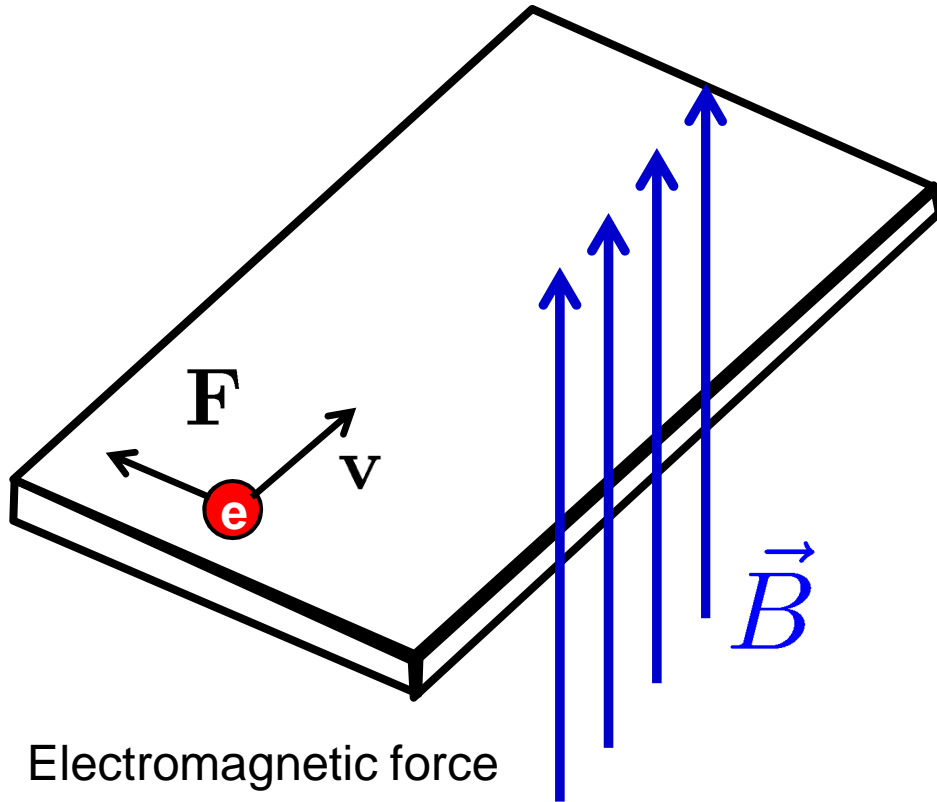


Electrons in a magnetic field



Electromagnetic force

$$\mathbf{F} = (-e)\mathbf{E} + (-e)\mathbf{v} \times \mathbf{B}$$

Electric potential and Vector potential.

$$\begin{cases} \mathbf{B} = \nabla_{\mathbf{r}} \times \mathbf{A}(\mathbf{r}) \\ \mathbf{E} = -\nabla_{\mathbf{r}} V(\mathbf{r}) - \frac{\partial}{\partial t} \mathbf{A}(\mathbf{r}, t) \end{cases}$$

$$m^* \frac{d\mathbf{v}}{dt} = +e\nabla_{\mathbf{r}} V + e \frac{\partial \mathbf{A}}{\partial t} + (-e)\mathbf{v} \times (\nabla_{\mathbf{r}} \times \mathbf{A})$$

We now use:

$$\begin{cases} \mathbf{v} \times (\nabla \times \mathbf{A}) = \nabla (\mathbf{v} \cdot \mathbf{A}) - (\mathbf{v} \cdot \nabla) \mathbf{A} \\ \frac{d\mathbf{A}}{dt} = \frac{\partial \mathbf{A}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{A} \end{cases}$$

to obtain: $\frac{d}{dt} (m^* \mathbf{v} - e\mathbf{A}) = +e\nabla_{\mathbf{r}} (V - \mathbf{v} \cdot \mathbf{A})$

