

Application of Wilson's NRG to Majorana-Kondo systems.

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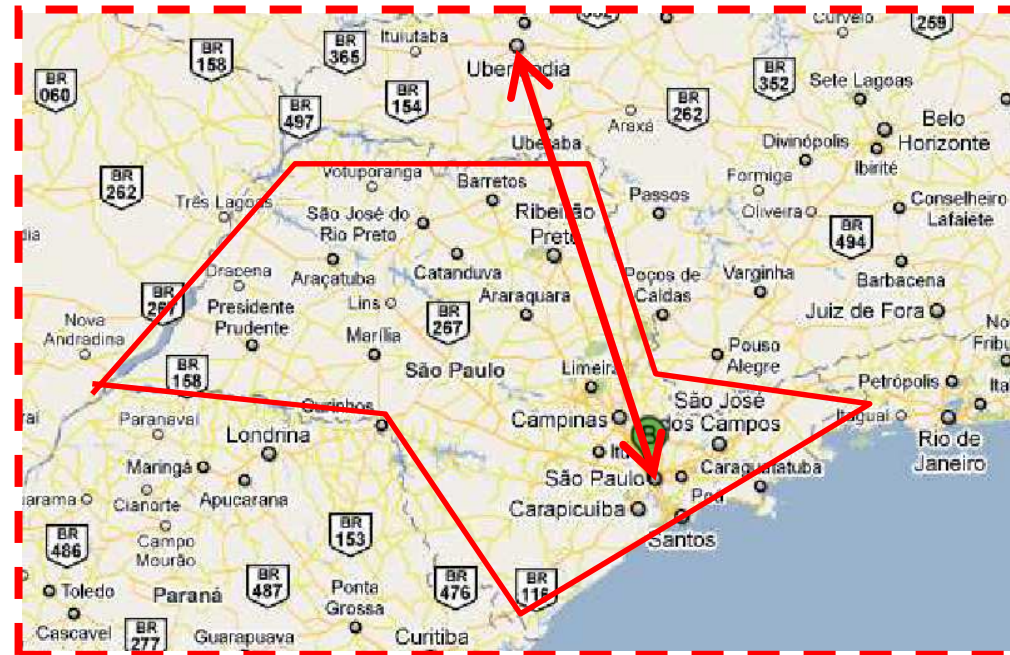
*NRG UFU/2019 Advanced Studies School
Uberlândia, Feb. 01, 2019.*

Physics @ USP – São Paulo.



Physics Institute-USP

- 6 departments
- ~130 active faculty
- ~250 grad students
- ~400 undergrad students



S.Paulo-Uberlândia ~600 km

Physics @ USP – São Paulo.



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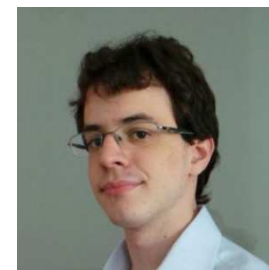
Group Members



Luis Gregório Dias da Silva
Professor



Marcos Medeiros
Doutorado



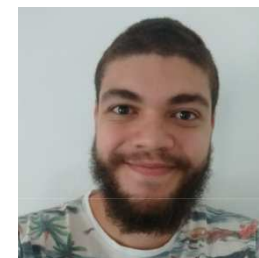
Raphael Levy
Doutorado



Jesus Cifuentes
Mestrado



Rafael Magaldi
Mestrado



João Victor Ferreira Alves
Mestrado

Outline

- Basics: Majorana bound states in condensed matter systems.
- *Detecting Majorana states with quantum dots.*
- Interacting quantum dots: Wilson's NRG
- *Kondo-Majoranana co-existence.*
- *Manipulating Majorana states with (double) quantum dots.*

What are Majorana fermions?

Majorana Fermions

Majorana solution: Representations of Dirac matrices with only imaginary non-zero elements while still satisfying



<http://www.giornalettismo.com/archives/255332/il-ritorno-di-ettore-majorana/>

$$\boxed{\begin{matrix} \tilde{\gamma}_0^\dagger = \tilde{\gamma}_0 \\ \tilde{\gamma}_i^\dagger = -\tilde{\gamma}_i \end{matrix}} \implies [i\tilde{\gamma}^\mu \partial_\mu - m] \Psi = 0$$

Real solutions: $[i\tilde{\gamma}^\mu \partial_\mu - m] \gamma = 0$ $\boxed{\gamma = \gamma^\dagger}$

- A Dirac fermion can be “written” in terms of two Majorana fermions


$$\begin{cases} \Psi = \frac{1}{2} (\gamma_1 + i\gamma_2) \\ \Psi^\dagger = \frac{1}{2} (\gamma_1 - i\gamma_2) \end{cases} \quad \text{or} \quad \boxed{\gamma_1 = (\Psi^\dagger + \Psi)}$$



E. Majorana, *Nuovo Cimento* **5**, 171 (1937)

Where do we find Majorana fermions?

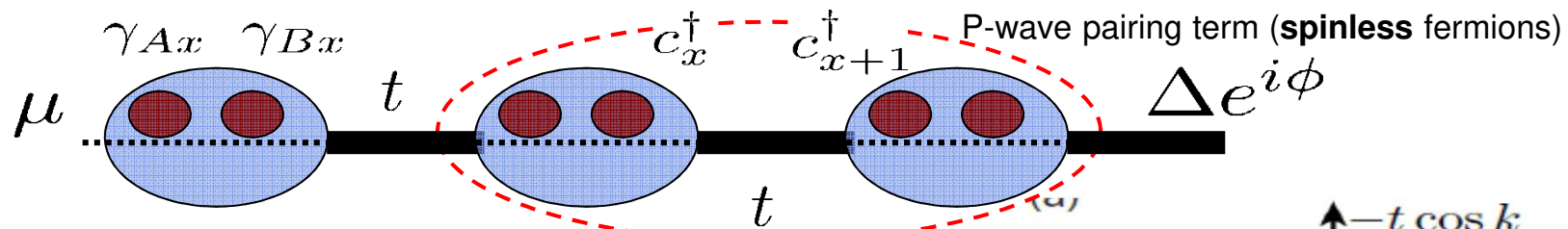
Majorana quasiparticles in condensed matter systems?

- Fractional Quantum Hall liquids ($\nu=5/2$): “non-Abelian anyons”. Moore and Read, *Nucl. Phys. B* (1991).
- Two-channel Kondo non-Fermi-liquid state.  Emery, Kivelson, *PRB* (1992).
Coleman, Ioffe, Tsvelik *PRB* (1995).
Maldacena, Ludwig, *Nucl. Phys. B.* (1997).
Zhang, Hewson, Bulla, *Solid State Comm.* (1999).
- Interface of topological insulators with BCS superconductors Fu and Kane, *Phys. Rev. Lett.* (2008).
- Spin-polarized (“spinless”) p-wave superconductors. Read and Green, *Phys. Rev. B* (2000).
Kitaev, *Phys. Usp.* (2001).

Motivation: entanglement of particles with non-abelian statistics (“Ising anyons”); topologically protected quantum computation.

1D p-wave superconductor (Kitaev model)

$$H = -\mu \sum_x c_x^\dagger c_x - \frac{1}{2} \sum_x (t c_x^\dagger c_{x+1} + \Delta e^{i\phi} c_x c_{x+1} + h.c.)$$



Energy spectrum:

$$E(k) = \pm \sqrt{(t \cos k + \mu)^2 + (\Delta \sin k)^2}$$

$$|\mu| > t$$

Gapped ($E_+ - E_- > 0$): **trivial**

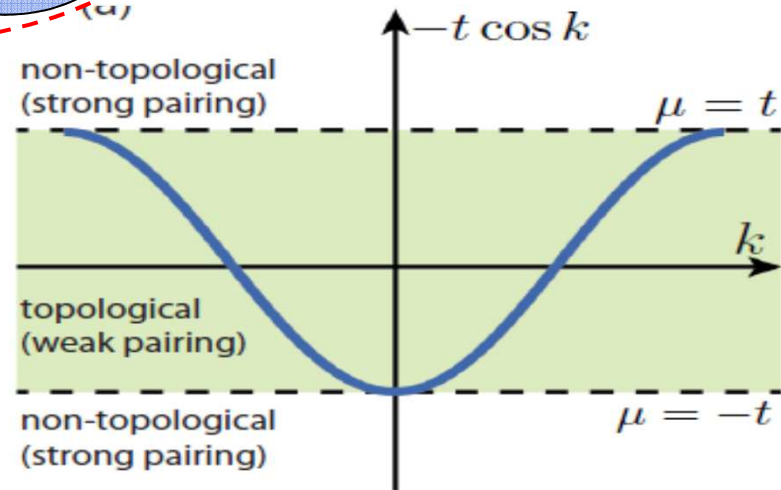
$$\mu = \pm t$$

Gapless modes ($E=0$):

$$k = \pm\pi \text{ or } k = 0$$

$$|\mu| < t$$

Gapped: **topological** ($\Delta \neq 0$)



Majorana states in the Kitaev model.

Map into a “chain of Majorana modes” using:

$$\begin{cases} c_x = \frac{e^{-i\phi/2}}{2} (\gamma_{B,x} + i\gamma_{A,x}) \\ c_x^\dagger = \frac{e^{+i\phi/2}}{2} (\gamma_{B,x} - i\gamma_{A,x}) \end{cases}$$

$$H = -\mu \sum_x c_x^\dagger c_x - \frac{1}{2} \sum_x (t c_x^\dagger c_{x+1} + \Delta e^{i\phi} c_x c_{x+1} + h.c.)$$



$$H = -\frac{\mu}{2} \sum_x (1 + i\gamma_{B,x}\gamma_{A,x}) - \frac{i}{4} \sum_x^{N-1} (\Delta + t) \gamma_{B,x}\gamma_{A,x+1} + (\Delta - t) \gamma_{A,x}\gamma_{B,x+1}$$

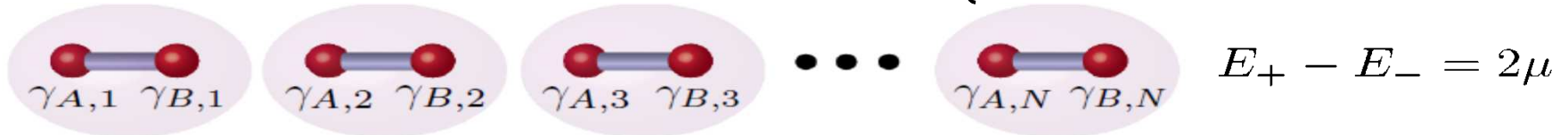
Majorana states in the Kitaev model.

$$H = -\frac{\mu}{2} \sum_x (1 + i\gamma_{B,x}\gamma_{A,x}) - \frac{i}{4} \sum_x^{N-1} (\Delta + t) \gamma_{B,x}\gamma_{A,x+1} + (\Delta - t) \gamma_{A,x}\gamma_{B,x+1}$$

$$|\mu| > t$$

Gapped: **trivial**. Special case:

$$\begin{cases} \mu \neq 0 \\ t = \Delta = 0 \end{cases}$$

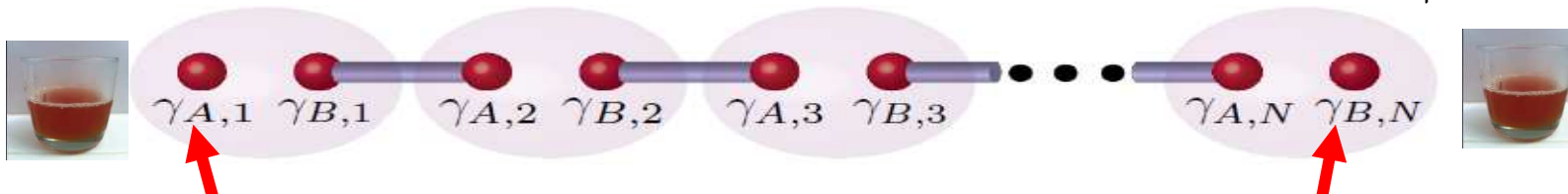


$$|\mu| < t$$

Gapped: **topological**. Special case:

$$\begin{cases} \mu = 0 \\ t = \Delta \neq 0 \end{cases}$$

$$E_+ - E_- = 2\Delta$$



Topological regime: Majorana modes ($e=\mu=0!!!$) at the *edges* of the chain!

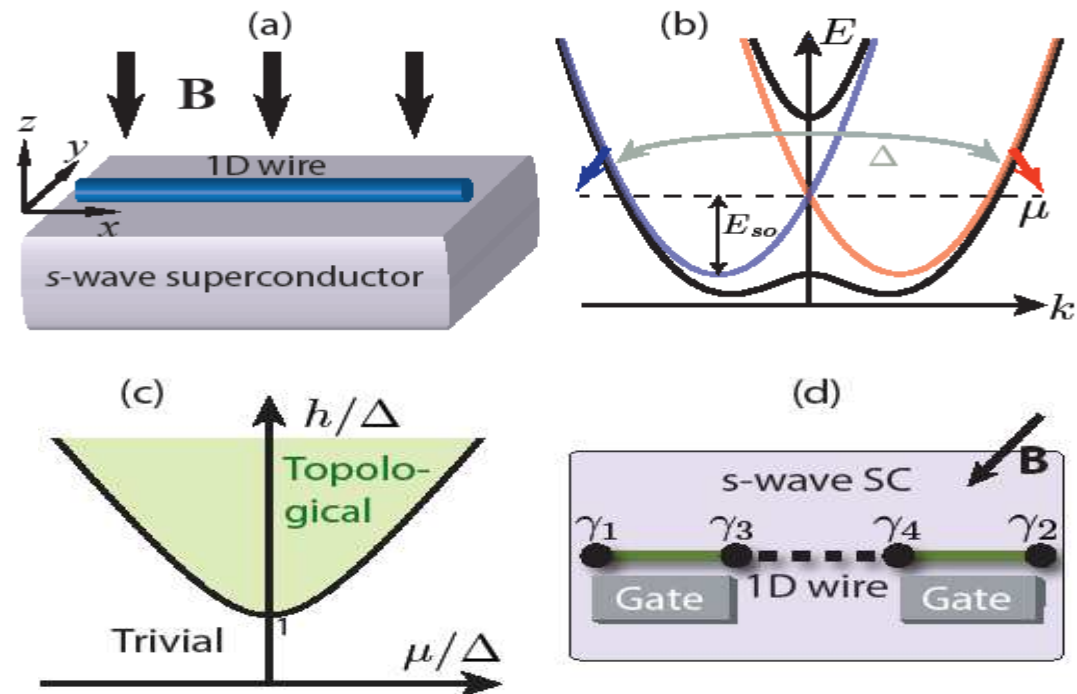
Can the Kitaev model be realized experimentally?

