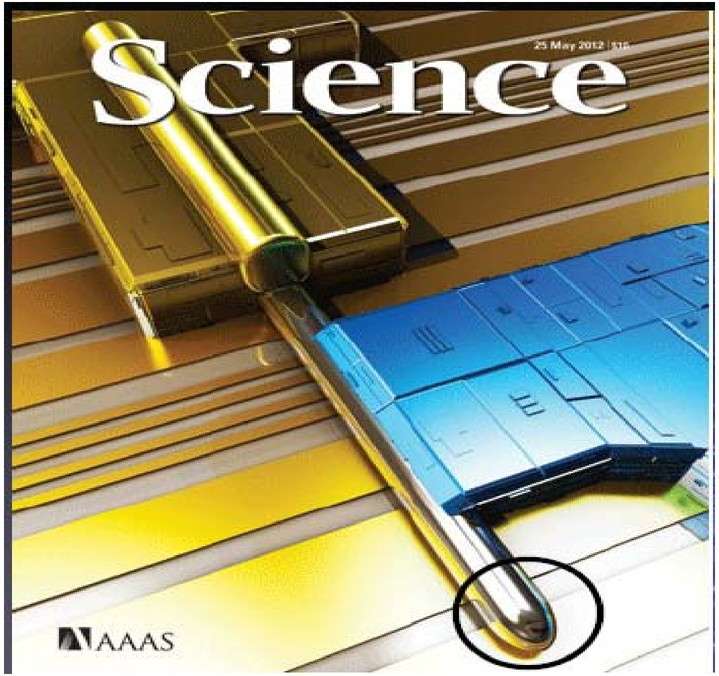
Signatures appear for:

- Large enough magnetic field (topological phase)
- Not too big (that it kills the induced superconductivity)
- Perpendicular to Rashba SO

A success story??

Theory: Lutchyn et al. PRL, **105**, 077001 (2010); Oreg et al. PRL,**105**, 077002 (2010);

Experiment: V. Mourik et al. Science **336** 1003 (2012)



Inside joke:

“Majorana found at the end of a quantum wire”

What do we do with them?

“Topological Quantum Computation”

Microsoft's high stakes game...

The New York Times

Microsoft Spends Big to Build a Computer Out of Science Fiction

https://www.nytimes.com/2016/11/21/technology/microsoft-spends-big-to-build-quantum-computer.html?smid=tw-share

https://www.microsoft.com/en-us/research/lab/quantum/

Microsoft | The AI Blog   The Official Microsoft Blog   Microsoft On the Issues   Transform

Microsoft doubles down on quantum computing bet

November 20, 2016 | Allison Linn

https://blogs.microsoft.com/ai/microsoft-doubles-quantum-computing-bet/

Microsoft / Features

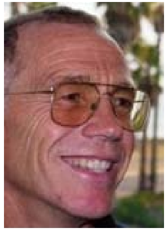
With new Microsoft breakthroughs, general purpose quantum computing moves closer to reality

By Allison Linn
25 September, 2017

https://news.microsoft.com/features/new-microsoft-breakthroughs-general-purpose-quantum-computing-moves-closer-reality/

Microsoft Quantum

https://www.microsoft.com/en-us/research/lab/quantum/



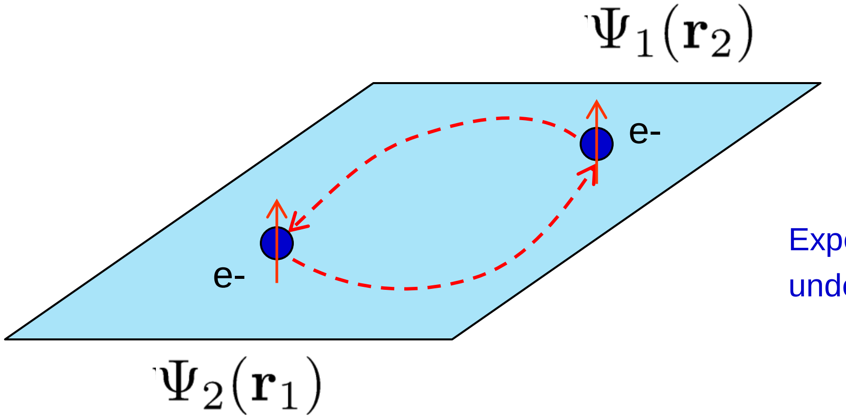
Michael Freedman
(Univ. of California - Santa Barbara)

Leo Kouwenhoven
(Delft University)

Charlie Marcus
(Univ. of Copenhaguen – Niels Bohr Inst)

The key: “braiding”

Fermions' exchange statistics



Indistinguishable particles :

$$|\psi_i(\mathbf{r}_1,\mathbf{r}_2)|^2 = |\psi_f(\mathbf{r}_1,\mathbf{r}_2)|^2$$

Experimental fact (3D): anti-symmetric wavefunction under particle exchange :

$$\psi(\mathbf{r}_1,\mathbf{r}_2) = -\psi(\mathbf{r}_2,\mathbf{r}_1)$$

Corolary (Pauli's principle): two fermions cannot occupy the same quantum state.

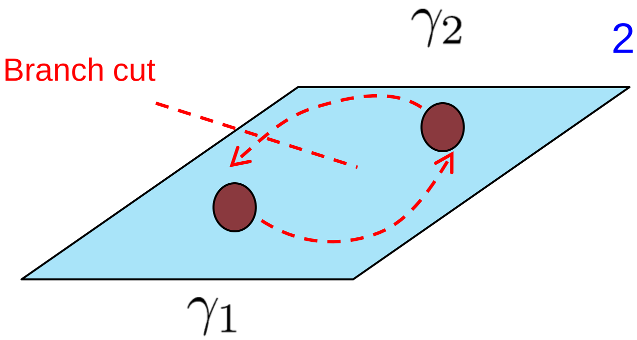
$$\boxed{\psi(\mathbf{r}_1,\mathbf{r}_1) = 0}$$

Constraint to Dirac fields:

$$\boxed{\Psi_1(\mathbf{r}_1)\Psi_2(\mathbf{r}_2) = -\Psi_1(\mathbf{r}_2)\Psi_2(\mathbf{r}_1)}$$

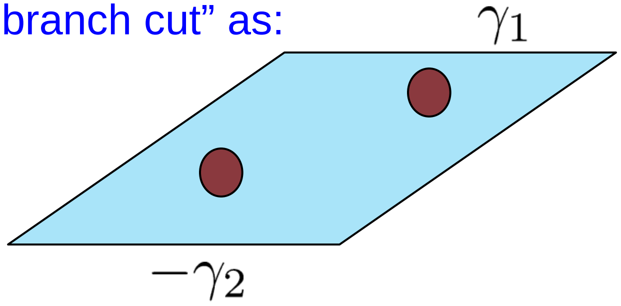
“Braiding”: fermions (including Majorana’s).

$$\boxed{\gamma_1\gamma_2 \to -\gamma_2\gamma_1}$$

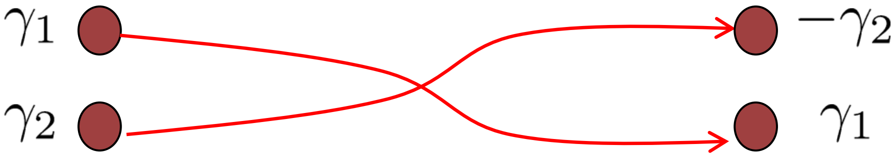


2D: we can choose the “branch cut” as:

$$\gamma_1 \to -\gamma_2$$
$$\gamma_2 \to +\gamma_1$$



Pictoric representation:



