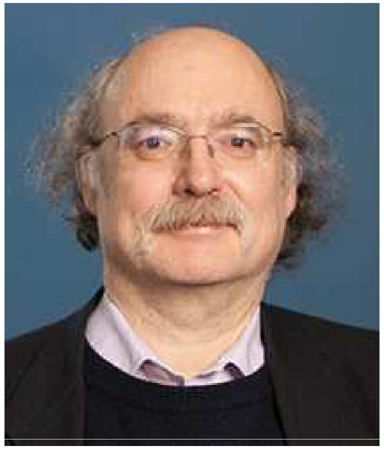
The road to the quantum spin Hall effect.

Haldane: Hall conductance with zero flux.



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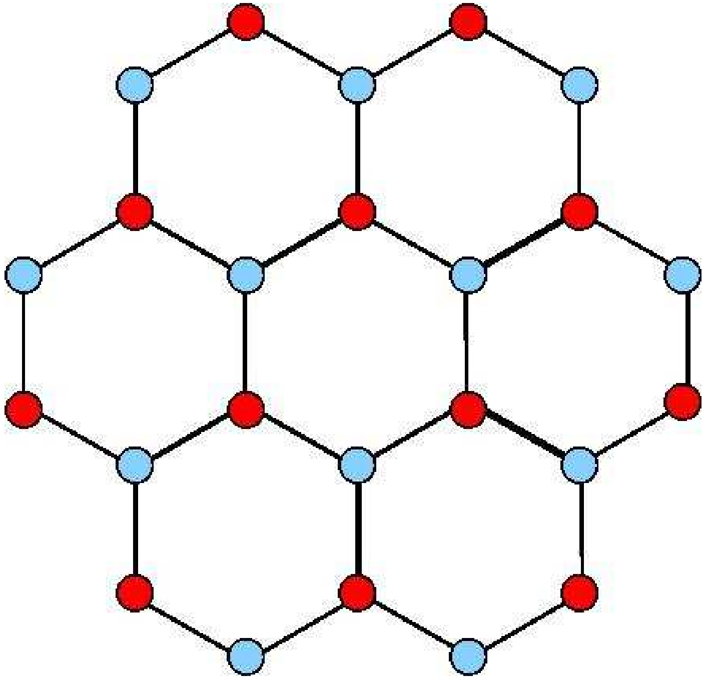

Spinless fermions in a Graphene-like lattice model (two triangular sublattices)

F.D.M. Haldane,
***Phys. Rev. Lett*. 61, 2015 (1988)**

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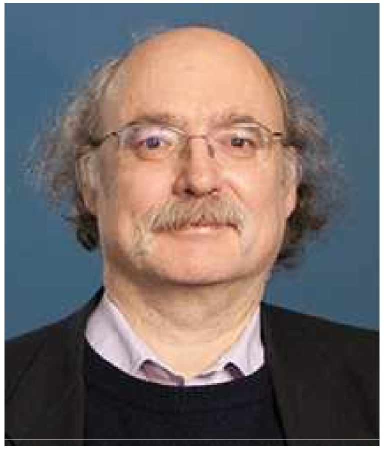

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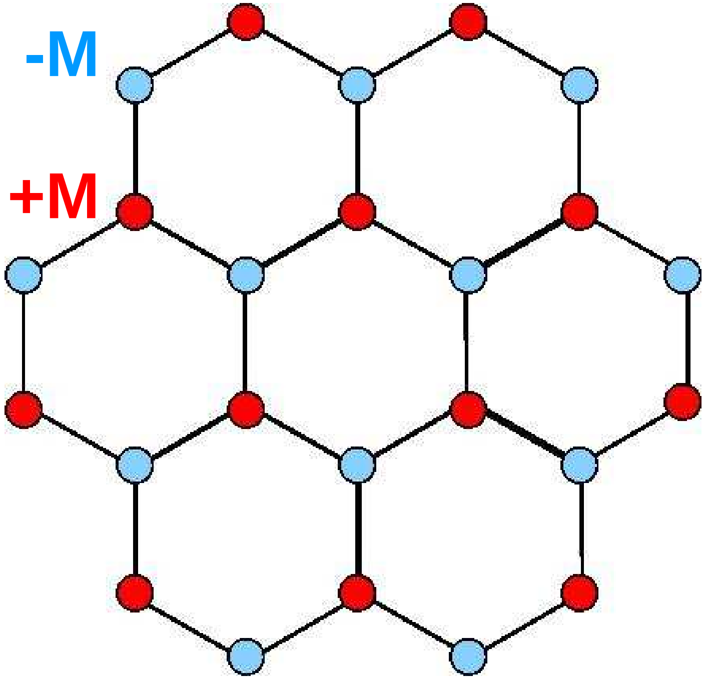