How to realize a p-wave SC: Quantum wires.

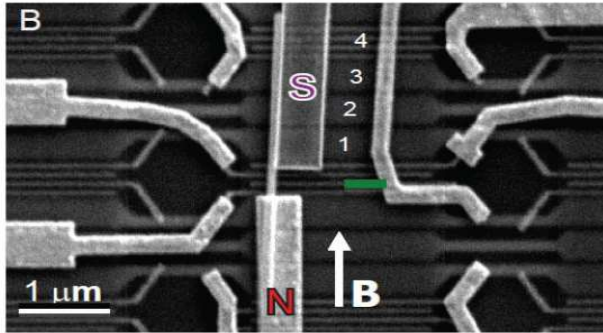
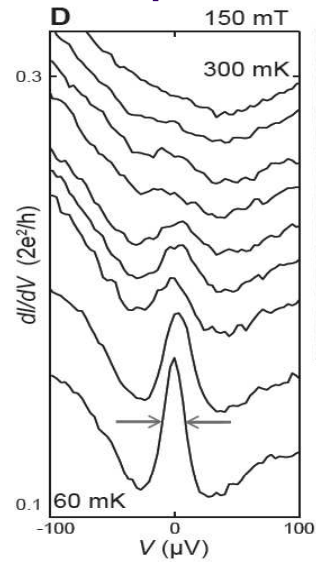
Theory: Lutchyn et al. PRL, **105**, 077001 (2010); Oreg et al. PRL, **105**, 077002 (2010);

- **Step 1:**
create spinless 1D fermions.
Ingredients: spin-orbit, B field.

- **Step 2:**
Introduce SC pairing.
Ingredients: proximity with a BCS SC



Experiment on InSb nanowires.

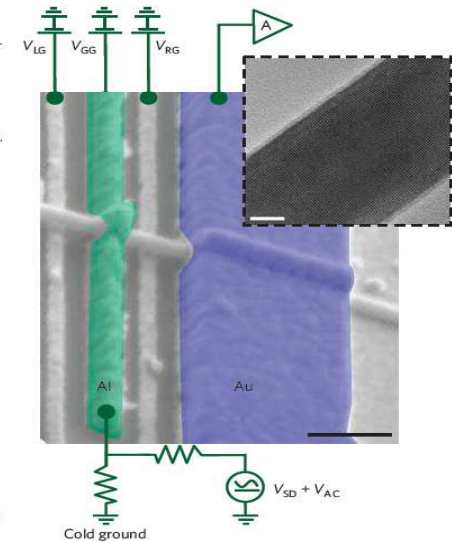
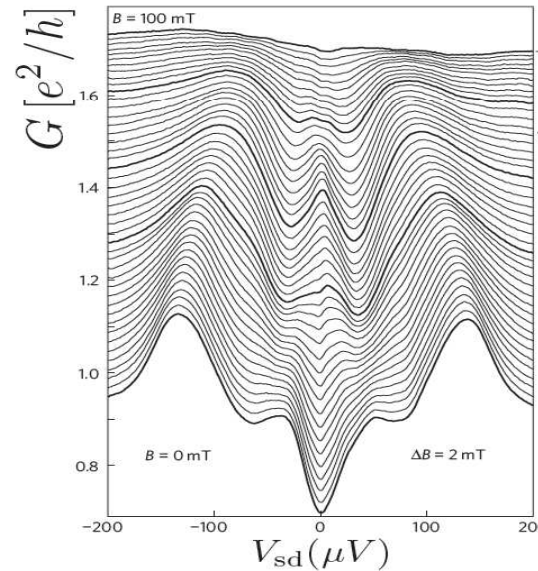


Zero-bias peak in tunneling spectroscopy

- ← Mourik *et al.*, *Science* **336**, 1003–1007 (2012)
- Deng *et al.*, *Nano Lett.* **12**, 6414 (2012)
- Das *et al.*, *Nature Phys.* **8**, 887 (2012)
- Prada *et al.*, *Phys. Rev. B* **86**, 180503 (2012)
- Churchill *et al.*, *Phys. Rev. B* **87**, 241401 (2013)

Signatures appear for:

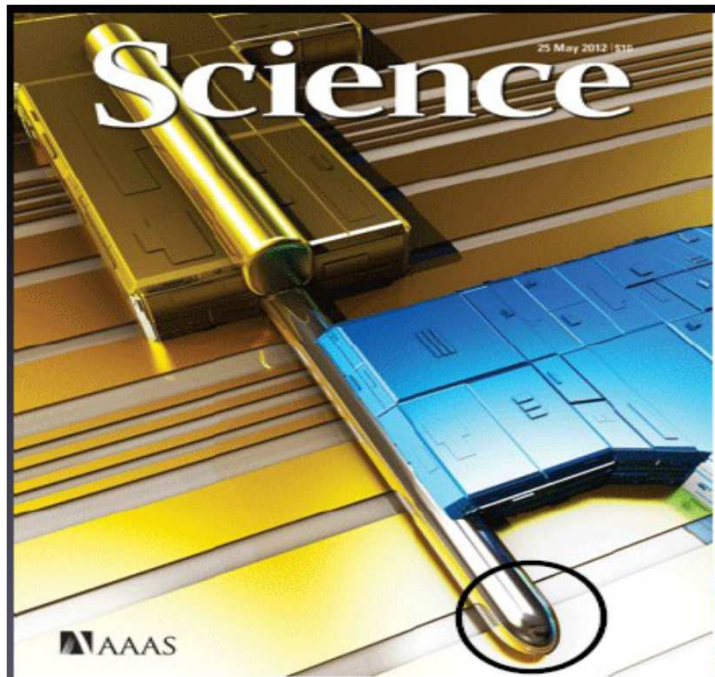
- Large enough magnetic field (topological phase)
- Not too big (that it kills the induced superconductivity)
- Perpendicular to Rashba SO



A success story??

Theory: Lutchyn et al. PRL, **105**, 077001 (2010); Oreg et al. PRL, **105**, 077002 (2010);

Experiment: V. Mourik et al. Science **336** 1003 (2012)



Inside joke:

“Majorana found at the end of a quantum wire”

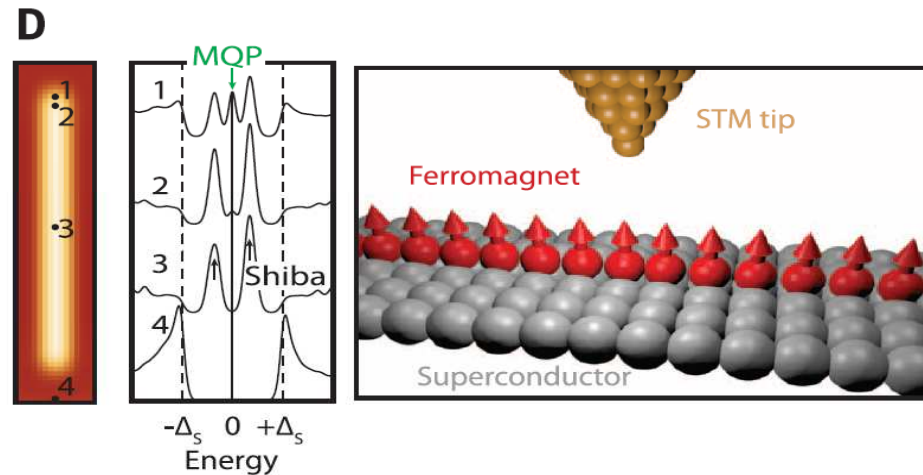
Alternative explanations for the zero-bias peak.

Skepticism:

- Tunneling spectroscopy probes the BULK too
- Possible origins of the zero-bias peak:
 - ▶ Localization due to disorder
 - ▶ Andreev reflection
 - ▶ Kondo effect

Solution*:

Local probing of the wire ends

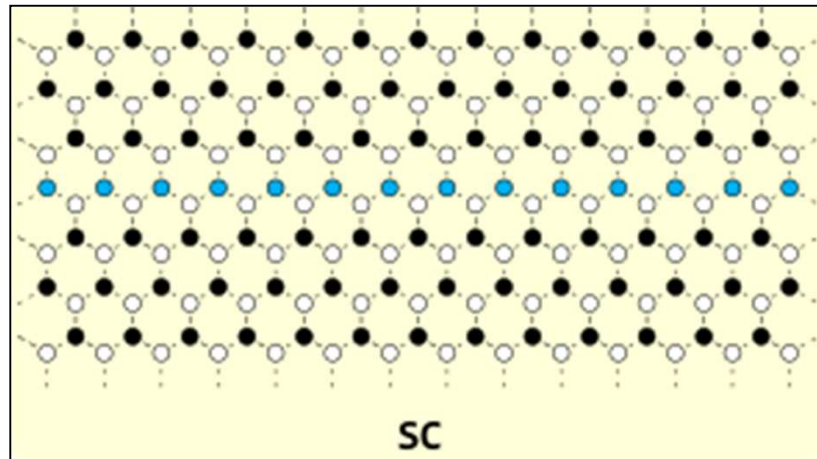


Nadj-Perje *et al.*, Science **346**, 602–607 (2014)

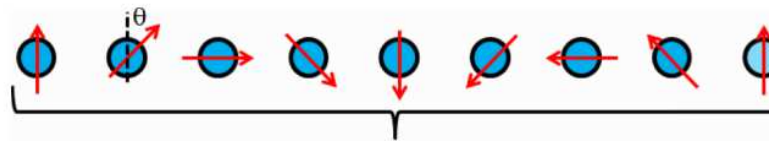
MBSs in magnetic chains on topological insulators

(See poster by Raphael Levy...)

Honeycomb lattices:
Silicene, Stanene...
Kane-Mele-type TIs



Magnetic chain: spiral angle θ



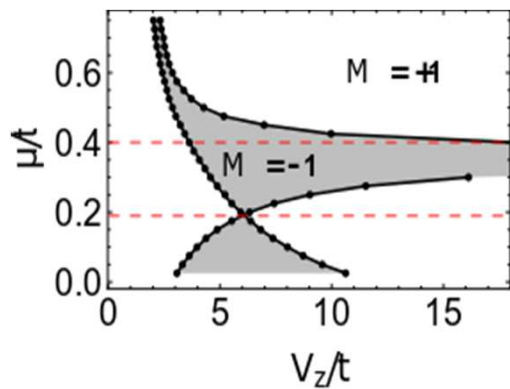
k turns: $N=k(2\pi/\theta)$

R. Teixeira *et al.* *Phys Rev B* **99** 035127 (2019).

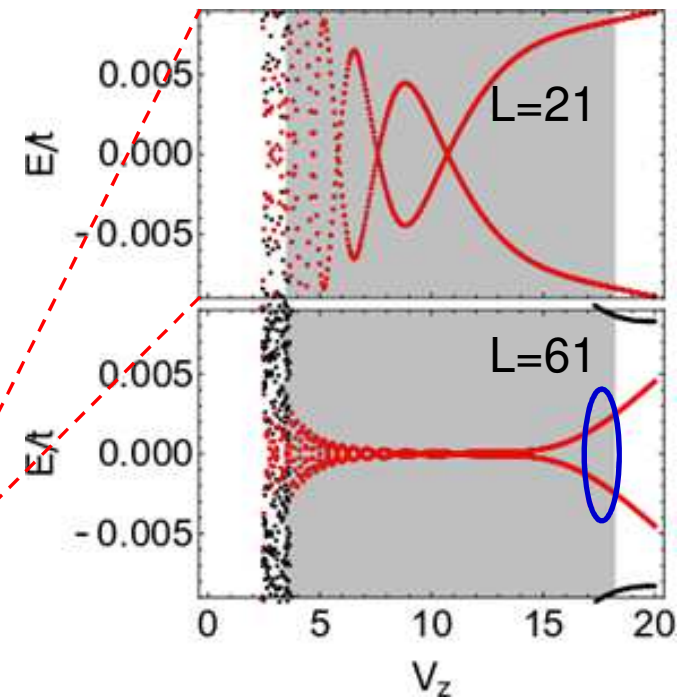
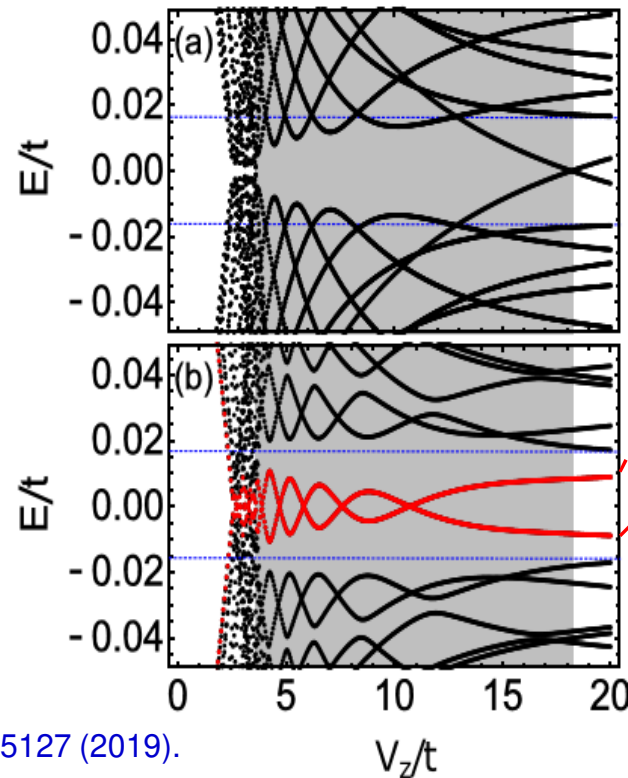
MBSs in magnetic chains on topological insulators

(See poster by Raphael Levy...)