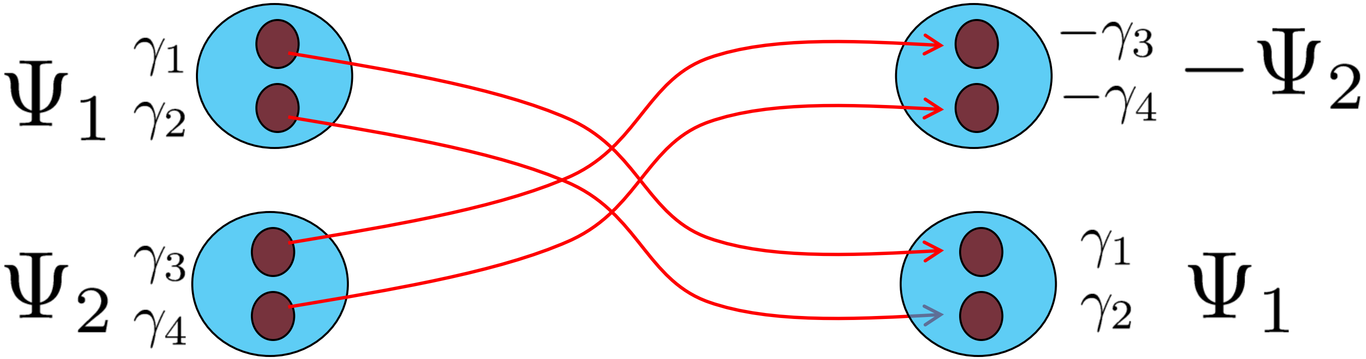
In general:

$$\gamma_i\gamma_j \to -\gamma_j\gamma_i$$

$$\boxed{\begin{aligned}\gamma_i &\to -\gamma_j\\ \gamma_j &\to +\gamma_i\end{aligned}}$$

“Braiding”: Dirac fermions.

$$\gamma_1 \to -\gamma_3 \qquad \gamma_2 \to -\gamma_4$$
$$\gamma_3 \to +\gamma_1 \qquad \gamma_4 \to +\gamma_2$$



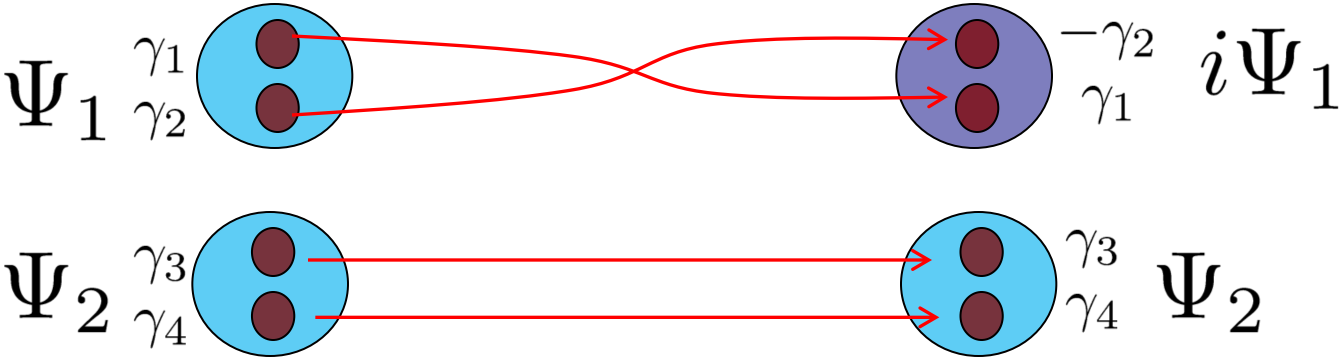
$$\boxed{\Psi_1 = \frac{1}{2}(\gamma_1 + i\gamma_2)}$$
$$\boxed{\Psi_2 = \frac{1}{2}(\gamma_3 + i\gamma_4)}$$

$$\Psi_1\Psi_2 \to -\Psi_2\Psi_1$$ Ok, it works!

“Braiding”: what if I exchange γ_1 and γ_2 only???

$$\gamma_1 \to -\gamma_2$$
$$\gamma_2 \to +\gamma_1$$

$$\frac{1}{2}(-\gamma_2 + i\gamma_1) = i\Psi_1$$

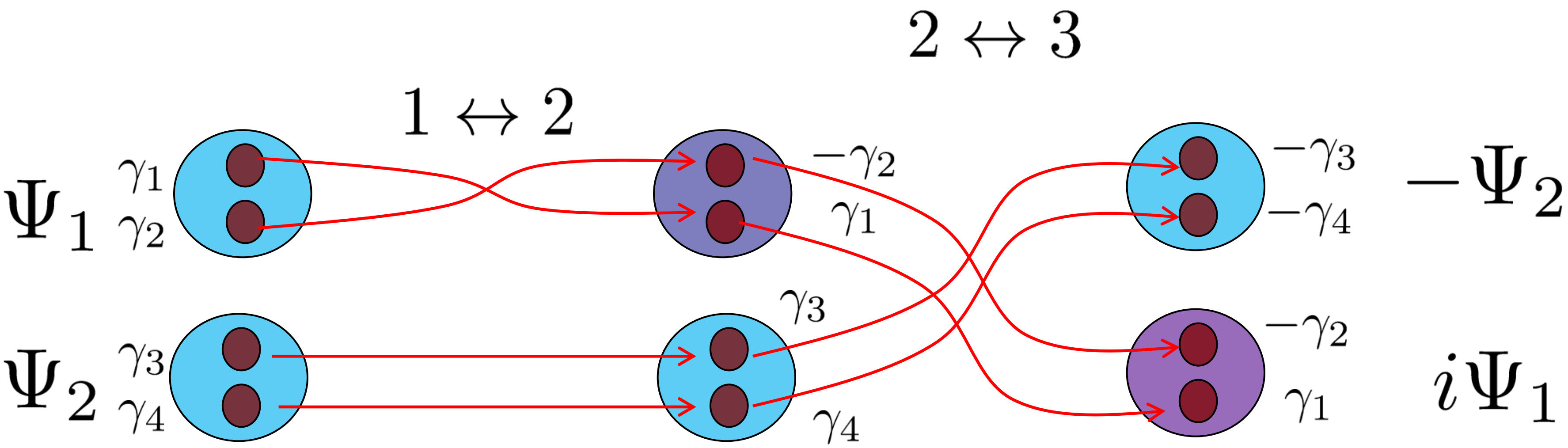


$$\boxed{\Psi_1 = \frac{1}{2}(\gamma_1 + i\gamma_2)}$$

$$\boxed{\Psi_2 = \frac{1}{2}(\gamma_3 + i\gamma_4)}$$

$$\Psi_1\Psi_2 \to i\Psi_1\Psi_2 = e^{i\frac{\pi}{2}}\Psi_1\Psi_2$$

“Anyonic” statistics!

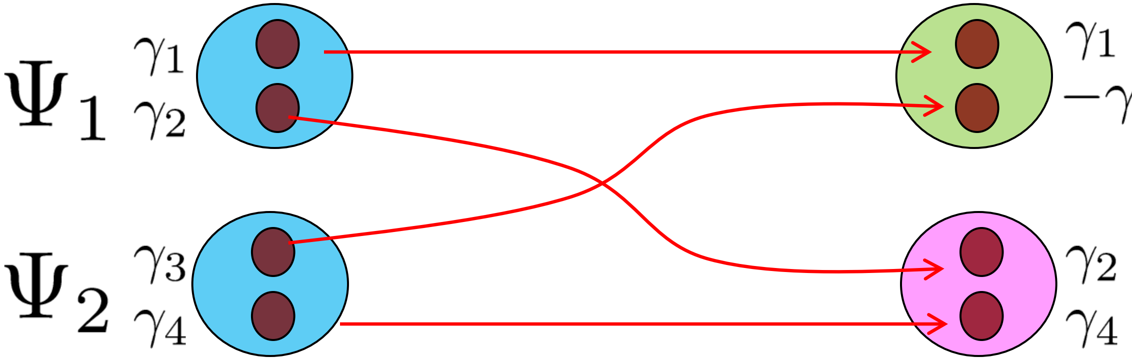


$$\Psi_1\Psi_2 \rightarrow -i\Psi_2\Psi_1 = e^{-i\frac{\pi}{2}}\Psi_2\Psi_1$$

Fermion? Bóson? *Anyon!*

"Braiding": what if I exchange γ_2 and γ_3 ???