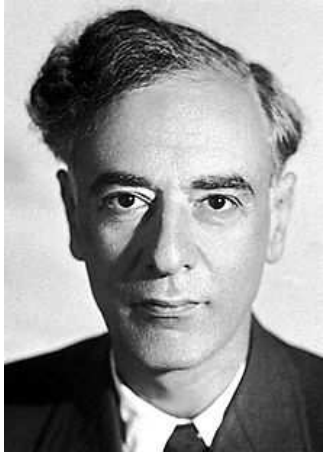
Canonical momentum:

$$\mathbf{p} = m^* \mathbf{v} - e\mathbf{A}$$

Kinetic energy:

$$K = \frac{1}{2} m^* \mathbf{v}^2 = \frac{(\mathbf{p} + e\mathbf{A})^2}{2m^*}$$

Electrons in a magnetic field: quantum solution.



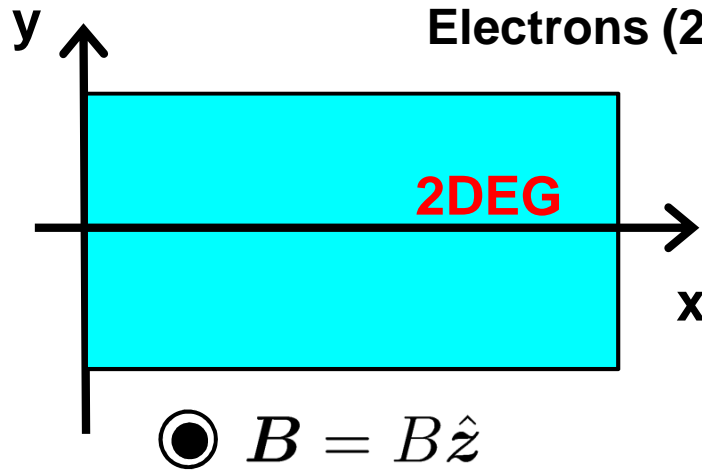
Lev Landau



1962

Landau and Lifshitz,
*Quantum Mechanics:
Non-Relativistic
Theory*

LD Landau *Z. Phys.* 64
31-38 (1930)



$$\left[\frac{1}{2m^*} (\mathbf{p} + e\mathbf{A})^2 \right] \Psi(\mathbf{r}) = E\Psi(\mathbf{r})$$

$$\mathbf{B} = \nabla_{\mathbf{r}} \times \mathbf{A} = B\hat{z}$$

Gauge choice: $\mathbf{A} = -By\hat{x}$

Hamiltonian: $H = \frac{p_x^2}{2m^*} + \frac{1}{2m^*} (p_y^2 - 2eBp_x y + (eB)^2 y^2)$

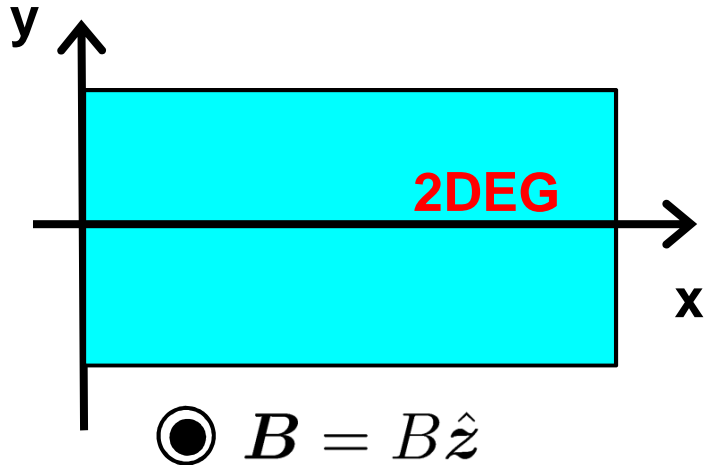
Canonical momentum in x is conserved:

$$[p_x, H] = 0 \Rightarrow \Psi(\mathbf{r}) = A e^{ik_x x} f(y)$$

We write:

$$H\Psi(\mathbf{r}) = \left[\frac{\hbar^2 k_x^2}{2m^*} + \frac{1}{2m^*} (p_y^2 - 2\hbar e B k_x y + (eB)^2 y^2) \right] \Psi(\mathbf{r})$$

Electrons in a magnetic field: quantum solution.