L=21



$$\theta = \pi/2$$



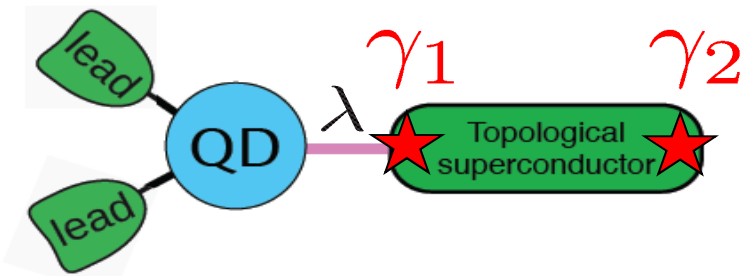
MBS or ABS?

R. Teixeira *et al. Phys Rev B* **99** 035127 (2019).

Detecting MBS with quantum dots.

Better way to measure?

- Quantum dot coupled to metallic leads coupled with at the end of the nanowire.

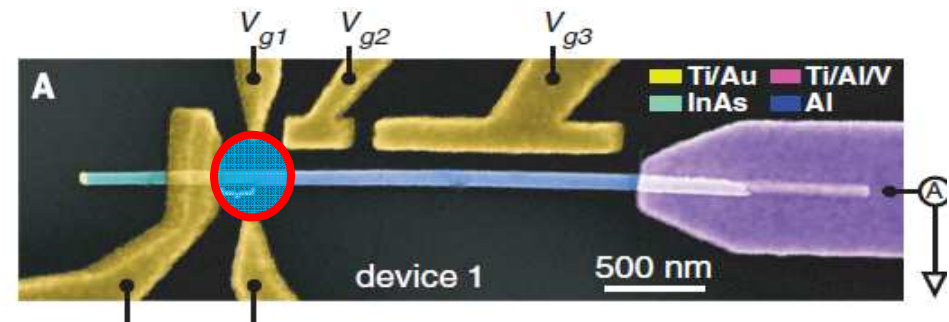


Theory

Liu and Baranger, *Phys Rev B* **84** 201308 (2011).

Vernek et al., *Phys Rev B* **89** 165314 (2014).

Ruiz-Tijerina et al. *Phys Rev B* **91** 115435 (2015).



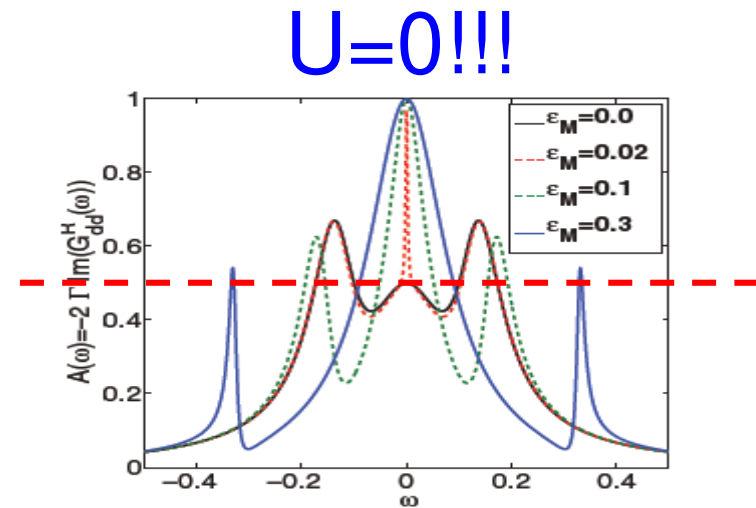
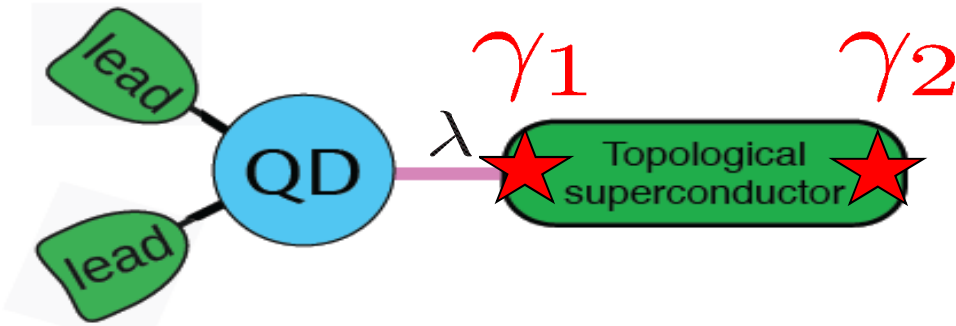
Experiment (Marcus' group)

M.T. Deng et al., *Science* **354** 1557 (2016).

Better way to measure?

Liu and Baranger, *Phys Rev B* **84** 201308 (2011).

Vernek et al., *Phys Rev B* **89** 165314 (2014).



- Connect a quantum dot + metallic leads at the end of the nanowire.
- Measure conductance through the dot
- $0.5 e^2/h$ = signature of the Majorana mode for $U=0$
- What happens for the (common) case of non-zero U ???

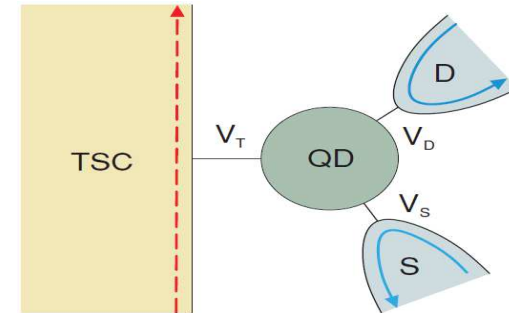
Ruiz-Tijerina et al. *Phys Rev B* **91** 115435 (2015).

Majoranas + interaction

- Kondo impurity + Majorana edge states (NRG)

R. Zitko, *Phys. Rev. B* **83**, 195137 (2011).

R. Zitko, P. Simon, *Phys. Rev. B* **84**, 195310 (2011).

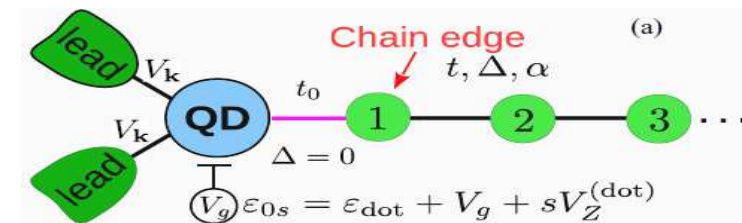


- Quantum dot + Kitaev (NRG)

M. Lee, et al., *Phys. Rev. B* **87**, 241402 (2013).

Chirla et al., *Phys. Rev. B* **90**, 195108 (2014).

Ruiz-Tijerina et al., *Phys. Rev. B* **91**, 115435 (2015).



- Quantum dot + Kitaev (DMRG)

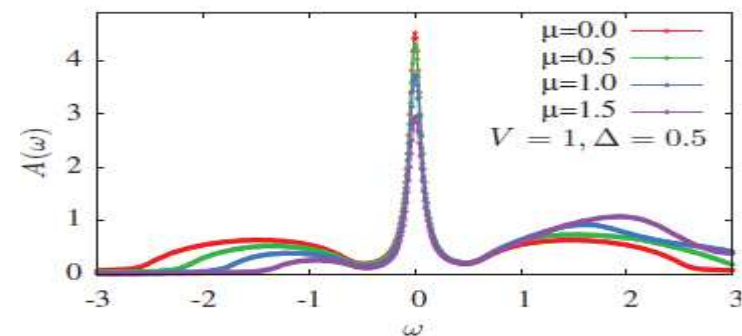
Korytár and Schmitteckert, *JPCM* **25** 475304 (2014).

Cheng et al., *Phys. Rev. X* **4**, 031051 (2014).

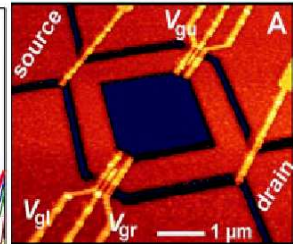
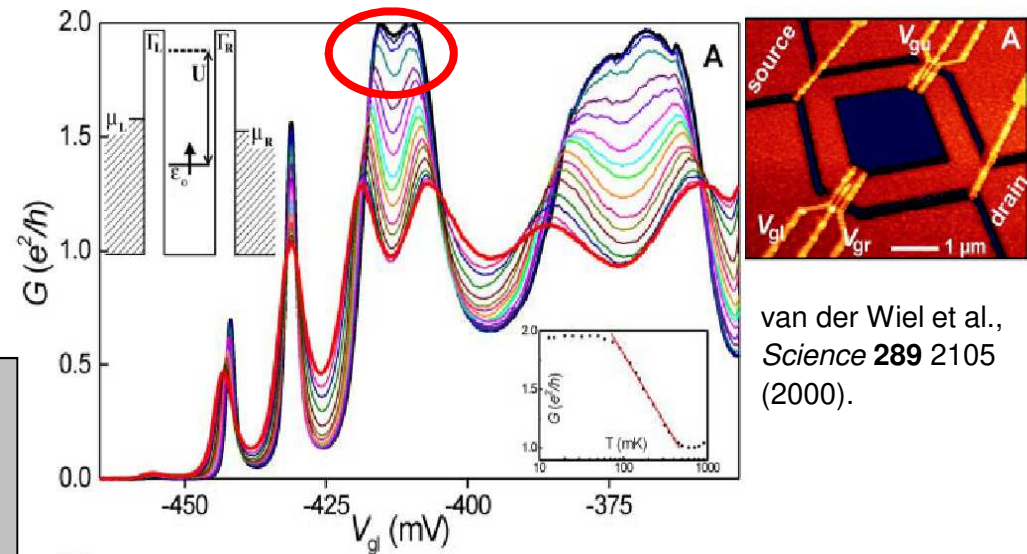
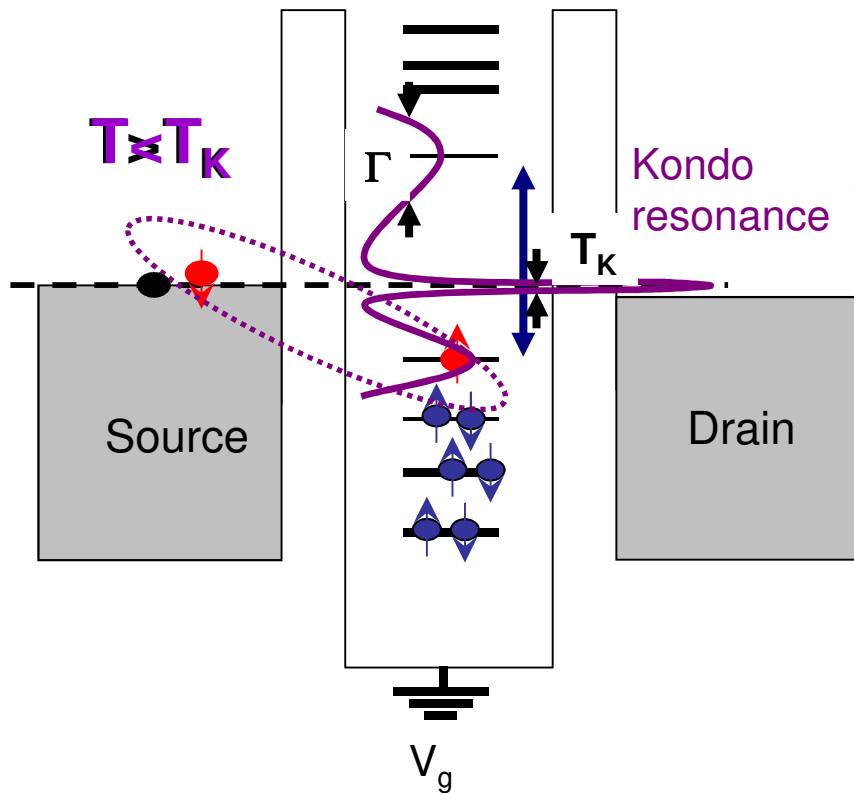
- Interacting Kitaev model (DMRG)

Stoudenmire et al., *Phys. Rev. B* **84** 014503 (2011).

Thomale et al., *Phys. Rev. B* **88** 161103(R) (2013).



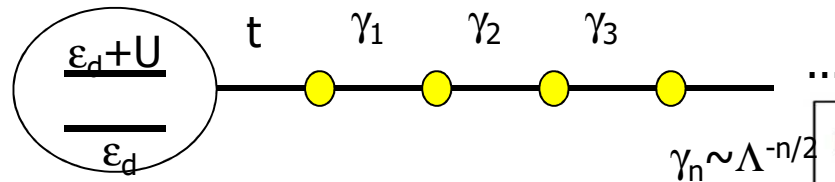
Kondo Effect in Quantum Dots: zero-bias transport.



van der Wiel et al.,
Science **289** 2105
(2000).

- $T > T_K$: Coulomb blockade (low G)
- $T < T_K$: Kondo singlet formation
- Kondo resonance at E_F (width T_K).
- New conduction channel at E_F :
Zero-bias enhancement of G ($\rightarrow 2e^2/h$!)