$$\gamma_2 \to -\gamma_3$$
$$\gamma_3 \to +\gamma_2$$



$$\frac{1}{2}(\gamma_1 - i\gamma_3) = \frac{1}{2}\left[\Psi_1 + \Psi_1^\dagger - i(\Psi_2 + \Psi_2^\dagger)\right]$$

$$\frac{1}{2}(\gamma_2 + i\gamma_4) = \frac{1}{2}\left[\Psi_2 - \Psi_2^\dagger - i(\Psi_1 - \Psi_1^\dagger)\right]$$

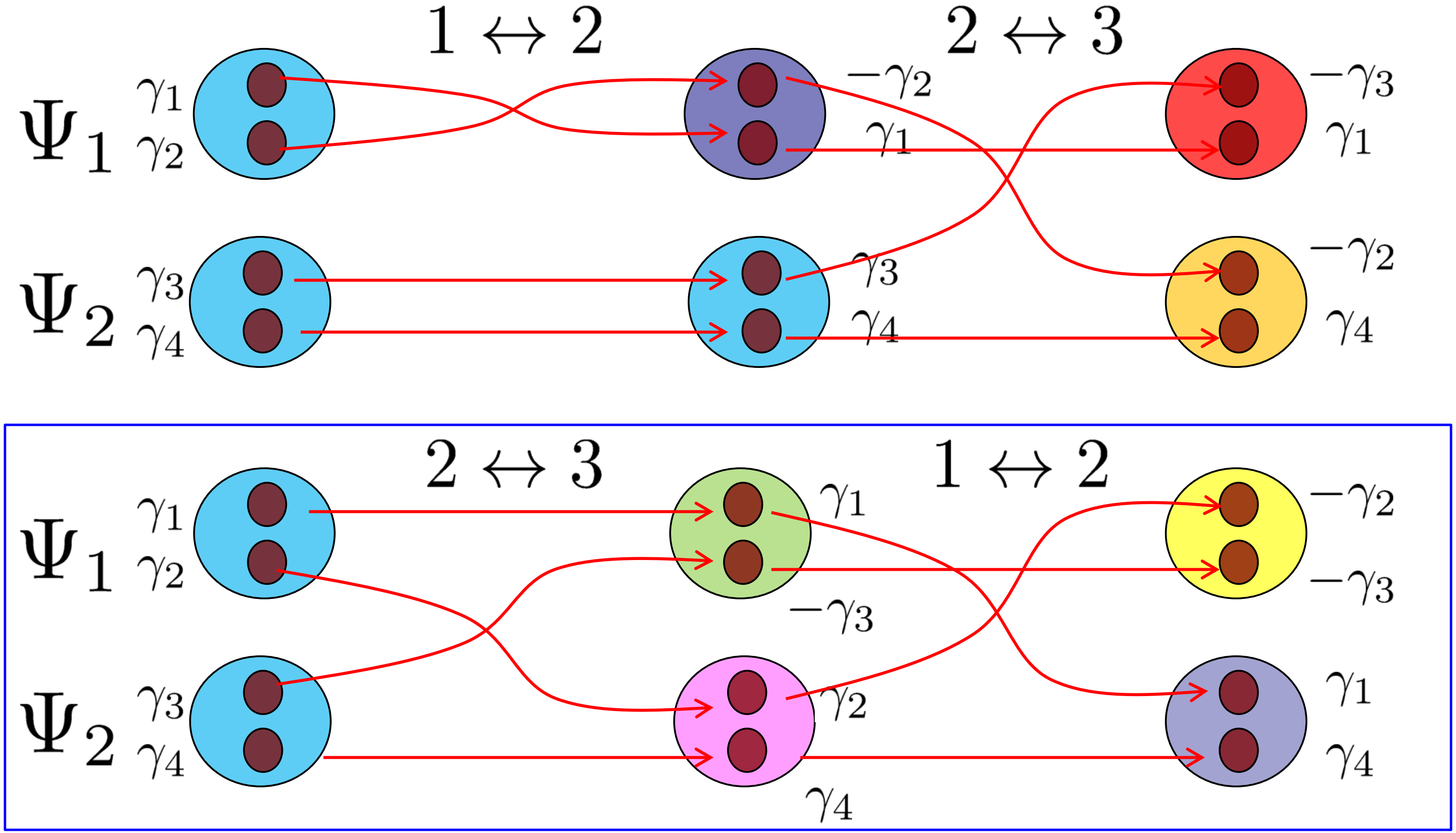
$$\boxed{\Psi_1 = \frac{1}{2}(\gamma_1 + i\gamma_2)}$$

$$\boxed{\Psi_2 = \frac{1}{2}(\gamma_3 + i\gamma_4)}$$

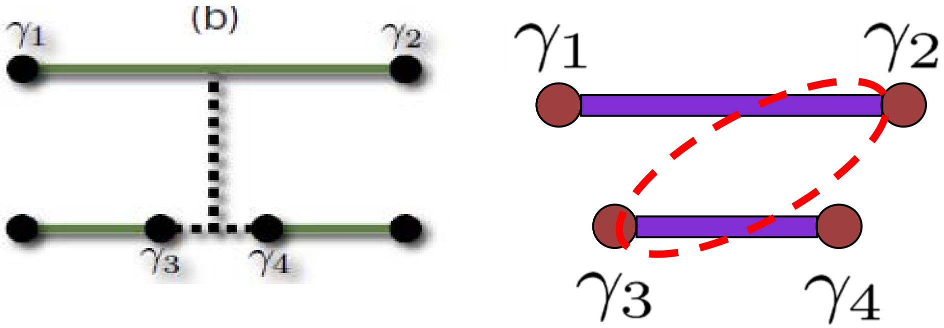
$$\Psi_1\Psi_2 \to ?? \cdots$$

Something else...

And the order matters! (=non-Abelian anyons)



Topological quantum computation: basics.



Braiding operations = Hilbert space rotations ("gates") U_{ij} :

Final result depends on the **order** which you apply the gates.

$$\begin{cases} |\Psi_0'\rangle = U_{12}U_{23}|\Psi_0\rangle \\ |\Psi_0''\rangle = U_{23}U_{12}|\Psi_0\rangle \neq |\Psi_0'\rangle \end{cases}$$

non-Abelian statistics.

(non-Abelian Ising anyons)

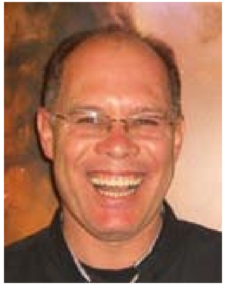
Braidings lead to topologically distinct final states.

Quantum information can be coded in a "topologically protected" manner.

Detecting MBS with quantum dots.

Collaborators
in this work:

David Ruiz-Tijerina
Post-doc IFUSP
(2013-2016)



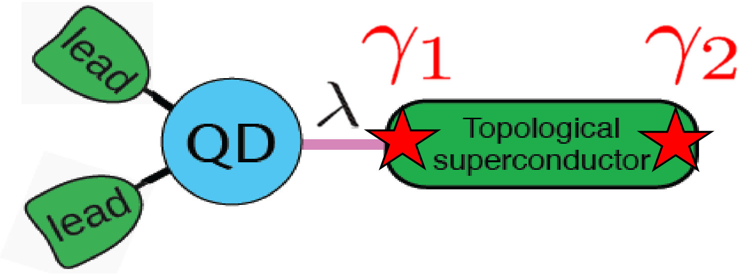
Carlos Egues
IFSC-USP

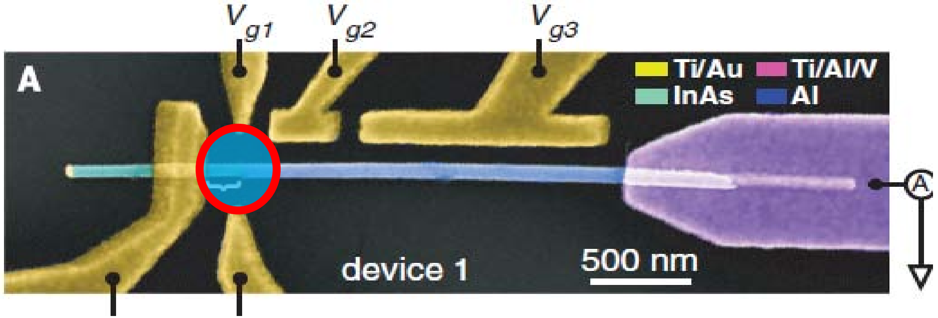
Edson Vernek
UFU

D. A. Ruiz-Tijerina et al. *Phys Rev B* **91** 115435 (2015).

How to positively identify an MBS?