Spinless fermions in a Graphene-like lattice model (two triangular sublattices)

Each sublatice has a different "mass term": Inversion symmetry breaking.

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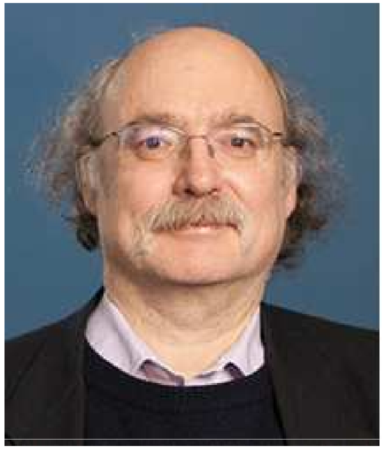

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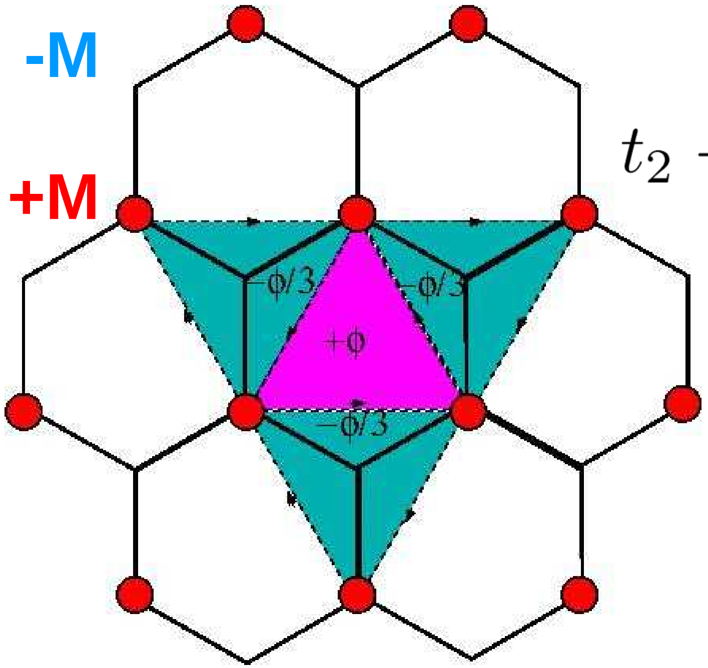


$$t_2 \rightarrow t_2 e^{\pm\phi}$$

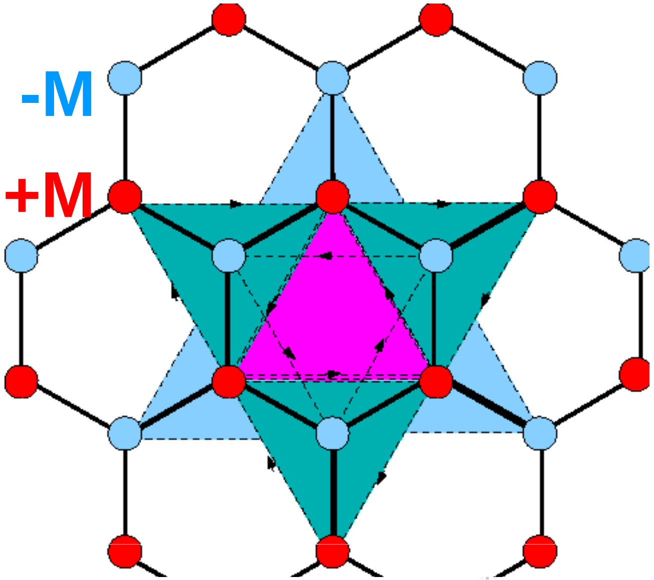
- Spinless fermions in a Graphene-like lattice model (two triangular sublattices)
- Each sublatice has a different “mass term”: Inversion symmetry breaking.
- Space-varying B(r) with **ZERO NET FLUX**: Time reversal symmetry breaking.