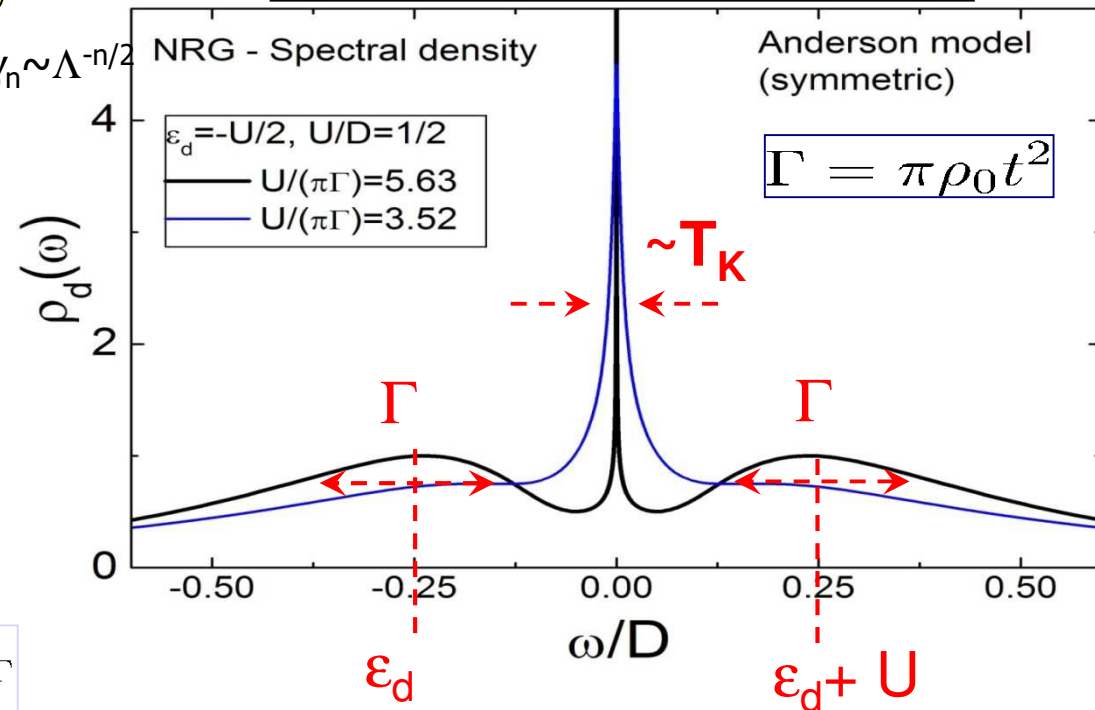
Kondo resonance with Wilson's NRG



$$\rho_d(\omega) = -\text{Im} G_d(\omega)$$

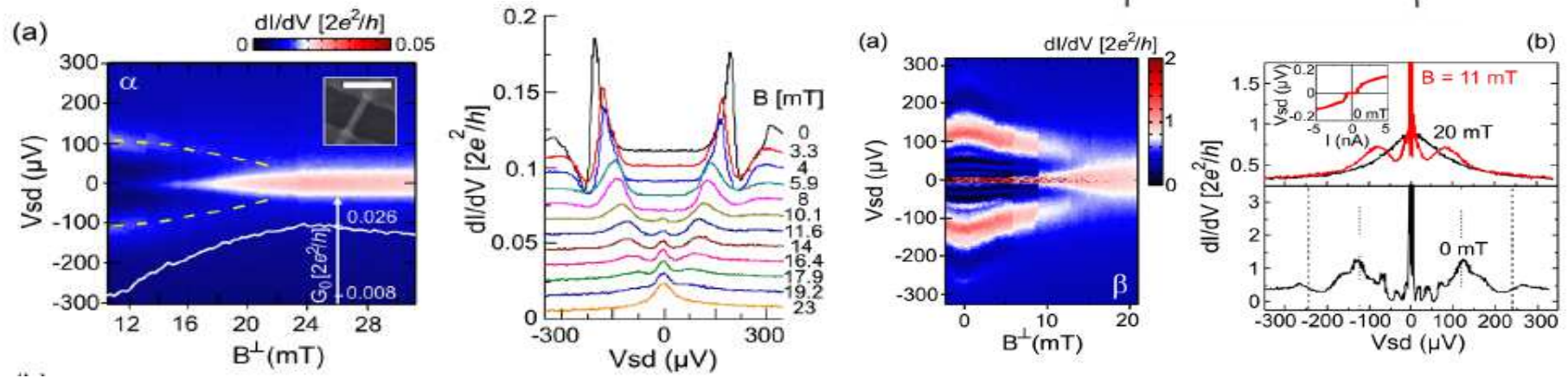
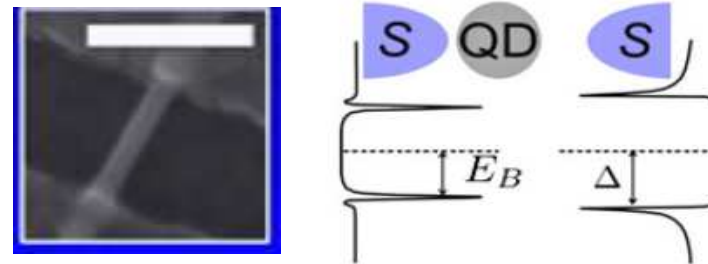
- Spectral density:
 - Single-particle peaks at ϵ_d and $\epsilon_d + U$.
 - *Many-body* peak at the Fermi energy: **Kondo resonance** (width $\sim T_K$).
- NRG: very good resolution at low ω .

$$T_K \sim \sqrt{\frac{U\Gamma}{2}} e^{-\pi|\epsilon_d + U|/\epsilon_d / 2U\Gamma}$$



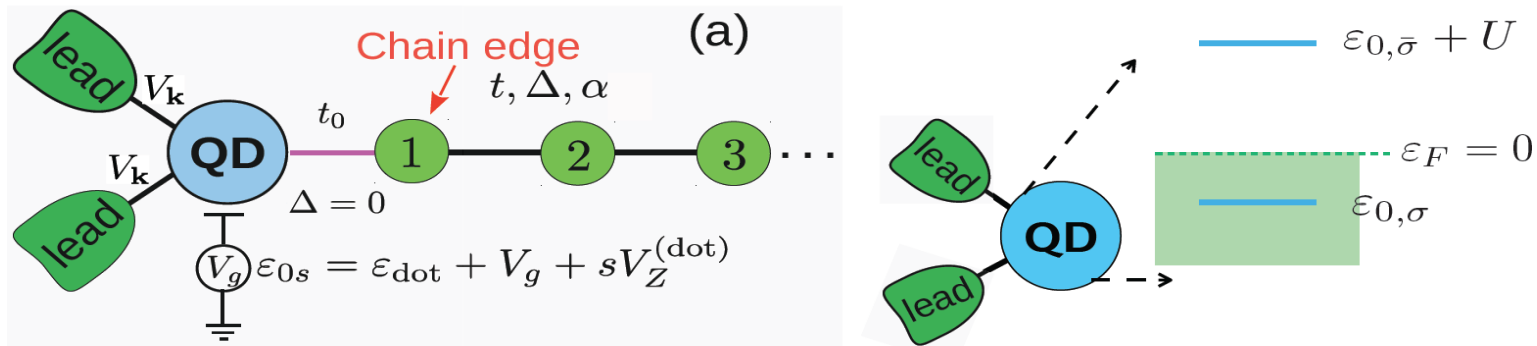
Kondo zero-bias peak in quantum wires coupled to SC leads.

E.J. Lee et al. PRL **109** 186802 (2012)



- Quantum dot defined in InAs/InP quantum wires coupled to superconducting leads.
- Kondo-like zero-bias peak emerges at a critical field B_c .

Model: Quantum dot + quantum wire + SC pairing.



Quantum wire:

$$H_{\text{wire}} = H_{\text{TB}}(\mu, t, V_Z) + H_{\text{Rashba}}(\alpha) + H_{\text{SC}}(\Delta)$$

Quantum dot:

$$H_{\text{dot}} = \sum_{s=\uparrow,\downarrow} \varepsilon_{0,s} n_{0,s} + U n_{0,\uparrow} n_{0,\downarrow}$$

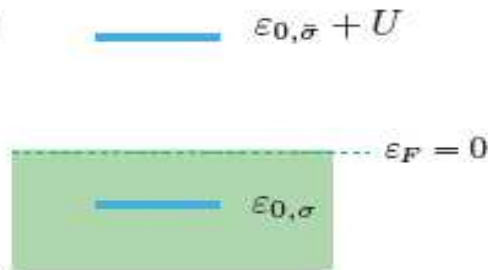
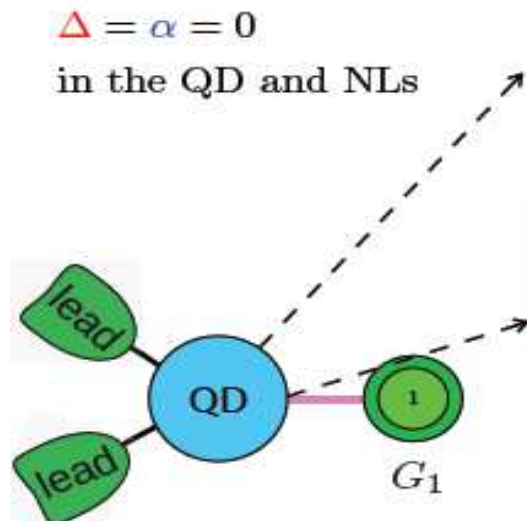
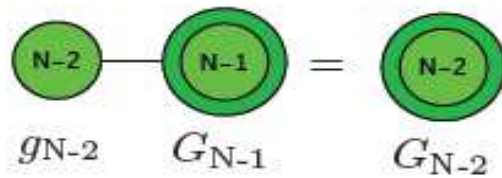
Topological phase for $|V_Z| > \sqrt{\mu^2 + \Delta^2}$

Rainis *et al.*, Phys. Rev. B **87**, 024515 (2013)

QD-wire coupling:

$$H_{\text{dot-wire}} = t_0 \sum_{s=\uparrow,\downarrow} \left[c_{0,s}^\dagger c_{1,s} + c_{1,s}^\dagger c_{0,s} \right]$$

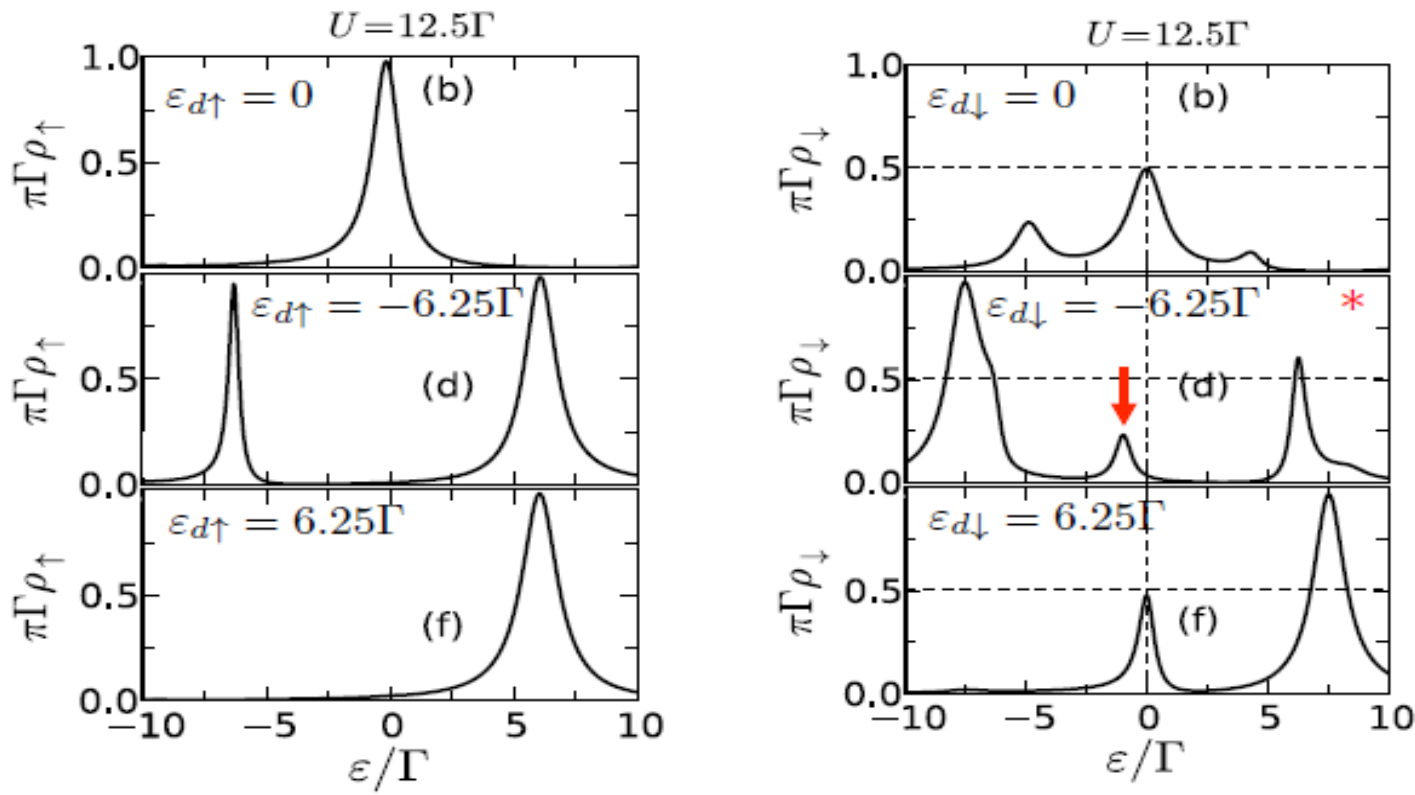
Iterative Green's functions + mean field (Hubbard I).



- Because of the Coulomb interaction, the **NL-QD** part displays many-body correlations (Kondo physics).
- We use an approximation based on the *Hubbard I method** to obtain the Green's function.