- Quantum dot coupled to metallic leads coupled with at the end of the nanowire.





Theory

Liu and Baranger, *Phys Rev B* **84** 201308 (2011).

Vernek et al., *Phys Rev B* **89** 165314 (2014).

Ruiz-Tijerina et al. *Phys Rev B* **91** 115435 (2015).

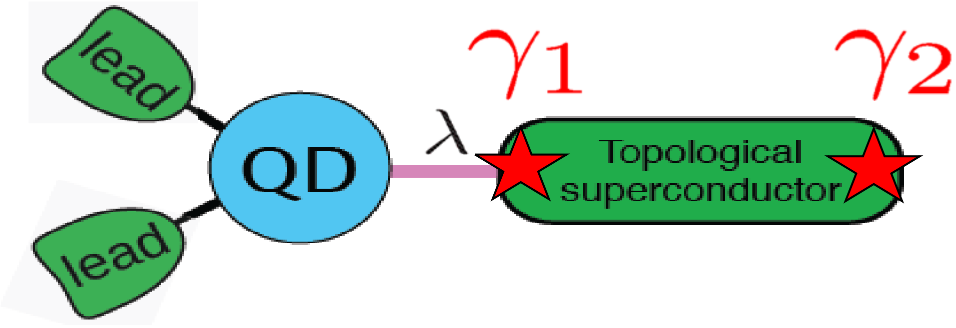
Experiment (Marcus' group)

M.T. Deng et al., *Science* **354** 1557 (2016).

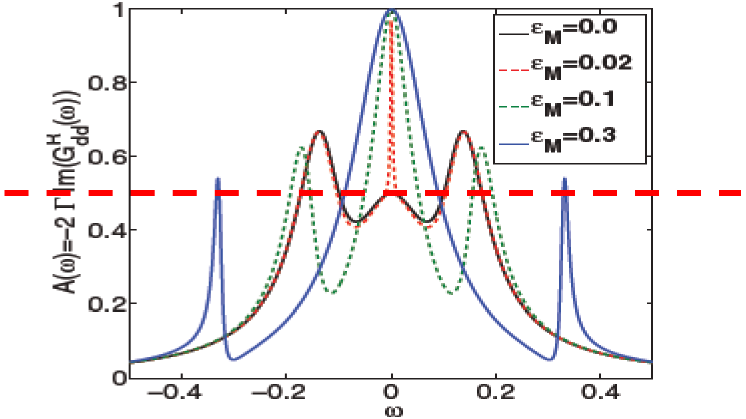
How to positively identify an MBS?

Liu and Baranger, *Phys Rev B* **84** 201308 (2011).

Vernek et al., *Phys Rev B* **89** 165314 (2014).



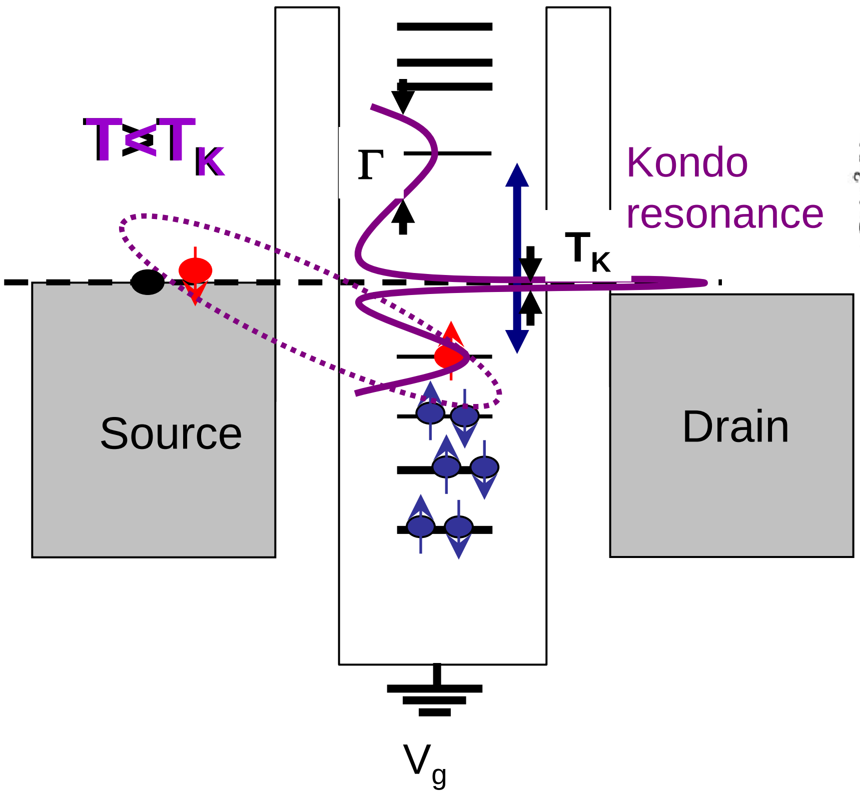
U=0!!!

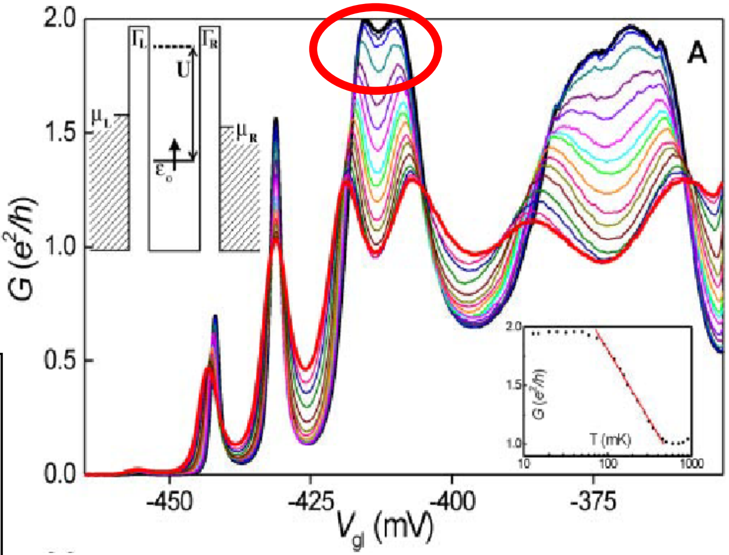


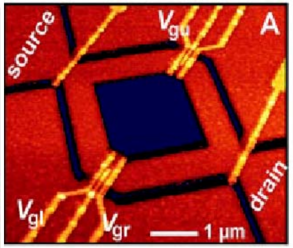
- Connect a quantum dot + metallic leads at the end of the nanowire.
- Measure conductance through the dot
- 0.5 e^2/h = signature of the Majorana mode for U=0
- What happens for the (common) case of non-zero U???

  Ruiz-Tijerina et al. *Phys Rev B* **91** 115435 (2015).

Kondo Effect in Quantum Dots: zero-bias transport.





