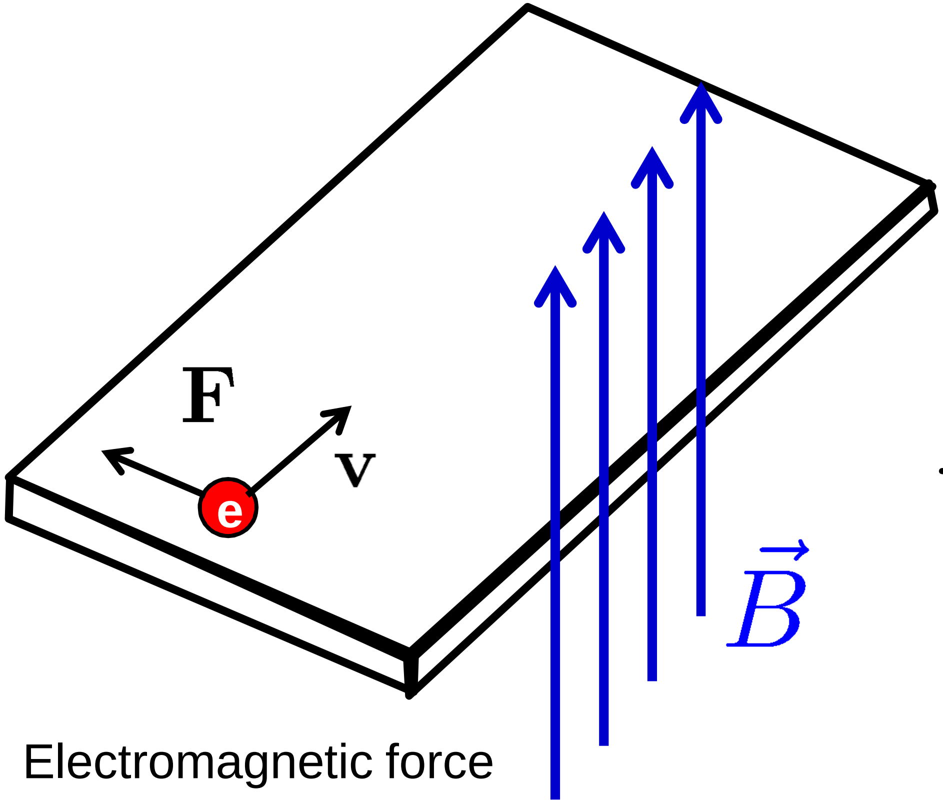
Electrons in a magnetic field



Electromagnetic force

$$\mathbf{F} = (-e)\mathbf{E} + (-e)\mathbf{v} \times \mathbf{B}$$

Electric potential and Vector potential.

$$\begin{cases} \mathbf{B} = \nabla_{\mathbf{r}} \times \mathbf{A}(\mathbf{r}) \\ \mathbf{E} = -\nabla_{\mathbf{r}} V(\mathbf{r}) - \dfrac{\partial}{\partial t}\mathbf{A}(\mathbf{r}, t) \end{cases}$$

$$m^* \frac{d\mathbf{v}}{dt} = +e\nabla_{\mathbf{r}} V + e\frac{\partial \mathbf{A}}{\partial t} + (-e)\mathbf{v} \times (\nabla_{\mathbf{r}} \times \mathbf{A})$$

We now use:

$$\begin{cases} \mathbf{v} \times (\nabla \times \mathbf{A}) = \nabla(\mathbf{v} \cdot \mathbf{A}) - (\mathbf{v} \cdot \nabla)\mathbf{A} \\ \dfrac{d\mathbf{A}}{dt} = \dfrac{\partial \mathbf{A}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{A} \end{cases}$$

to obtain: $$\frac{d}{dt}(m^*\mathbf{v} - e\mathbf{A}) = +e\nabla_{\mathbf{r}}(V - \mathbf{v} \cdot \mathbf{A})$$

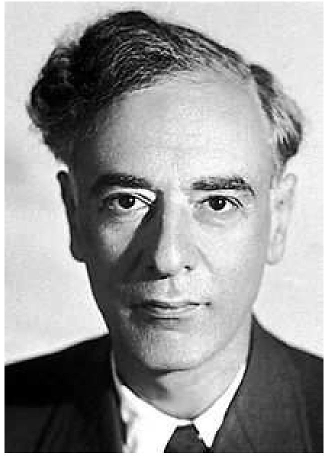
Canonical momentum:

$$\boxed{\mathbf{p} = m^*\mathbf{v} - e\mathbf{A}}$$

Kinetic energy:

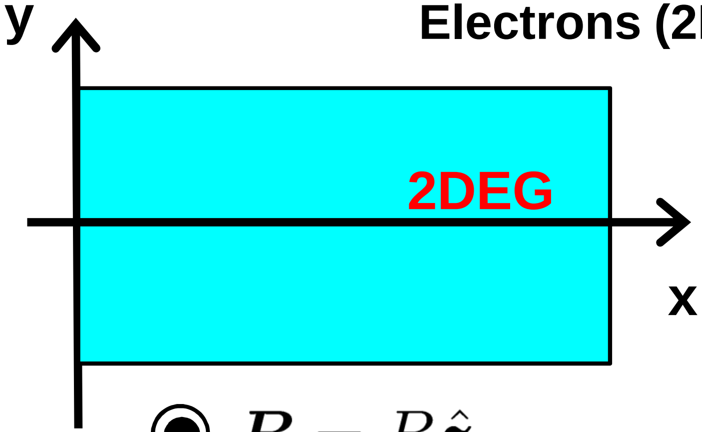
$$\boxed{K = \frac{1}{2}m^*\mathbf{v}^2 = \frac{(\mathbf{p} + e\mathbf{A})^2}{2m^*}}$$

Electrons in a magnetic field: quantum solution.