*J. Hubbard, Proc. Roy. Soc. (London) **A276**, 238 (1963)

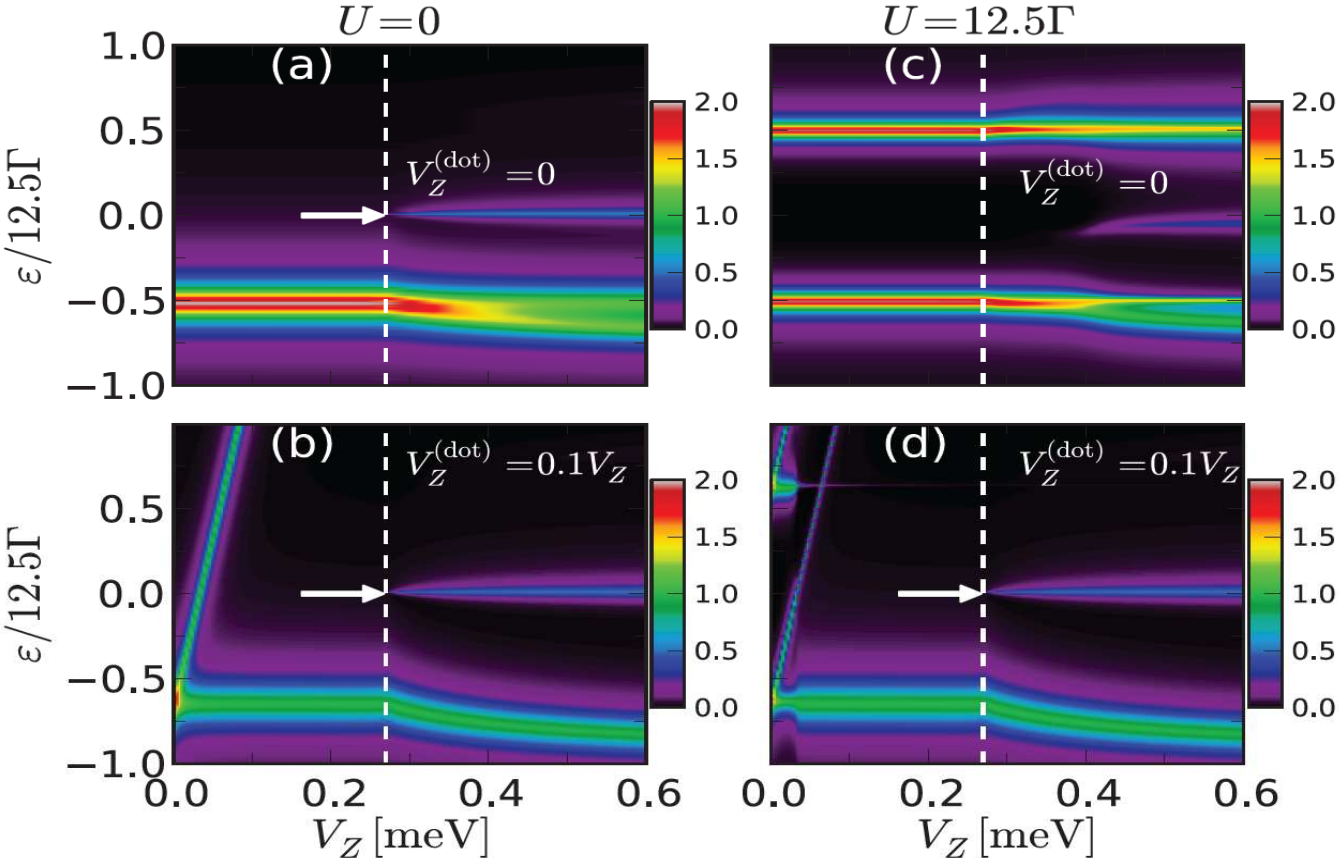
Iterative Green's functions + mean field (Hubbard I).



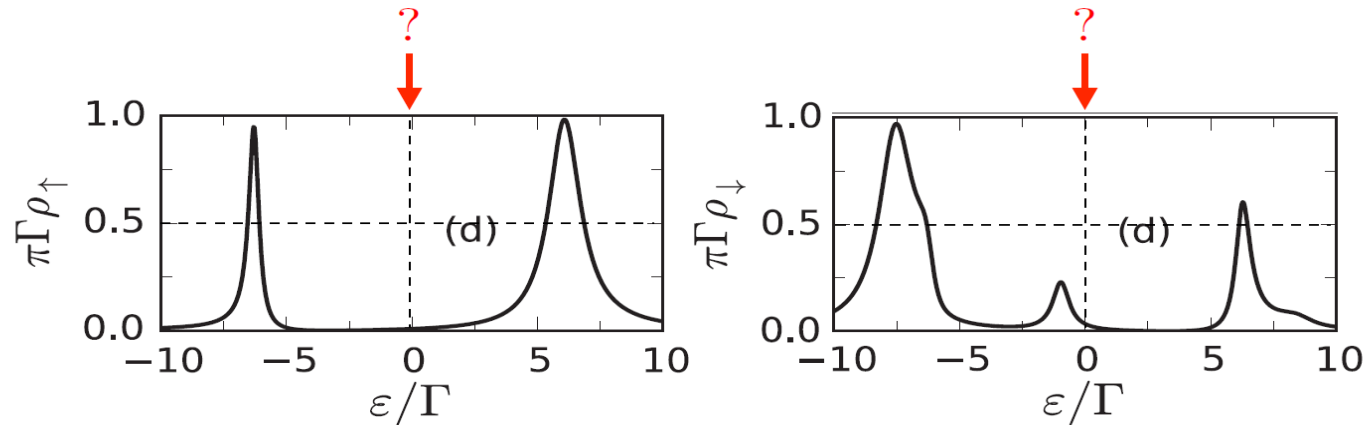
*Particle-hole symmetry

Ruiz-Tijerina, et al. *Phys. Rev. B* **91**, 115435 (2015)

Iterative Green's functions + mean field (Hubbard I).



Shortcomings of the mean-field approximation.

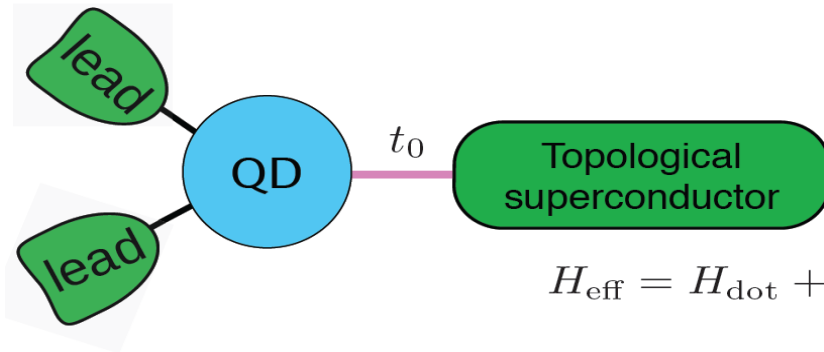


- The Hubbard I approximation captures the Majorana physics outside of the Kondo regime
- It doesn't capture the Kondo correlations
- What if there is a strong [Kondo-Majorana](#) interplay?

*Particle-hole symmetry

Ruiz-Tijerina, *et al. Phys. Rev. B* **91**, 115435 (2015)

Effective low-energy Anderson model



Lee *et al.*, Phys. Rev. B **87**, 241402 (2013)

- Effective model: **MZM** couples directly to the QD spin-down ($V_Z > 0$).

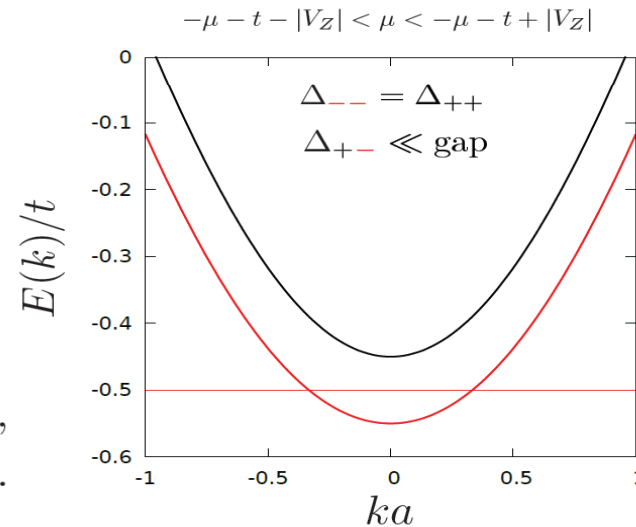
$$H_{\text{eff}} = H_{\text{dot}} + H_{\text{leads}} + H_{\text{dot-leads}} + \lambda \gamma (d_{\downarrow} - d_{\downarrow}^{\dagger})$$

$$H_{\text{dot}} = \sum_{\sigma} \varepsilon_{0\sigma} (\varepsilon_d, V_Z^{(\text{dot})}) n_{0\sigma} + U n_{0\uparrow} n_{0\downarrow}$$

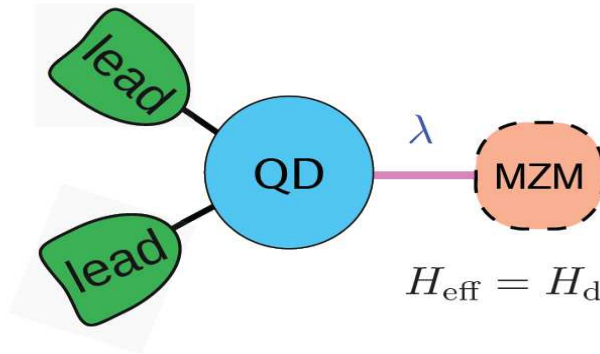
$$H_{\text{leads}} = \sum_{\vec{k}\sigma} \varepsilon_k c_{\vec{k}\sigma}^{\dagger} c_{\vec{k}\sigma}$$

$$H_{\text{dot-leads}} = \sum_{\vec{k}\sigma} [V_{\vec{k}} d_{\sigma}^{\dagger} c_{\vec{k}\sigma} + \text{H. c.}]$$

For a positive Zeeman splitting V_Z , the wire couples only to the QD spin-dn.



Effective low-energy Anderson model



Lee *et al.*, Phys. Rev. B **87**, 241402 (2013)

- Effective model: **MZM** couples directly to the QD spin-down ($V_Z > 0$).

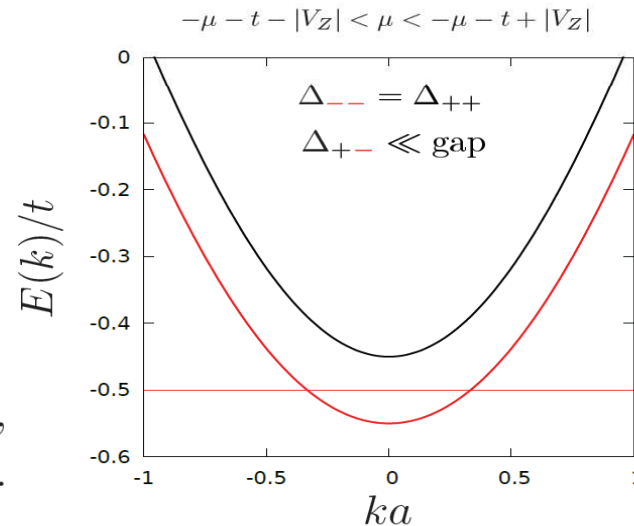
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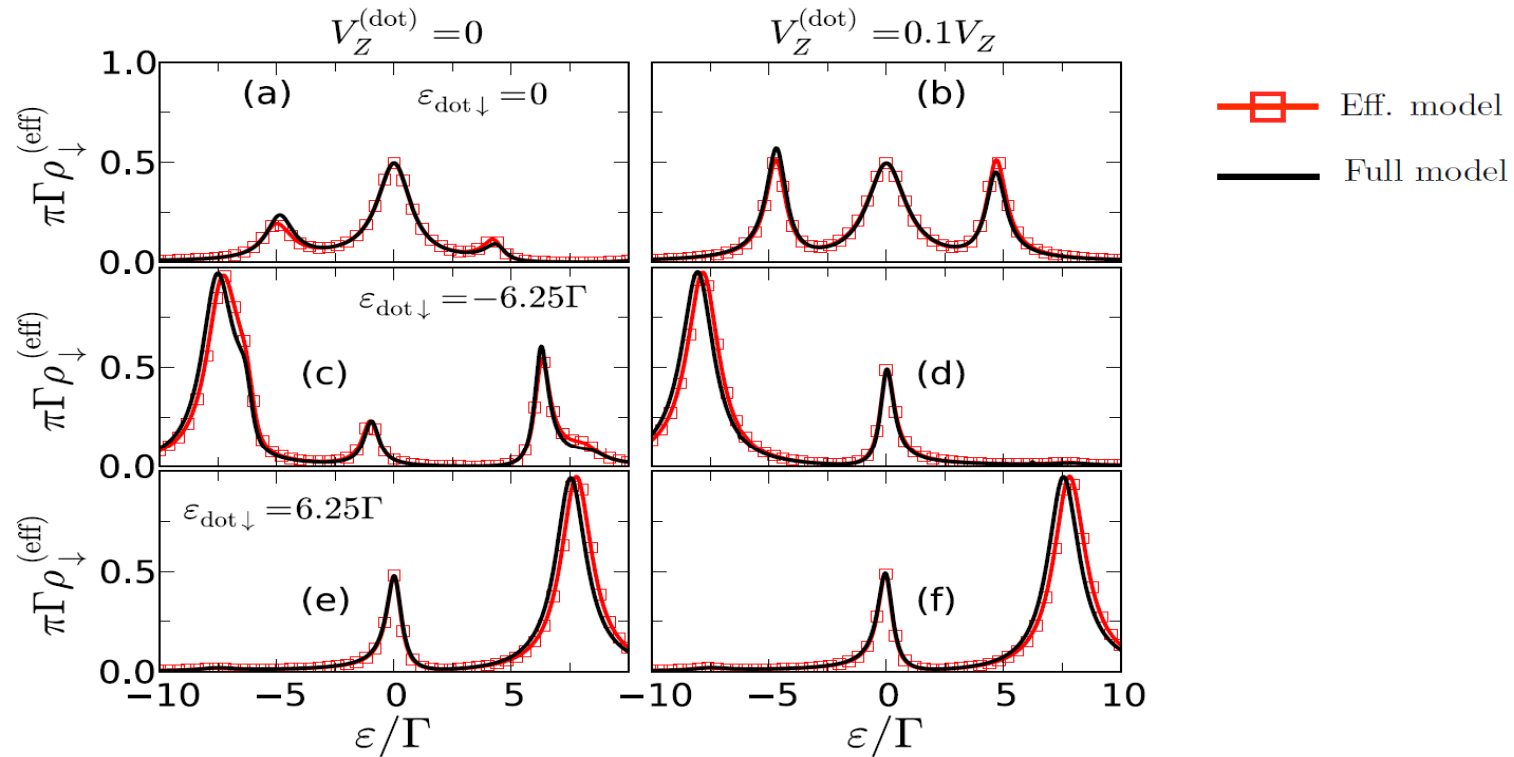
$$H_{\text{leads}} = \sum_{\vec{k}\sigma} \varepsilon_k c_{\vec{k}\sigma}^{\dagger} c_{\vec{k}\sigma}$$

$$H_{\text{dot-leads}} = \sum_{\vec{k}\sigma} [V_{\vec{k}} d_{\sigma}^{\dagger} c_{\vec{k}\sigma} + \text{H. c.}]$$

For a positive Zeeman splitting V_Z , the wire couples only to the QD spin-dn.



Effective low-energy Anderson model



With the right choice of λ , we reproduce the numerical results for a given t_0 .