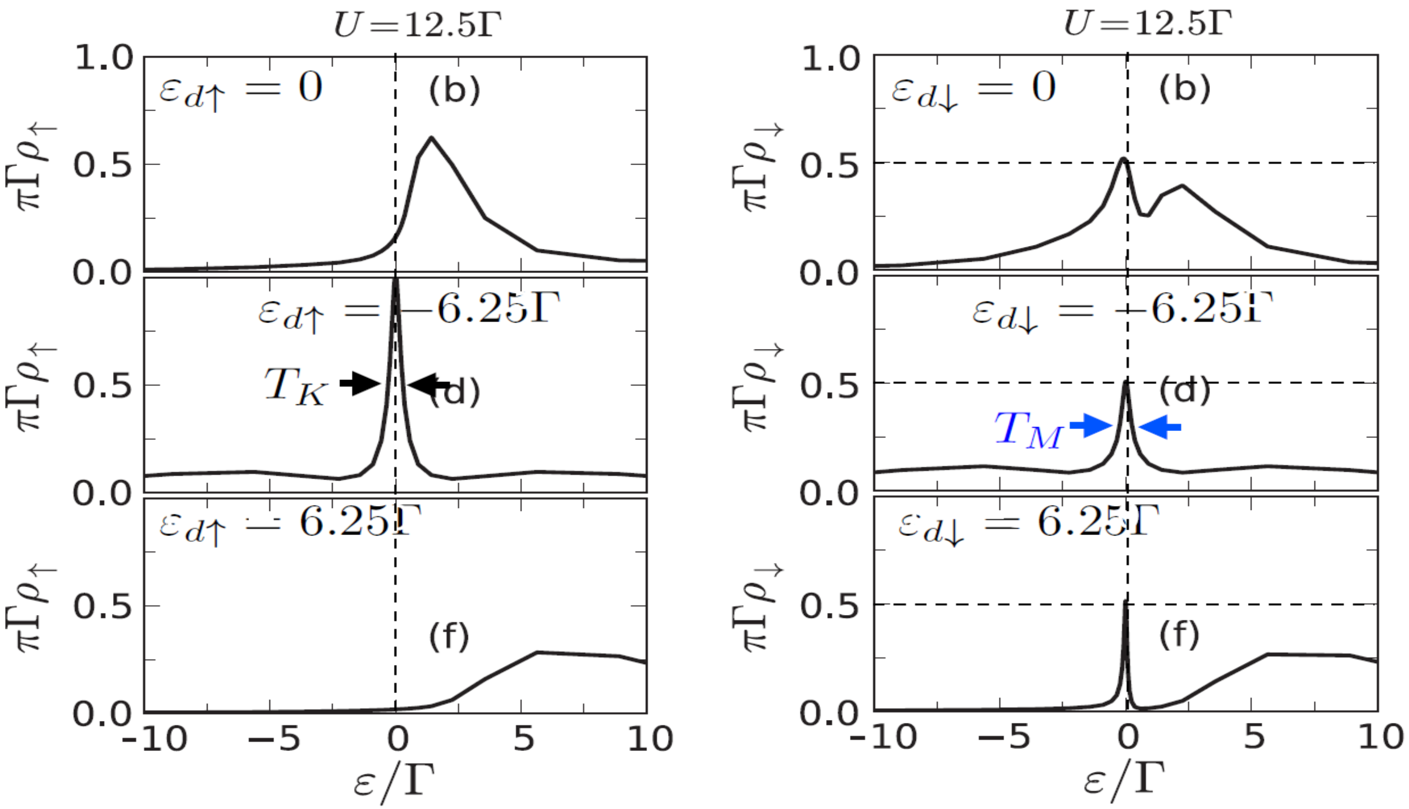


van der Wiel et al., *Science* **289** 2105 (2000).

- $T>T_K$: Coulomb blockade (low G)
- $T<T_K$: Kondo singlet formation
- Kondo resonance at E_F (width T_K).
- New conduction channel at E_F:

Zero-bias enhancement of G ($\rightarrow 2e^2/h$!)

Majorana-Kondo co-existence



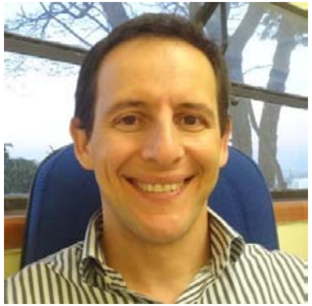
D. A. Ruiz-Tijerina et al. *Phys Rev B* **91** 115435 (2015).

Consistent with:

M. Lee, et al., *Phys. Rev. B* **87**, 241402 (2013).

Cheng et al., *Phys. Rev. X* **4**, 031051 (2014).

Membros do grupo



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Raphael Levy
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Bruna Mendonça
Doutorado

Jesus Cifuentes
Mestrado

Rafael Magaldi
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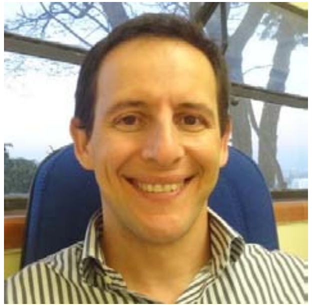
João Victor Ferreira Alves
Mestrado

Lucas Baldo
Mestrado

Membros do grupo



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Manipulation of MBS using *double* quantum dots

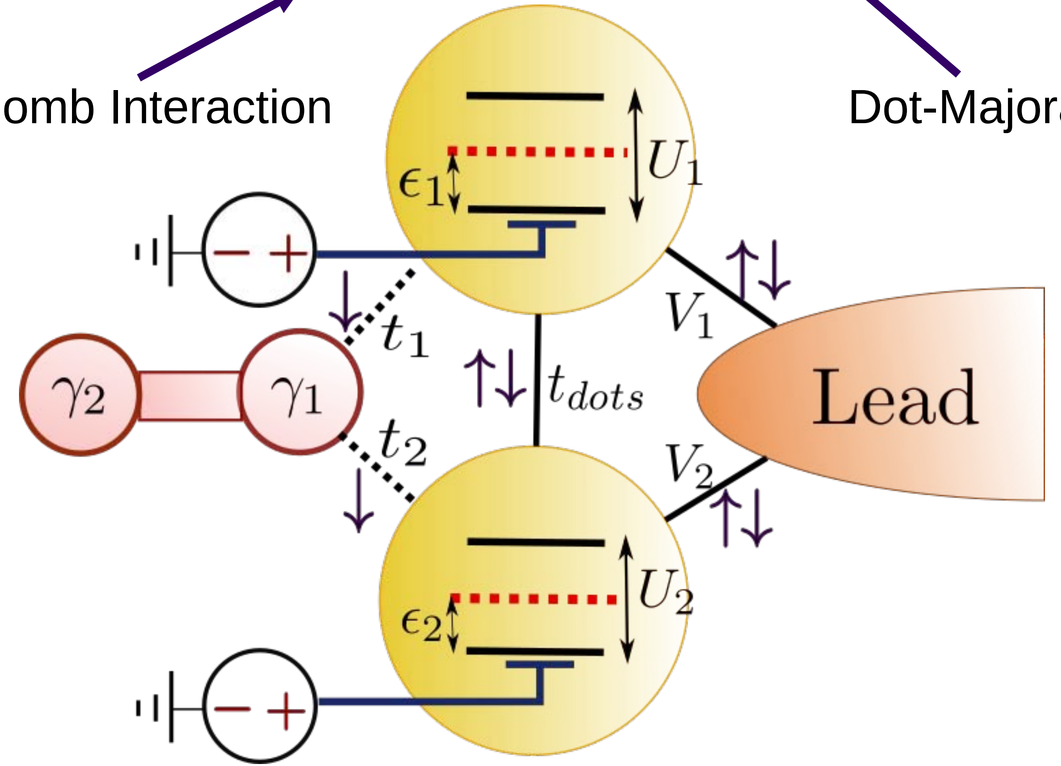
$$H = \sum_{i=1}^{2} \sum_{k,\sigma} \left(\epsilon_i + \frac{U_i}{2} \right) d_{i\sigma}^\dagger d_{i\sigma} + \frac{U_i}{2} (d_{i\sigma}^\dagger d_{i\sigma} - 1)^2 + t_i \gamma_1 d_{i,\downarrow} + t_i^* d_{i,\downarrow}^\dagger \gamma_1 + V_i d_{i\sigma}^\dagger c_{k\sigma} + V_i^* c_{k\sigma}^\dagger d_{i\sigma} .$$