We can re-write the Hamiltonian as:

$$H\Psi(\mathbf{r}) = \left[\frac{p_y^2}{2m^*} + \frac{m^*}{2} \omega_c^2 (y - y_k)^2 \right] \Psi(\mathbf{r})$$

where: $y_k = \frac{\hbar k_x}{eB}$

Harmonic oscillator with a “center” that depends on k_x !!

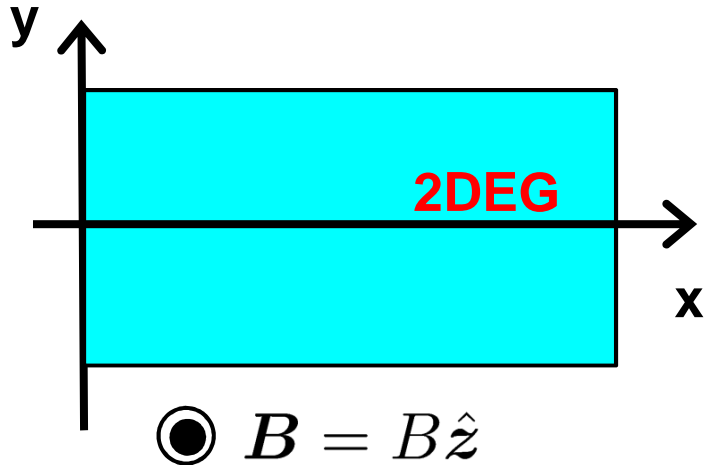
Solution: $\Psi_{n,k_x}(\mathbf{r}) = \frac{1}{\sqrt{L_x}} e^{ik_x x} f_{n,k_x}(y - y_k)$

Eigenfunctions of the harmonic oscillator centered in y_k :

$$f_{n,k_x}(y - y_k) = \frac{1}{\sqrt{2^n n!}} \left(\frac{eB}{\pi \hbar} \right)^{\frac{1}{4}} e^{-\frac{eB(y-y_k)^2}{2\hbar}} H_n \left[\sqrt{\frac{eB}{\hbar}} (y - y_k) \right]$$

Hermite polynomials

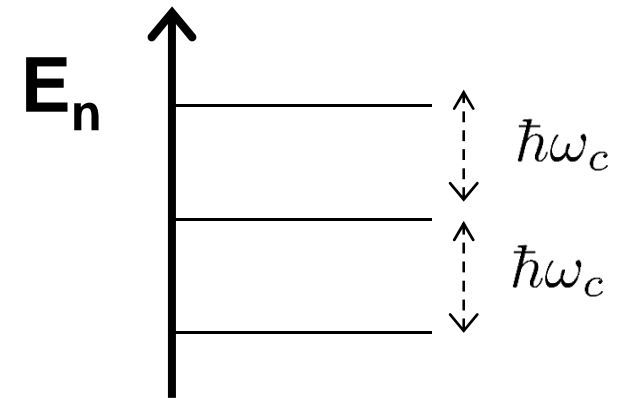
Landau levels.