Effective model

c_{α}^{\dagger} : creates a fermion in state α

$\hat{n}_{\alpha} \equiv c_{\alpha}^{\dagger} c_{\alpha}$: number operator (=0,1)

$$c_{E=0}^{\dagger} = (\gamma_1 - i\gamma_2) \text{ zero energy mode}$$

$$\gamma_1(2) = \gamma_1^{\dagger}(2)$$

Majorana operators

Quantum dot (V_Z : Zeeman ; U : e-e interaction)

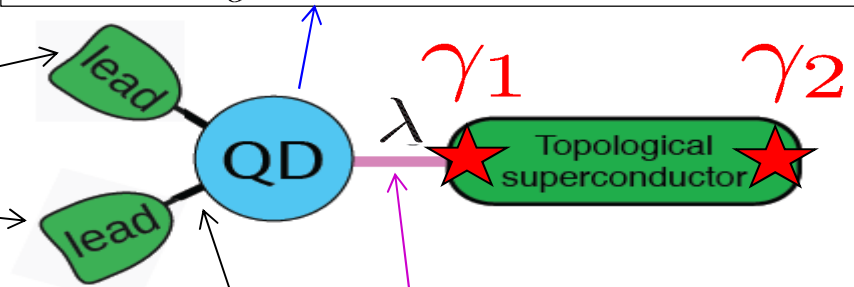
$$H_{\text{dot}} = \sum_{\sigma} (\varepsilon_d + \sigma \cdot V_Z) \hat{n}_{d\sigma} + U \hat{n}_{d\uparrow} \hat{n}_{d\downarrow}$$

Metallic leads

$$H_{\text{leads}} = \sum_{\mathbf{k}, \sigma} \varepsilon_{\mathbf{k}l\sigma} \hat{n}_{\mathbf{k}l\sigma}$$

Coupling to the metallic leads

$$H_{\text{dot-leads}} = \sum_{\mathbf{k}} V_{\mathbf{k}} c_{d\sigma}^{\dagger} c_{\mathbf{k}\sigma} + \text{H.c.}$$



$$H_{\text{dot-}\gamma} = \lambda (c_{d\downarrow} - c_{d\downarrow}^{\dagger}) \gamma_1 + \text{H.c.}$$

Coupling to one Majorana

NRG: spectral function and conductance

D. A. Ruiz-Tijerina et al. *Phys Rev B* **91** 115435 (2015).

NRG formulation: quantum numbers



$$\gamma_{1(2)} = \gamma_{1(2)}^\dagger \quad \text{Majorana operators}$$

$$f_\downarrow^\dagger = (\gamma_1 - i\gamma_2) \quad \text{Fermion operators}$$

$$\gamma_1 = \frac{1}{2} (f_\downarrow^\dagger + f_\downarrow) \Rightarrow$$

$$H_{\text{dot-}\gamma} = \frac{\lambda}{2} (c_{d\downarrow}^\dagger f_\downarrow + c_{d\downarrow}^\dagger f_\downarrow^\dagger) + \text{H.c.}$$

$$N_\uparrow = n_{d\uparrow} \quad \text{OK!}$$

$$N_\downarrow = n_{d\downarrow} + n_{f\downarrow} \quad \text{not a good QN!}$$

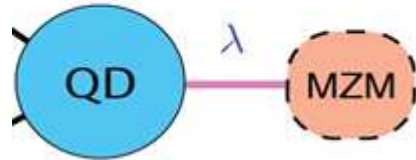
$$\langle \downarrow f \downarrow d | c_{d\downarrow}^\dagger f_\downarrow^\dagger | 0_d 0_f \rangle \neq 0$$

However: $P_\downarrow = (-1)^{N_\downarrow} \quad \text{OK!}$

Build blocks such as: $[N_\uparrow = 0, P_\downarrow = +1] \left\{ \begin{array}{l} |0_d 0_f\rangle \\ |\downarrow d \downarrow f\rangle \end{array} \right. \quad \text{etc,}$

See also: M. Lee, et al., *Phys. Rev. B* **87**, 241402 (2013).

NRG formulation: quantum numbers



H_{-1} : block-diagonal:

$$N_{\uparrow} = n_{d\uparrow}$$

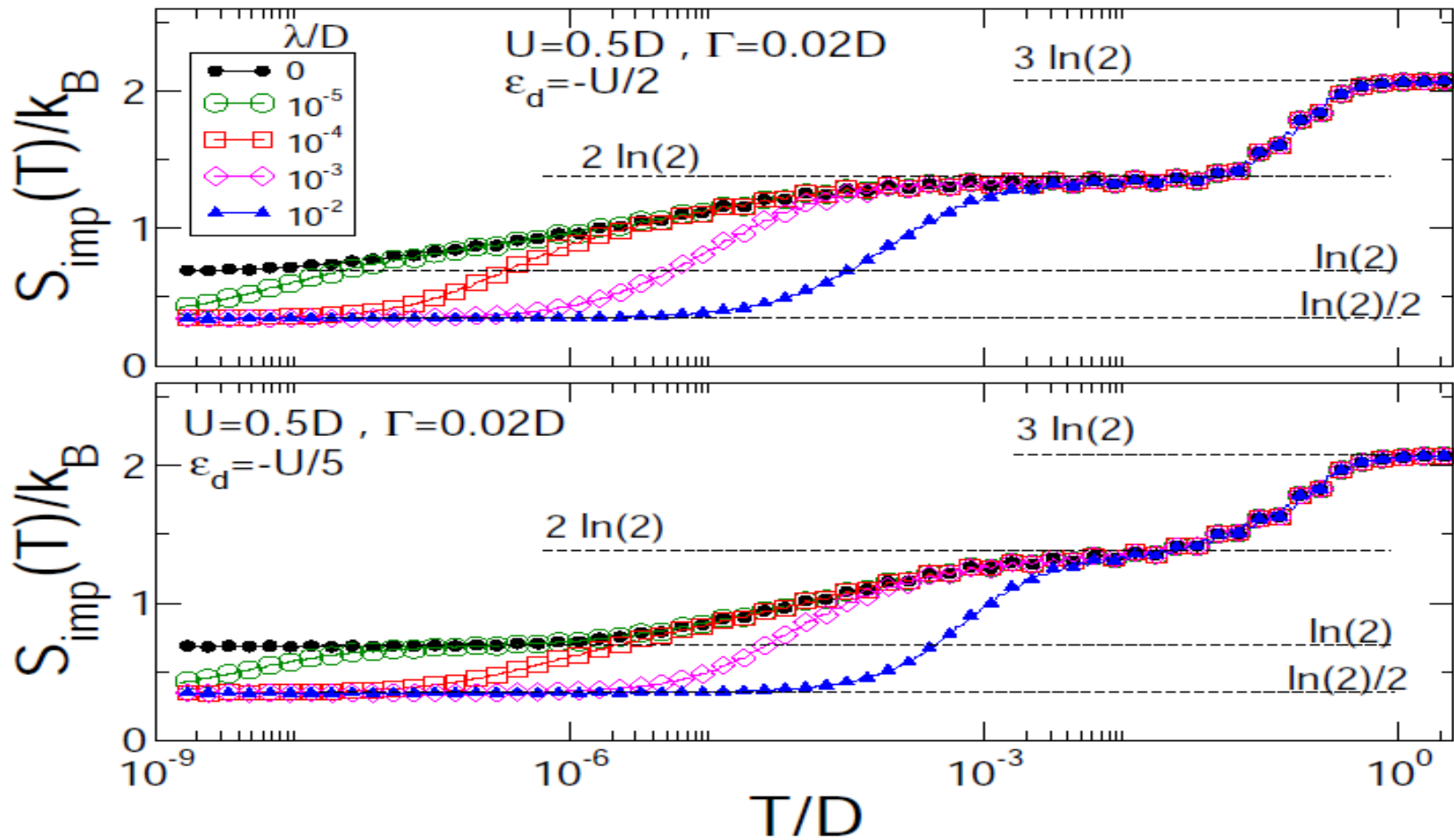
$$P_{\downarrow} = (-1)^{N_{\downarrow}}$$

$$\begin{aligned}
 [N_{\uparrow} = 0, P_{\downarrow} = -1] & \left\{ \begin{array}{l} |0_d \downarrow_f\rangle \\ |\downarrow_d 0_f\rangle \end{array} \right. \\
 [N_{\uparrow} = 0, P_{\downarrow} = +1] & \left\{ \begin{array}{l} |0_d 0_f\rangle \\ |\downarrow_d \downarrow_f\rangle \end{array} \right. \\
 [N_{\uparrow} = 1, P_{\downarrow} = -1] & \left\{ \begin{array}{l} |\uparrow_d \downarrow_f\rangle \\ |(\uparrow\downarrow)_d 0_f\rangle \end{array} \right. \\
 [N_{\uparrow} = 1, P_{\downarrow} = +1] & \left\{ \begin{array}{l} |\uparrow_d 0_f\rangle \\ |(\uparrow\downarrow)_d \downarrow_f\rangle \end{array} \right.
 \end{aligned}$$

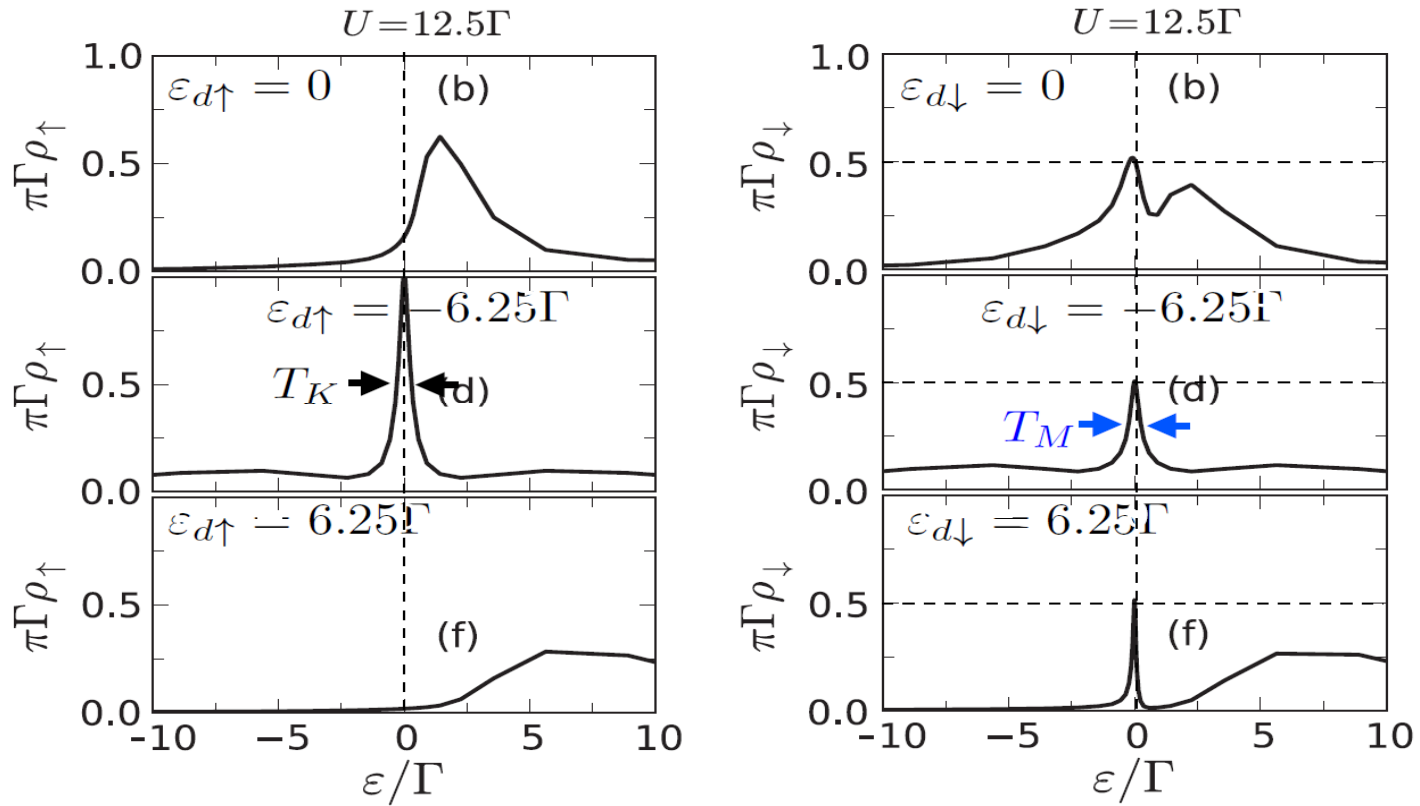
H_{-1} matrix

	$n_{\uparrow}=0, P_{\downarrow}=-1$	$n_{\uparrow}=0, P_{\downarrow}=1$	$n_{\uparrow}=1, P_{\downarrow}=-1$	$n_{\uparrow}=1, P_{\downarrow}=1$
$ 0_d \downarrow_f\rangle$	$\frac{1}{2} + \epsilon_m$			
$ \downarrow_d 0_f\rangle$	$\bar{\epsilon}_-$			
$ 0_d 0_f\rangle$		$\frac{1}{2} - \epsilon_m$		
$ \downarrow_d \downarrow_f\rangle$		$\bar{\epsilon}_+$		
$ \uparrow_d \downarrow_f\rangle$			$\epsilon_d + \frac{1}{2} + h$	
$ (\uparrow\downarrow)_d 0_f\rangle$			$-\bar{\epsilon}_-$	
$ \uparrow_d 0_f\rangle$			$2\epsilon_d + 3\frac{1}{2}$	
$ (\uparrow\downarrow)_d \downarrow_f\rangle$			$-\bar{\epsilon}_+$	
				$\epsilon_d + \frac{1}{2} + h$
				$-\bar{\epsilon}_+$
				$2\epsilon_d + 3\frac{1}{2}$
				$+\epsilon_m$

NFL behavior: $\ln(2)^{1/2}$ residual entropy



Majorana-Kondo co-existence



D. A. Ruiz-Tijerina et al. *Phys Rev B* **91** 115435 (2015).

Consistent with:

M. Lee, et al., *Phys. Rev. B* **87**, 241402 (2013).

Cheng et al., *Phys. Rev. X* **4**, 031051 (2014).

Manipulating MBS with quantum dots.