Energy level

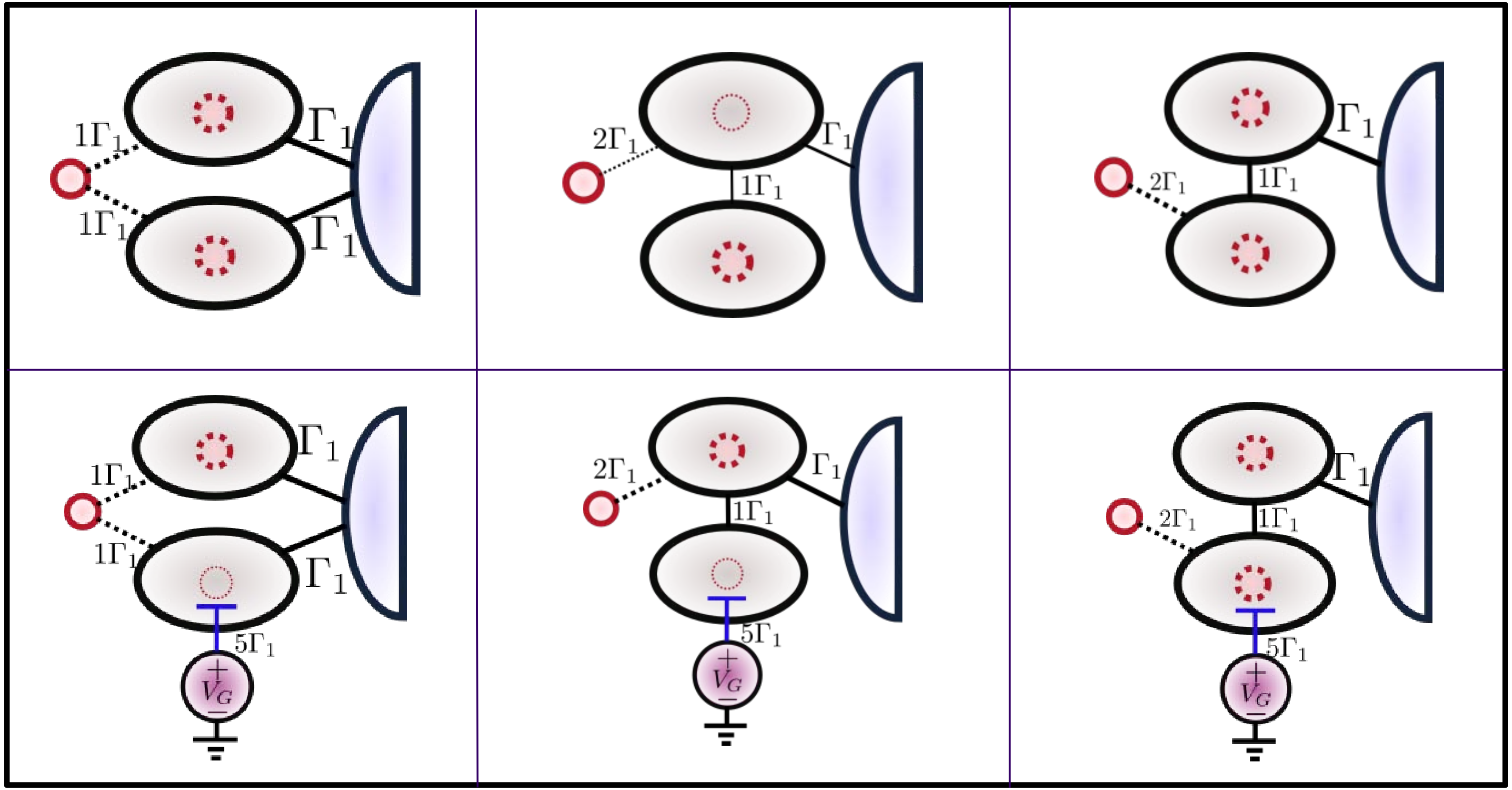
Coulomb Interaction

Dot-Majorana

Dot-Lead

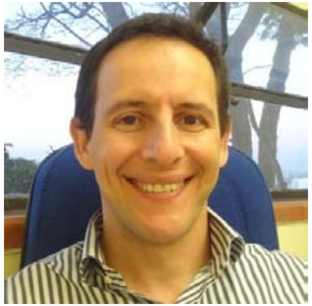


Manipulation of MBS using *double* quantum dots



Cifuentes, LDS, arXiv 1905.09140 (2019) – Submitted to *Phys Rev B*.

Membros do grupo



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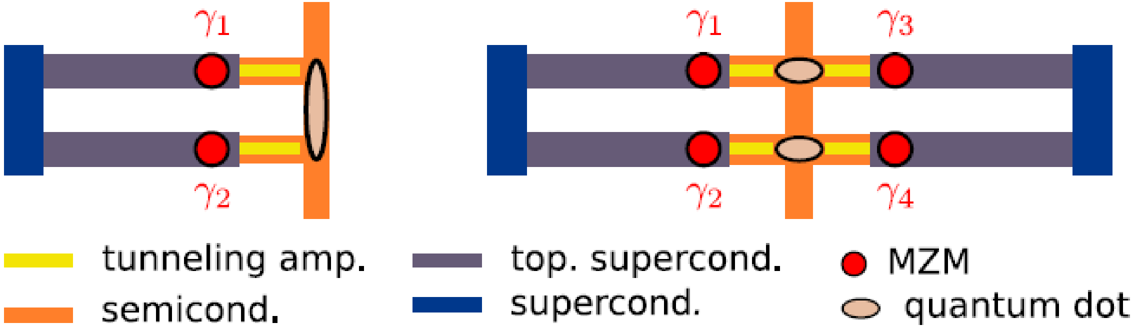
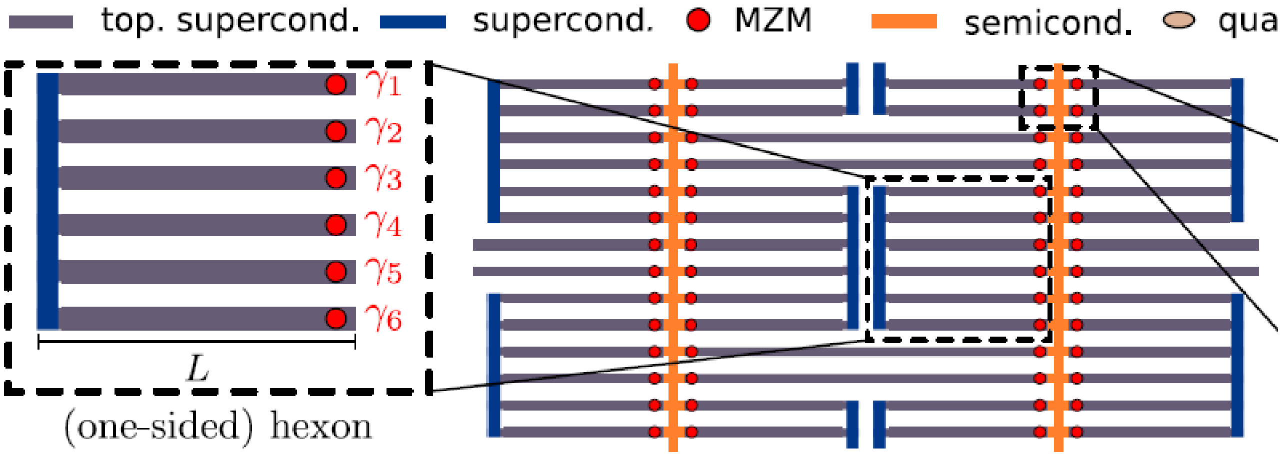
Scalable circuits for topological quantum computation

