

## Tarefa 3: Derive as seguintes passagens:

$$N = \frac{V_{\mathbf{r}}}{2\pi^2} \left( \frac{2m^*}{\hbar^2} \right)^{\frac{3}{2}} \int_0^{\infty} \frac{\varepsilon^{\frac{1}{2}}}{(e^{(\varepsilon-\mu)/k_B T} + 1)} d\varepsilon \xrightarrow{\mu \gg k_B T} \boxed{\mu \approx \varepsilon_F \left( 1 - \frac{\pi^2 (k_B T)^2}{12\mu^2} \right)}$$

- Dicas: use a expansão de Sommerfeld, a definição de  $\varepsilon_F$  e expansão de polinômio.

$$\int_0^{\infty} \frac{x^j}{(e^{(x-x_0)} + 1)} dx \approx \frac{x_0^{j+1}}{j+1} \left( 1 + \frac{\pi^2 j(j+1)}{6x_0^2} + \mathcal{O}(x_0^{-4}) \right)$$

$$\varepsilon_F = \left( \frac{\hbar^2 (3\pi^2)^{\frac{2}{3}}}{2m^*} \right) n^{\frac{2}{3}} \quad (1+x)^n \approx 1 + nx \text{ para } (x \ll 1)$$