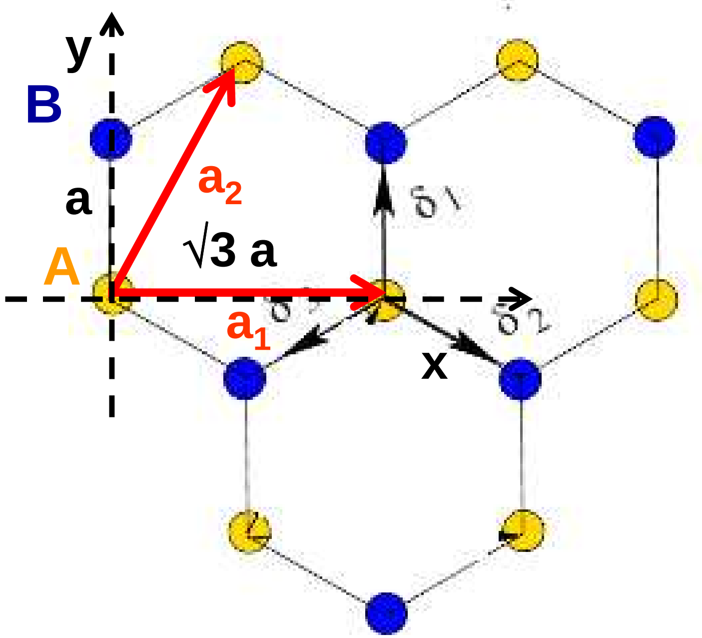
F.D.M. Haldane,
Phys. Rev. Lett. 61, 2015 (1988)

$$\hat{H}_{\mathrm{Haldane}} = -t_1 \sum_{\langle \mathrm{i,j} \rangle} c_{\mathrm{i}}^{\dagger} c_{\mathrm{j}} - t_2 \sum_{\langle\langle \mathrm{i,j} \rangle\rangle} e^{i\phi_{ij}} c_{\mathrm{i}}^{\dagger} c_{\mathrm{j}} + M \sum_{\mathrm{i}} \varepsilon_i c_{\mathrm{i}}^{\dagger} c_{\mathrm{i}}$$