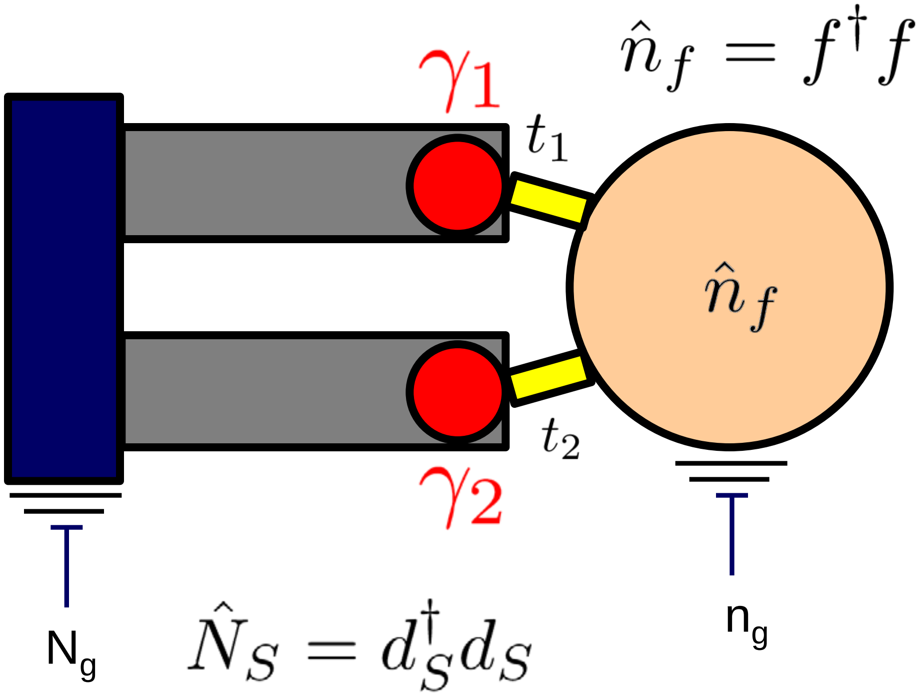




Karzig et al. *Phys. Rev. B* **95** 235305 (2017).

Charging energy model for the circuit

Example: two MZMs and 1 dot



$$H = H_C + H_{\rm dot} + H_{\rm coup}$$

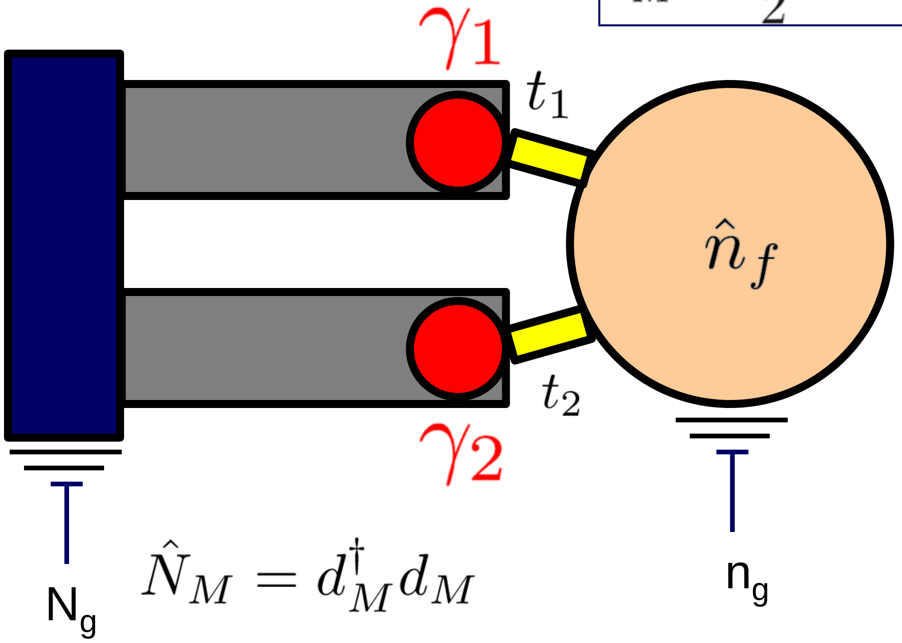
$$\begin{cases} H_C = E_C\left(\hat{N}_S - N_g\right)^2 \\ H_{\rm dot} = \varepsilon_c\left(\hat{n}_f - n_g\right)^2 \\ \boxed{H_{\rm coup} = -i\left(t_1 f^\dagger \gamma_1 + t_2 f^\dagger \gamma_2\right) + \text{H.c.}} \end{cases}$$

Quantum dot and “Majorana island” are “small capacitors” with charging energies(=“Capacitance”).

$$\varepsilon_C \gg E_C$$

Charging energy model for the circuit

Two MZMs define a Dirac fermion: $\boxed{d_M^\dagger = \frac{1}{2}(\gamma_1 + i\gamma_2)}$

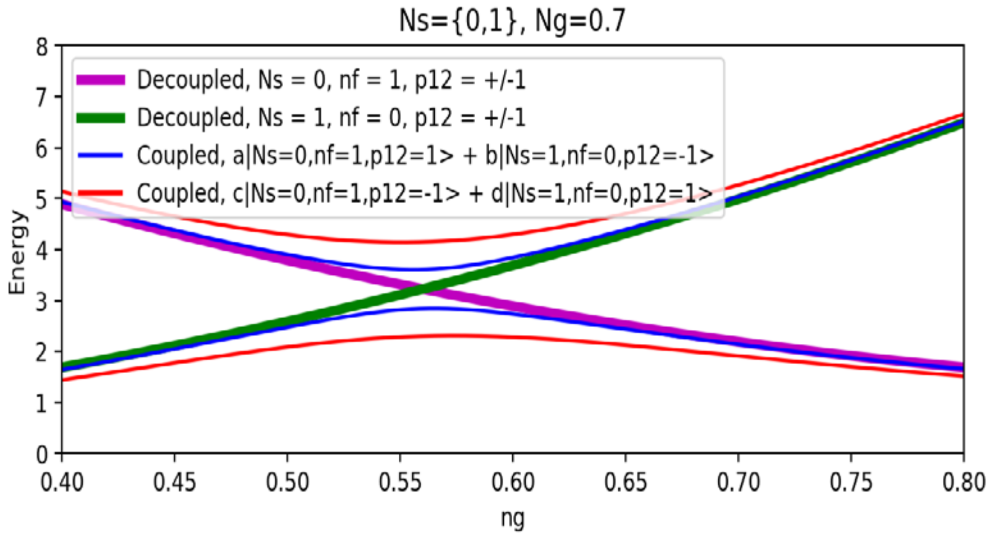


$$\hat{N}_M = d_M^\dagger d_M$$

$$\boxed{\hat{p}_{12} = i\gamma_1\gamma_2 = 2\hat{N}_M - 1}$$

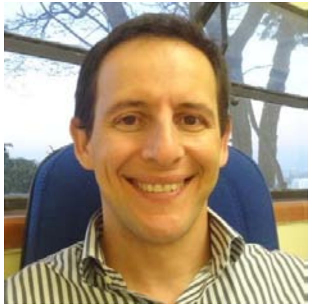
qbit states are **parity eigenstates**:

$$N_M = 0 \rightarrow p_{12} = -1$$
$$N_M = 1 \rightarrow p_{12} = +1$$



Tuning n_g allows for “inicialization” of the qbit!
Braiding allows for “gates”

Membros do grupo



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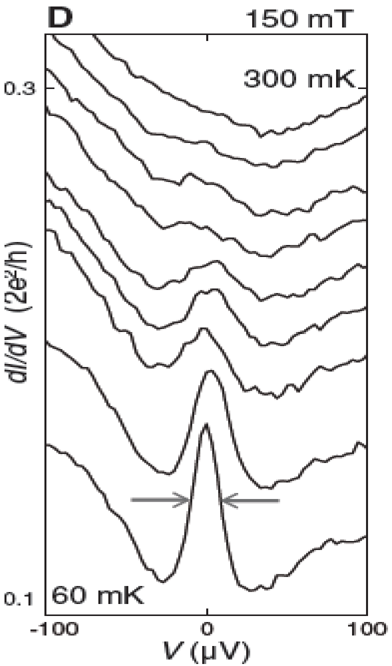
Alternative explanations for the zero-bias peak.