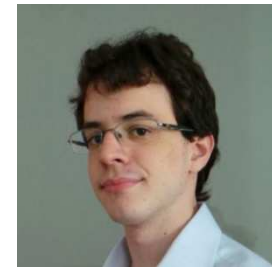
Group Members



Luis Gregório Dias da Silva
Professor



Marcos Medeiros
Doutorado



Raphael Levy
Doutorado



Jesus Cifuentes
Mestrado



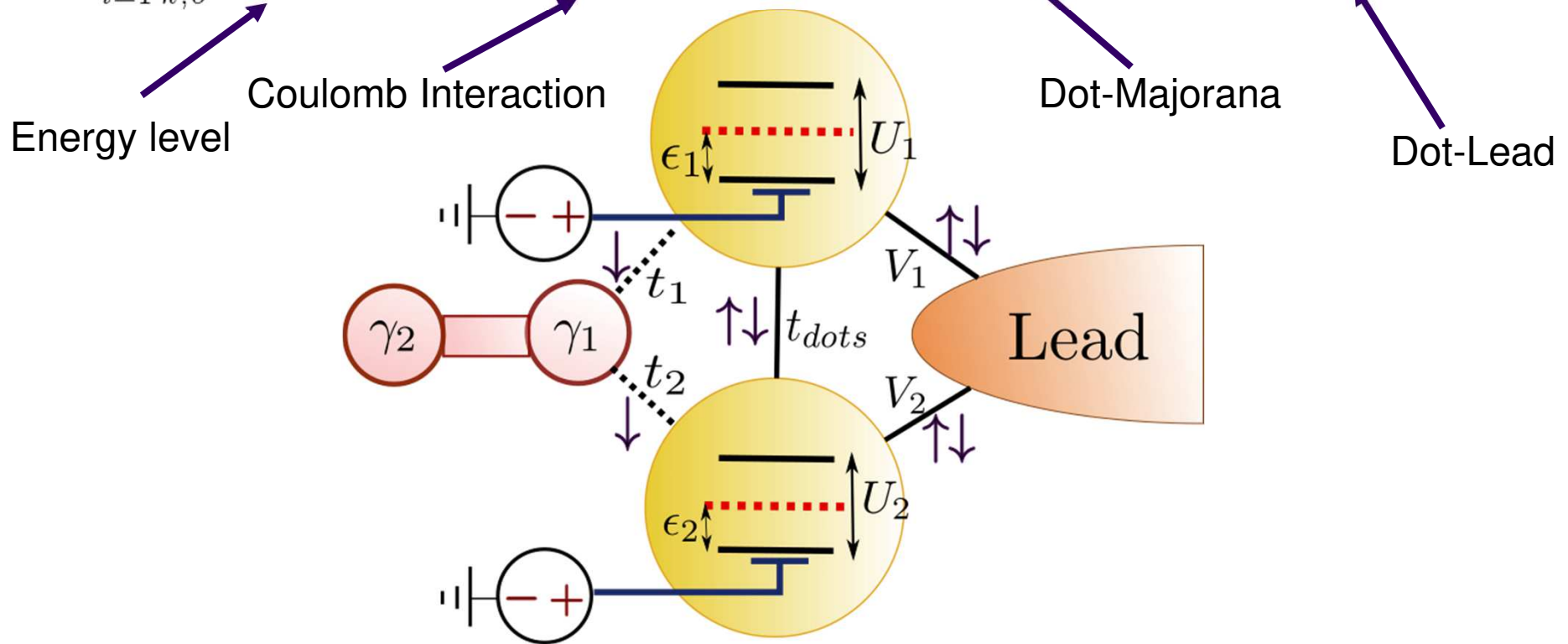
Rafael Magaldi
Mestrado



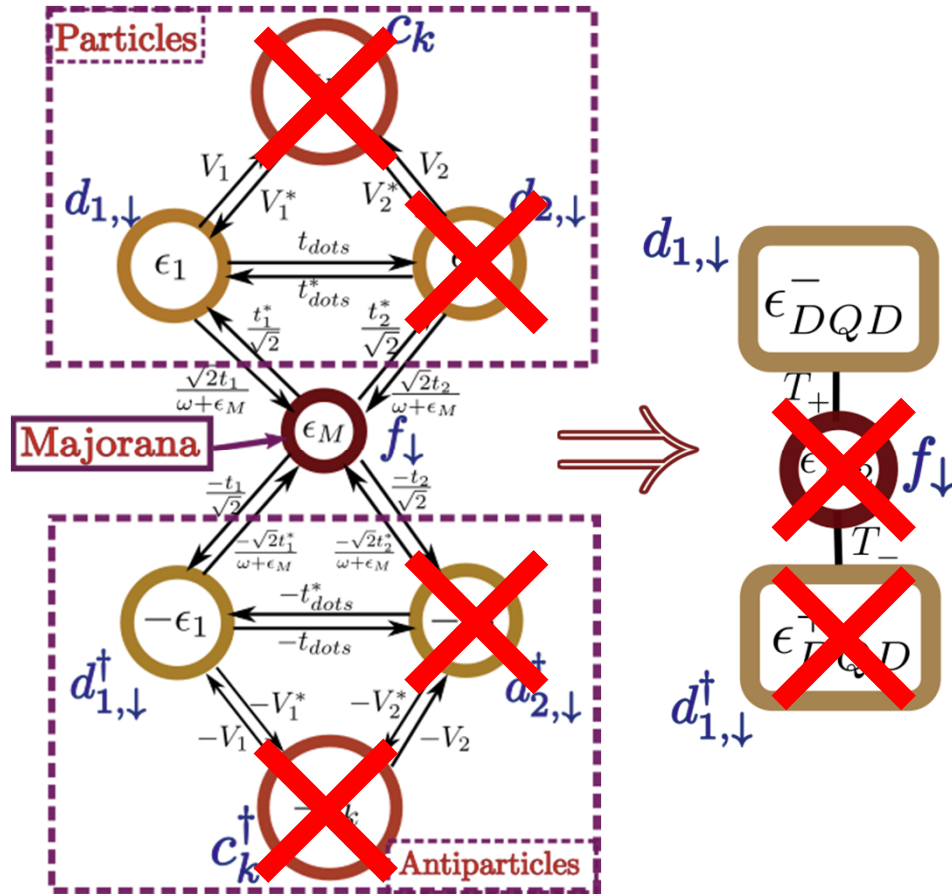
João Victor Ferreira Alves
Mestrado

Manipulation of Majorana fermions in Double Quantum Dots

$$H = \sum_{i=1}^2 \sum_{k,\sigma} \left(\epsilon_i + \frac{U_i}{2} \right) d_{i\sigma}^\dagger d_{i\sigma} + \frac{U_i}{2} (d_{i\sigma}^\dagger d_{i\sigma} - 1)^2 + t_i \gamma_1 d_{i,\downarrow} + t_i^* d_{i,\downarrow}^\dagger \gamma_1 + V_i d_{i\sigma}^\dagger c_{k\sigma} + V_i^* c_{k\sigma}^\dagger d_{i\sigma}.$$

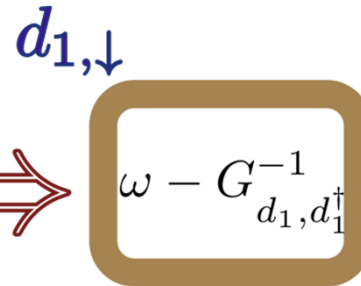


Non-interacting case: spectral densities



Green's Function

$$G_{d_{1\downarrow}, d_{1\downarrow}^\dagger}(\omega) = \frac{1}{\omega - \epsilon_{DQD}^+ + \frac{\|T_+\|^2}{\omega - \epsilon_{M2} - \frac{\|T_-\|^2}{\epsilon_{DQD}^-}}}$$

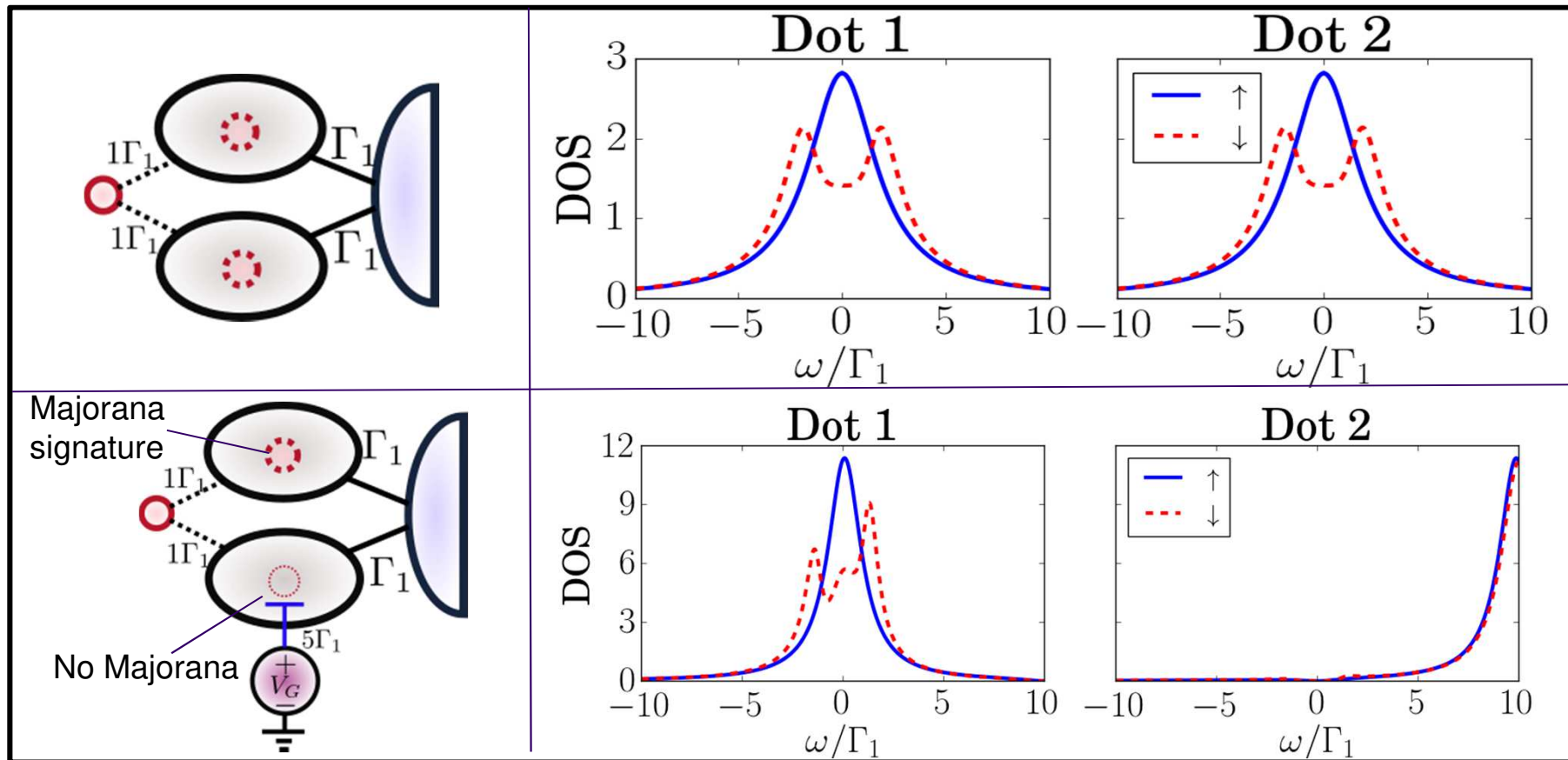


Density of States (DOS)

$$\rho_1(\omega) = -\frac{1}{\pi} \text{Im} \left[G_{d_1, d_1^\dagger}(\omega) \right].$$