https://topocondmat.org/w4_haldane/haldane_model.html

Haldane model: eigenvalues



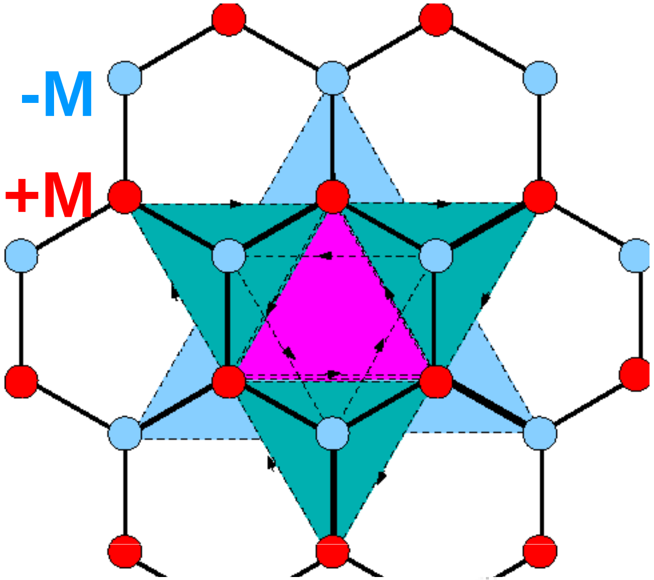


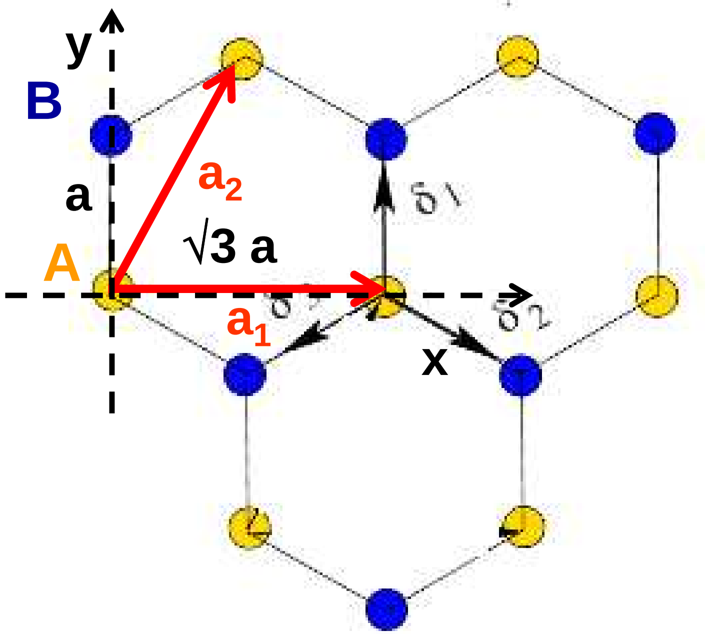
$$\frac{\mathcal{H}_{\mathbf{q}}}{N} = \begin{pmatrix} M + 2t_2 f(\mathbf{q}, \phi) & t_1 \gamma_{\mathbf{q}} \\ t_1 \gamma^*_{\mathbf{q}} & -M + 2t_2 f(\mathbf{q}, -\phi) \end{pmatrix}$$

$$\gamma_{\mathbf{q}} = 1 + e^{i\mathbf{q}\cdot\mathbf{a}_2} + e^{i\mathbf{q}\cdot(\mathbf{a}_2 - \mathbf{a}_1)}$$

$$\begin{aligned} f(\mathbf{q}, \phi) &= \cos(\mathbf{q}\cdot\mathbf{a}_1 + \phi) + \cos(\mathbf{q}\cdot\mathbf{a}_2 - \phi) + \\ &\quad \cos(\mathbf{q}\cdot(\mathbf{a}_2 - \mathbf{a}_1) + \phi) \end{aligned}$$

Tarefa 19: Haldane model





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Consider: t1=1 , $\phi=\pi/2$, and $\mathbf{a}_1$ and $\mathbf{a}_2$ as in the left.

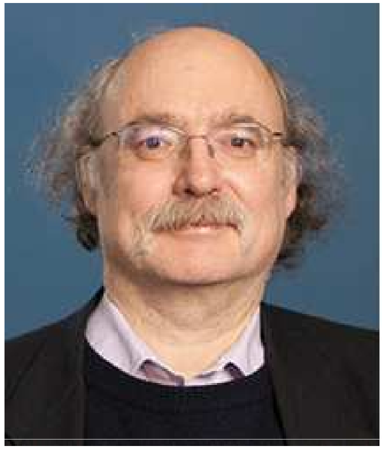
1) Calculate the Hamiltonian matrix for the Brillouin zone vertices **q**=K and **q**=K'. (remember Lista 03!)

2) Show that the gap *vanishes* for

$$t_2 = \pm M/(3\sqrt{3})$$

but not in K and K' at the same time!

Haldane: Hall conductance with zero flux.