Spectrum: Landau levels

$$E_n = \left(n + \frac{1}{2} \right) \hbar\omega_c$$

$$\omega_c = \frac{eB}{m^*}$$



- Separation between levels are linear with the applied field.

$$e = 1,6 \times 10^{-19} \text{ Coulomb}$$

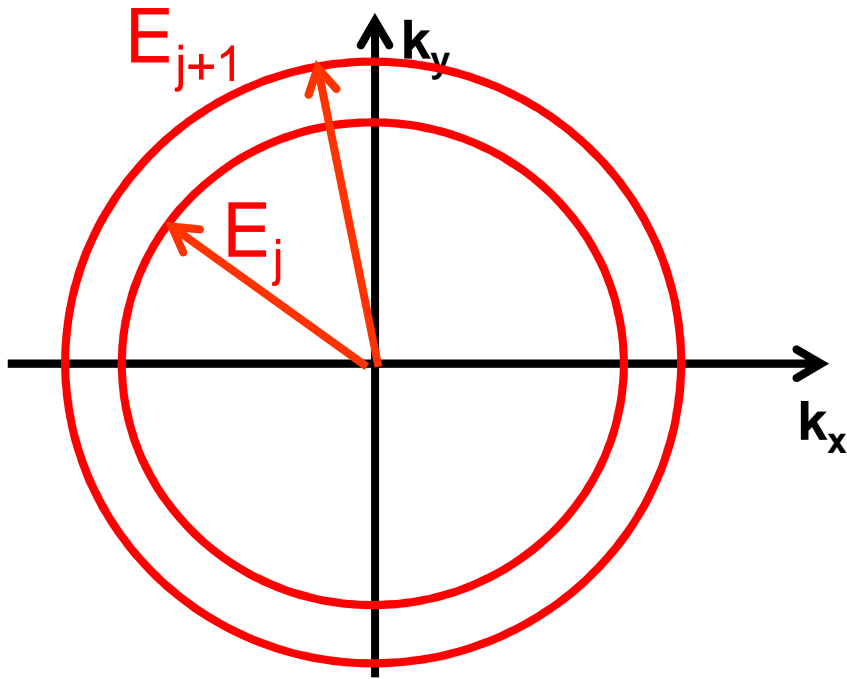
$$m_0 = 9,11 \times 10^{-31} \text{ kg}$$

$$\hbar = 6,58212 \times 10^{-16} \text{ eV.s}$$

$$\omega_c = \frac{1.756 \times 10^{11}}{(m^*/m_0)} \left(\frac{B}{1 \text{ Tesla}} \right) \text{ s}^{-1}$$

$$\hbar\omega_c = \frac{1.157 \times 10^{-4}}{(m^*/m_0)} \left(\frac{B}{1 \text{ Tesla}} \right) \text{ eV}$$

Landau level density of states.



- Number of states per area unit:

B=0: Number of states between \mathbf{k} and $\mathbf{k}+\Delta\mathbf{k}$:

$$\Delta N_{st}^{\Delta\mathbf{k}} = 2 \frac{\Delta A_{\mathbf{k}}}{A_{\mathbf{k}}^{1st}} = 2A_{\mathbf{r}} \frac{2\pi k dk}{(2\pi)^2}$$

B≠0: Number of states between LL j and $j+1$:

$$E_j = \left(j + \frac{1}{2} \right) \hbar\omega_c \equiv \frac{\hbar^2 k_j^2}{2m^*} \quad \omega_c = \frac{eB}{m^*}$$

$$\Delta N_{st}^{\Delta j=1} = 2 \frac{\Delta A_{\mathbf{k}}}{A_{\mathbf{k}}^{1st}} = 2A_{\mathbf{r}} \frac{\pi k_{j+1}^2 - \pi k_j^2}{(2\pi)^2}$$

$$\boxed{\frac{\Delta N_{st}^{\Delta j=1}}{A_{\mathbf{r}}} = \frac{2eB}{h}}$$

Landau level density of states.

- Number of states per area unit: $\frac{\Delta N_{st}^{\Delta j=1}}{A_r} = \frac{2eB}{h}$

- “Filled” j th Landau level (given density n):

$$n = j_{\max} \frac{2eB}{h} \Rightarrow j_{\max} = \frac{1}{2} \frac{nh}{eB}$$

- “Filling factor” ν (per spin): $\nu = \frac{nh}{eB}$

- “Half-filled” j th Landau level.

$$n = \left(j + \frac{1}{2} \right) \frac{2eB}{h}$$