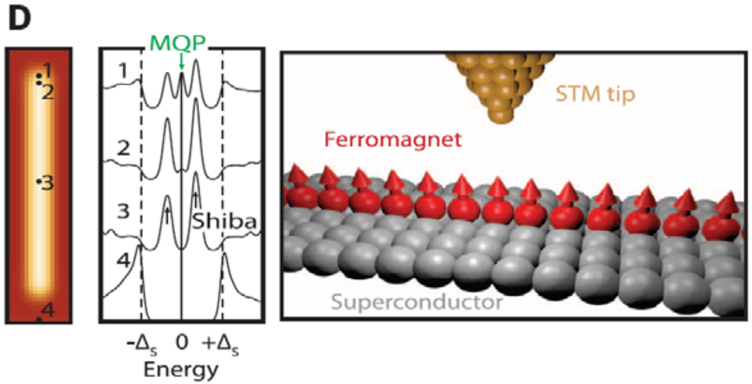
V. Mourik et al. Science **336** 1003 (2012)

Skepticism:

- Tunneling spectroscopy probes the BULK too
- Possible origins of the zero-bias peak:
  - Localization due to disorder
  - Andreev reflection
  - Kondo effect

Solution*:

Local probing of the wire ends

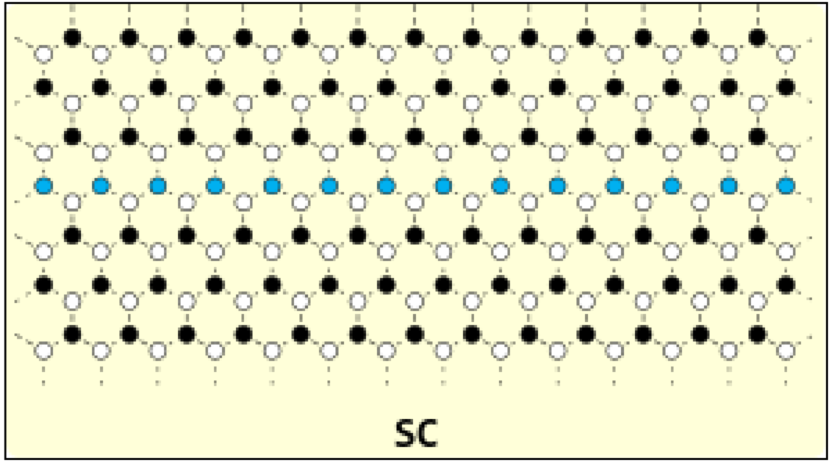


Nadj–Perje *et al.*, Science **346**, 602–607 (2014)

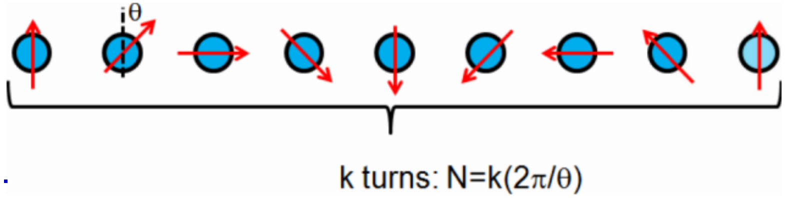
MBSs in magnetic chains on topological insulators

Honeycomb lattices:
Silicene, Stanene…

Kane-Mele-type TIs

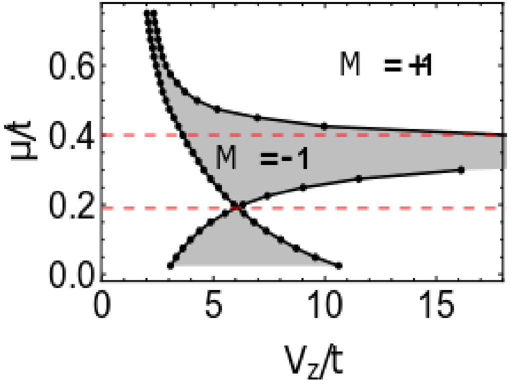


Magnetic chain: spiral angle θ



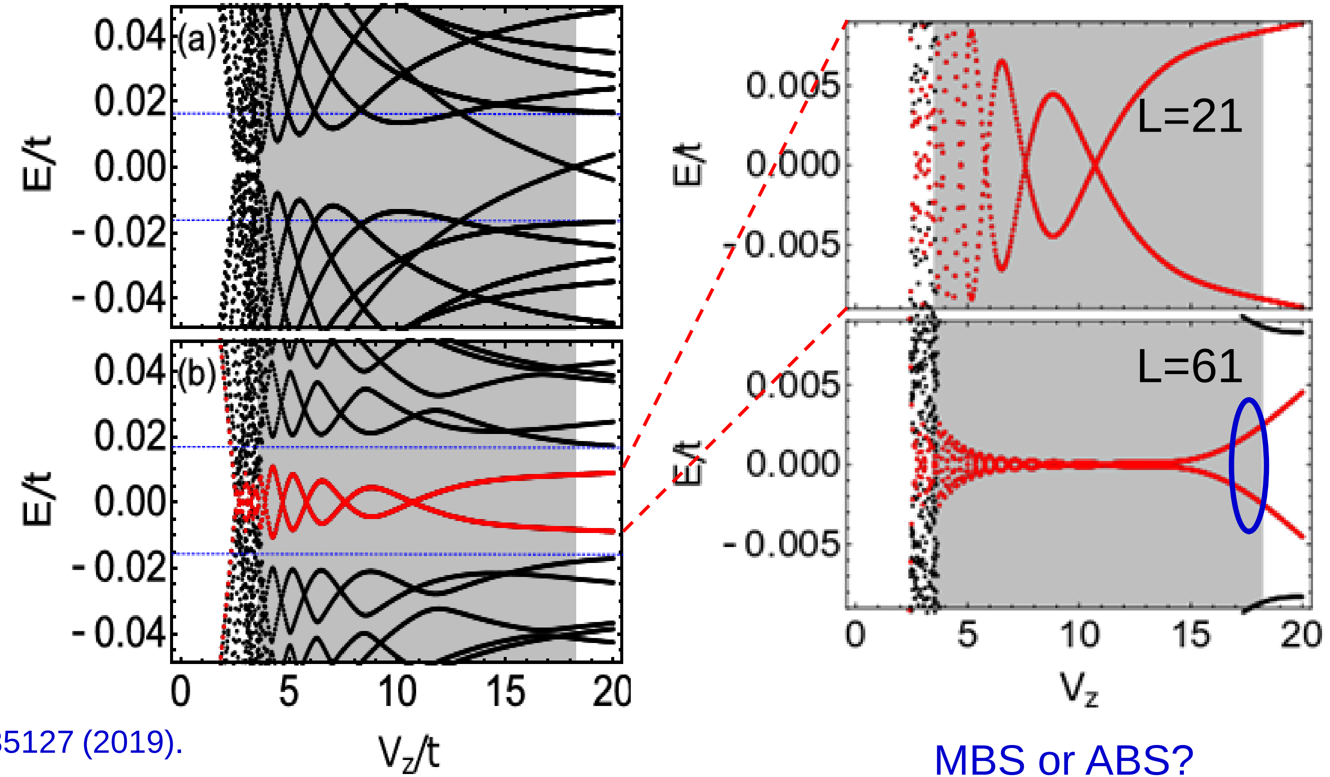
R. Teixeira *et al.* *Phys Rev B* **99** 035127 (2019).

MBSs in magnetic chains on topological insulators



$$\theta = \pi/2$$

L=21



R. Teixeira *et al. Phys Rev B* **99** 035127 (2019).

MBS or ABS?

Thank you for your attention!