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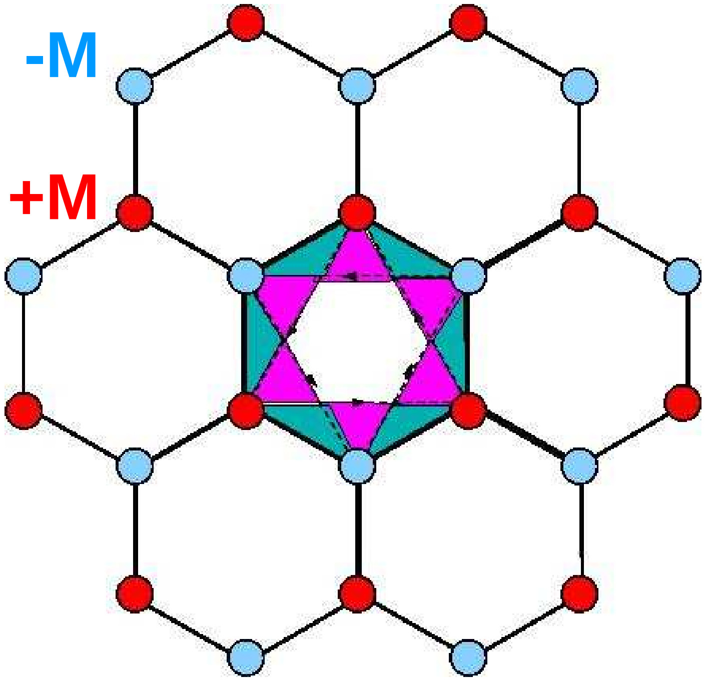
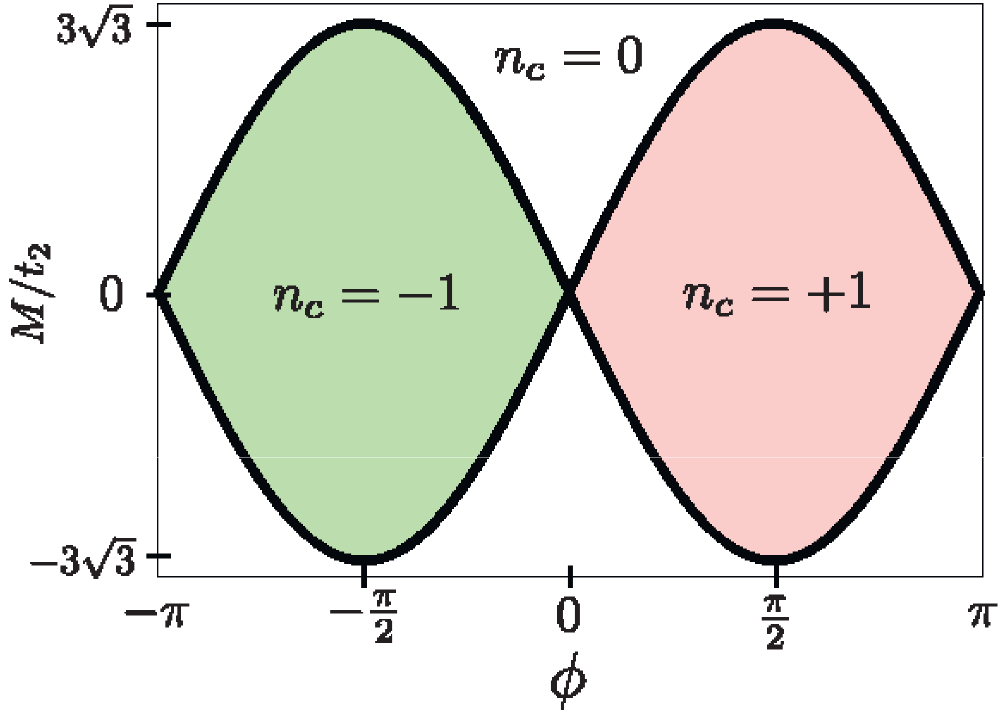
2016



Gap: $\bar{M}_{\pm} = \pm M - 3\sqrt{3}t_2 \sin\phi$



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Hall conductance also given by a Chern number:

$$n_c = \frac{1}{2}\left[\mathrm{sgn}\left(\bar{M}_+\right) + \mathrm{sgn}\left(\bar{M}_-\right)\right]$$

$$\sigma_{xy} = n_c \frac{e^2}{h}$$

$n_c = \pm 1$: Topological phases