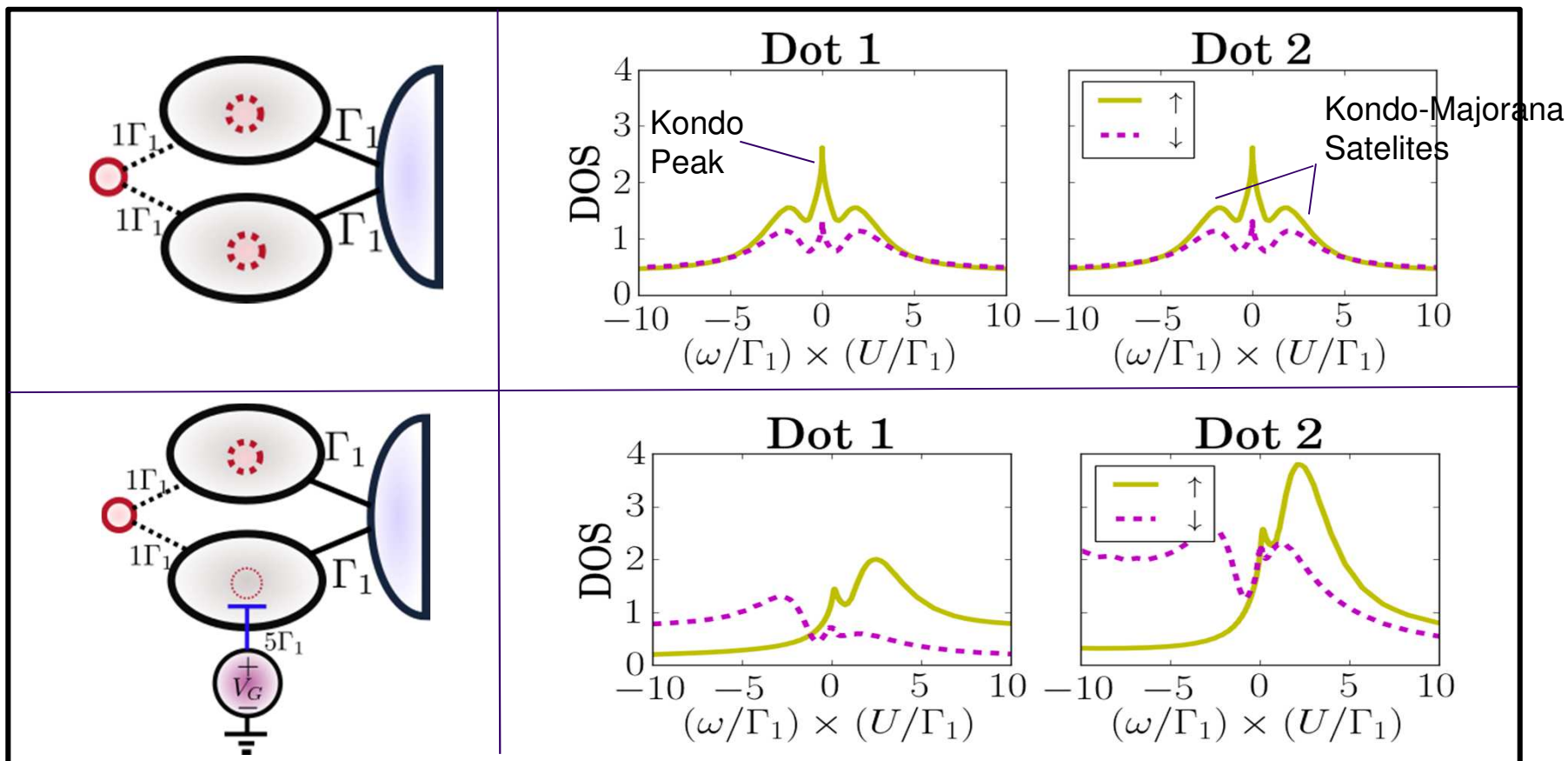
Symmetric coupling

Non-Interacting $U=0$



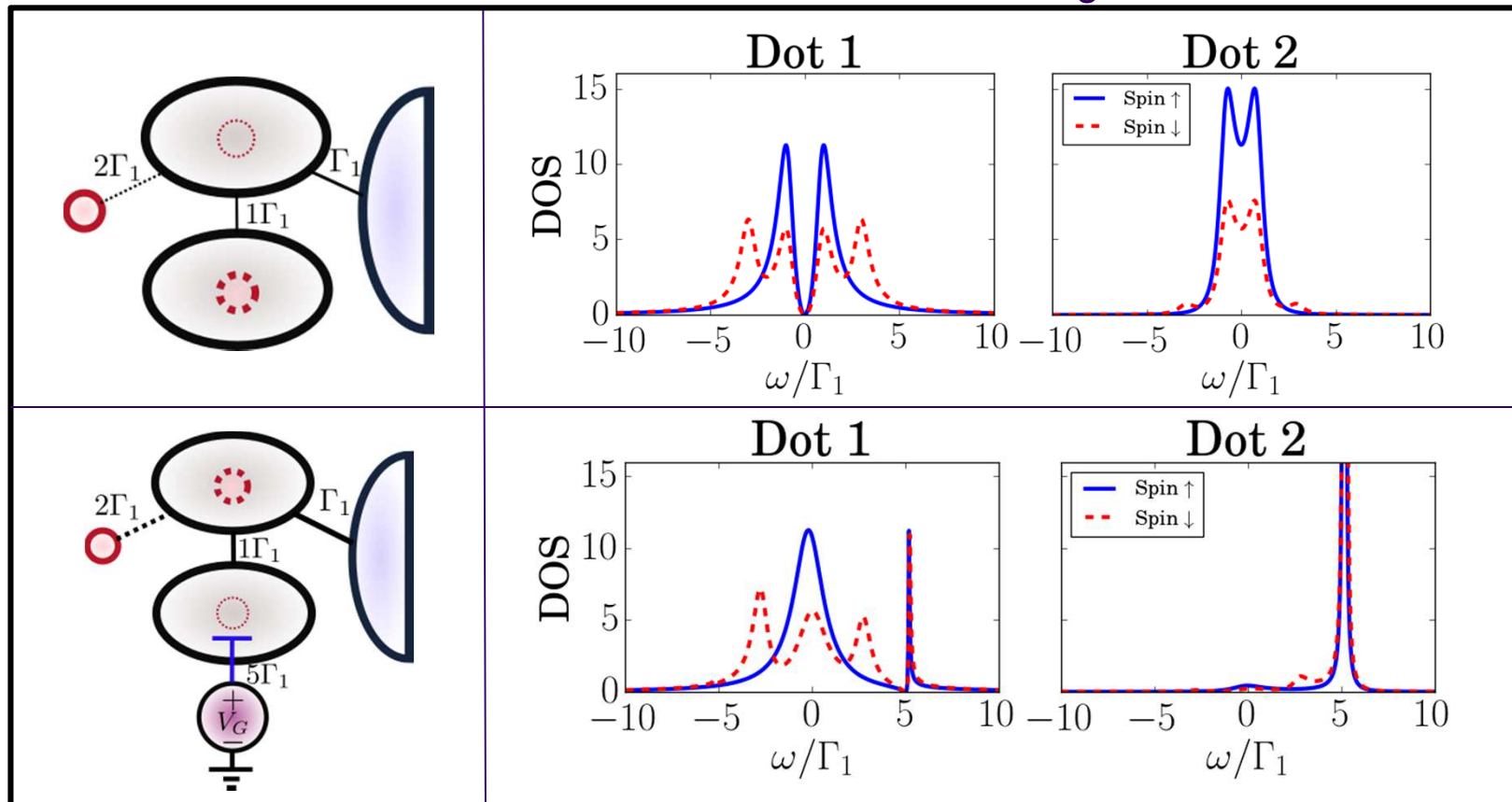
Symmetric coupling

Interacting $U > 0$ (NRG)



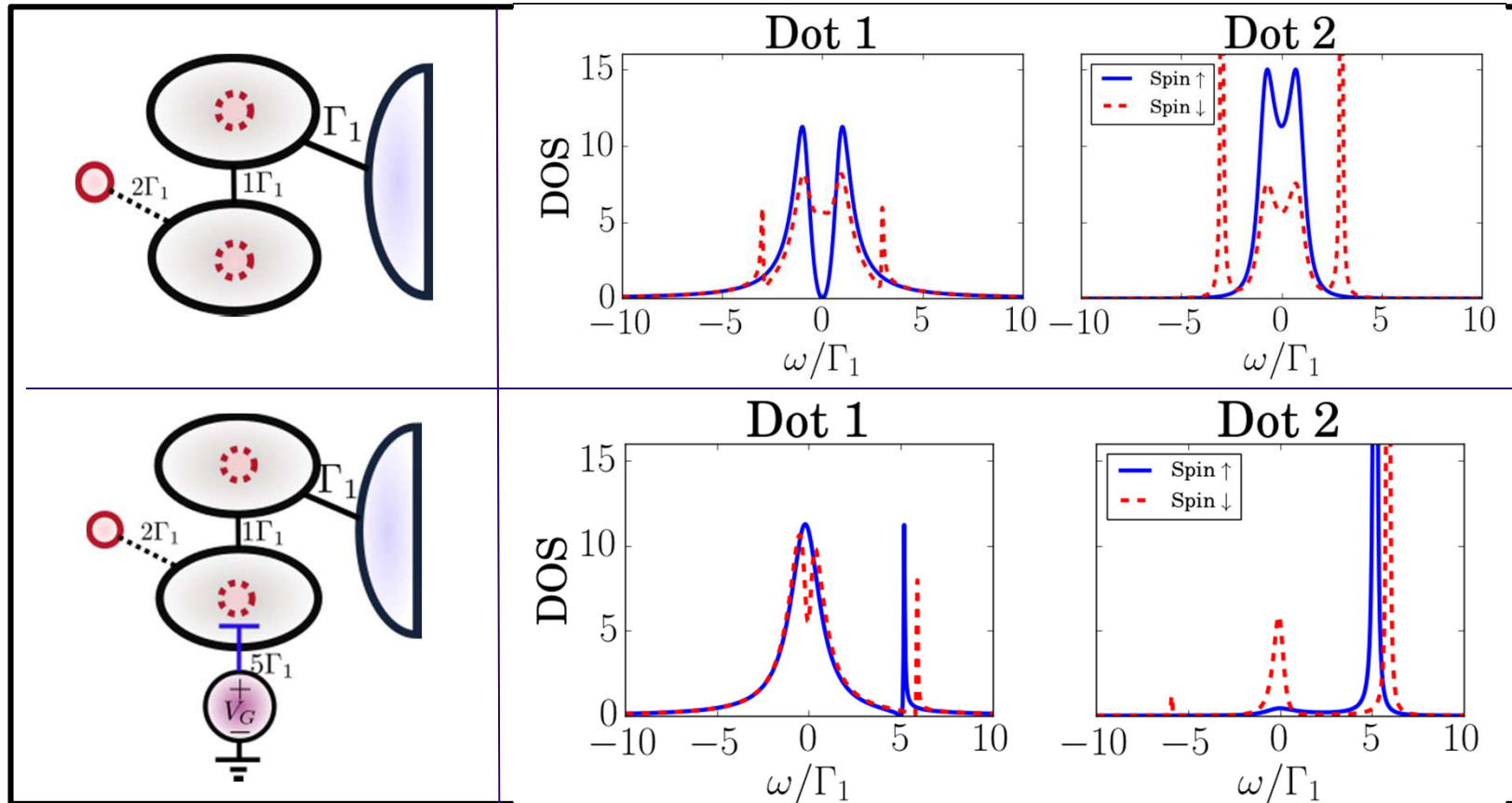
Interference destroying Majorana signature

Non-Interacting

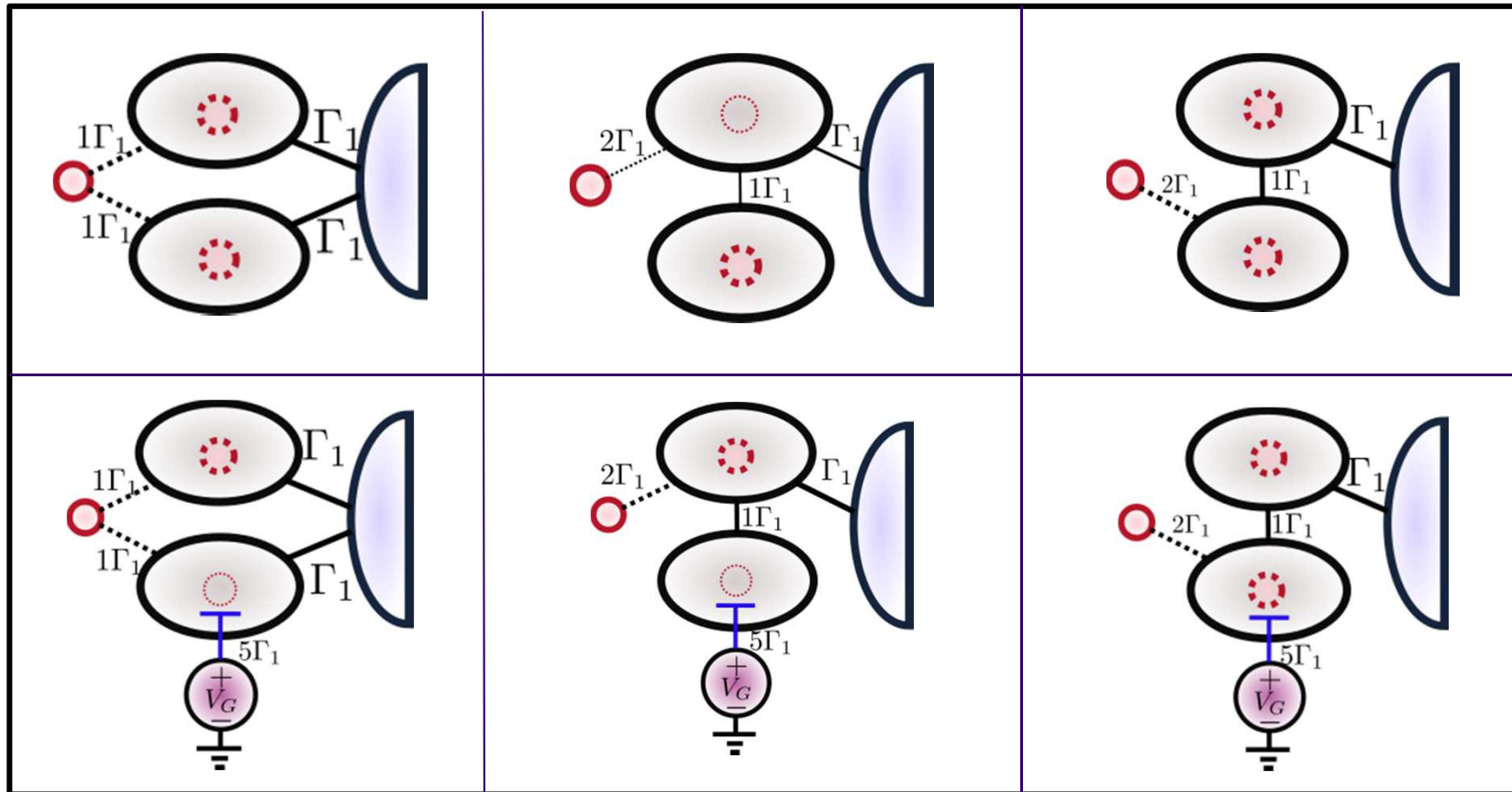


Indirect Majorana Coupling

Non-Interacting

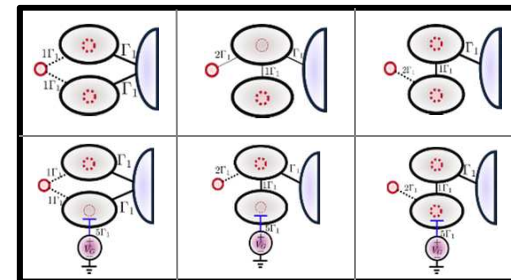
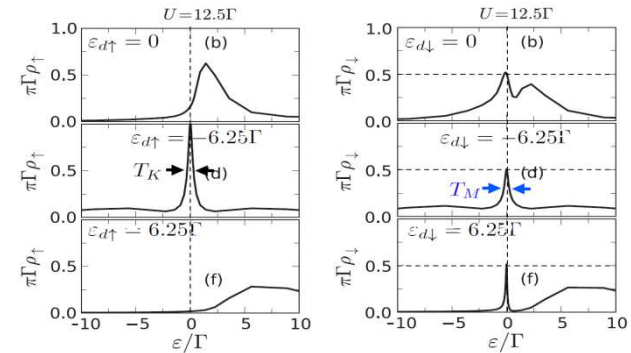


Manipulating with couplings/gate voltages.



Summary

- *Coexistence of MZM and Kondo states in interacting quantum dots*
- *Detecting MZMs using quantum dots: signature in the spin-resolved density of states* → large ($e^2/2$) reduction in the conductance.
- *Manipulating MZMs using double quantum dots using only gate voltages and couplings.*



Collaborators in these works



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