

Tarefa 18: identity for the velocity

Using:

$$\left\{ \begin{array}{l} \mathbf{v}(\mathbf{k}) = \frac{1}{\hbar} \nabla_{\mathbf{k}} H(\mathbf{k}) \\ H|n, \mathbf{k}(t)\rangle = E_n[\mathbf{k}(t)]|n, \mathbf{k}(t)\rangle \\ i\hbar \frac{d}{dt} |n, \mathbf{k}(t)\rangle = i\hbar \frac{d\mathbf{k}(t)}{dt} \cdot \nabla_{\mathbf{k}} |n, \mathbf{k}(t)\rangle \\ H|\Psi(t)\rangle = i\hbar \frac{d}{dt} |\Psi(t)\rangle \\ \nabla_{\mathbf{k}} (H|n, \mathbf{k}\rangle) = (\nabla_{\mathbf{k}} H)|n, \mathbf{k}(t)\rangle + H(\nabla_{\mathbf{k}} |n, \mathbf{k}\rangle) \end{array} \right.$$

Show that:

$$\hbar \mathbf{v} |n, \mathbf{k}(t)\rangle = \nabla_{\mathbf{k}} (E_n[\mathbf{k}] |n, \mathbf{k}\rangle) - i\hbar \frac{d\mathbf{k}(t)}{dt} \cdot \nabla_{\mathbf{k}} (\nabla_{\mathbf{k}} |n, \mathbf{k}\rangle)$$