Lev Landau

1962



⊙ $\boldsymbol{B} = B\hat{\boldsymbol{z}}$

Electrons (2D) in an uniform magnetic field.

$$\left[\frac{1}{2m^*}\left(\boldsymbol{p} + e\boldsymbol{A}\right)^2\right]\Psi(\boldsymbol{r}) = E\Psi(\boldsymbol{r})$$

$$\mathbf{B} = \nabla_{\mathbf{r}} \times \mathbf{A} = B\hat{\boldsymbol{z}}$$

Gauge choice: $\boldsymbol{A} = -By\hat{\boldsymbol{x}}$

Hamiltonian: $$H = \frac{p_x^2}{2m^*} + \frac{1}{2m^*}\left(p_y^2 - 2eBp_x y + (eB)^2 y^2\right)$$

Canonical momentum in x is conserved:

$$[p_x, H] = 0 \Rightarrow \Psi(\boldsymbol{r}) = Ae^{ik_x x} f(y)$$

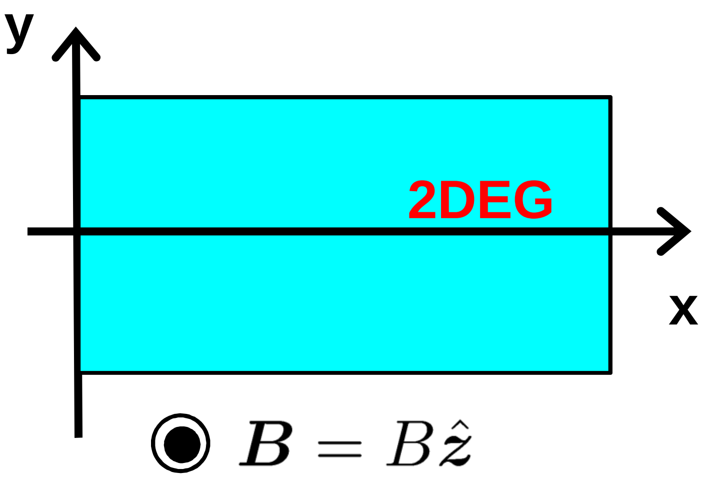
We write:

$$H\Psi(\boldsymbol{r}) = \left[\frac{\hbar^2 k_x^2}{2m^*} + \frac{1}{2m^*}\left(p_y^2 - 2\hbar eBk_x y + (eB)^2 y^2\right)\right]\Psi(\boldsymbol{r})$$

Landau and Lifshitz, *Quantum Mechanics: Non-Relativistic Theory*

LD Landau *Z. Phys.* 64 31-38 (1930)

Electrons in a magnetic field: quantum solution.



We can re-write the Hamiltonian as:

$$H\Psi(\boldsymbol{r}) = \left[\frac{p_y^2}{2m^*} + \frac{m^*}{2}\omega_c^2\left(y - y_k\right)^2\right]\Psi(\boldsymbol{r})$$

where: $$y_k = \frac{\hbar k_x}{eB}$$

Harmonic oscillator with a “center” that depends on k_x !!

Solution: $$\Psi_{n,k_x}(\boldsymbol{r}) = \frac{1}{\sqrt{L_x}} e^{ik_x x} f_{n,k_x}\left(y - y_k\right)$$

Eigenfunctions of the harmonic oscillator centered in y_k:

$$f_{n,k_x}(y - y_k) = \frac{1}{\sqrt{2^n n!}}\left(\frac{eB}{\pi\hbar}\right)^{\frac{1}{4}} e^{-\frac{eB(y-y_k)^2}{2\hbar}} H_n\left[\sqrt{\frac{eB}{\hbar}}(y - y_k)\right]$$

Hermite polinomials

LD Landau *Z. Phys.* 64 31-38 (1930)

Landau and Lifshitz, *Quantum Mechanics: Non-Relativistic Theory*

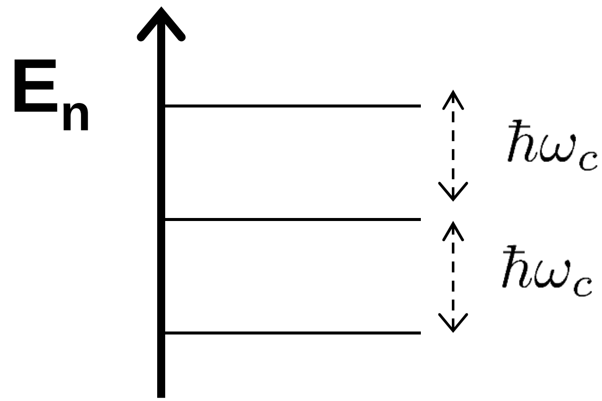
Landau levels.

$\odot\ \boldsymbol{B} = B\hat{\boldsymbol{z}}$

Spectrum: Landau levels

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega_c$$

$$\omega_c = \frac{eB}{m^*}$$



- Separation between levels are linear with the applied field.

$$\omega_c = \frac{1.756\times 10^{11}}{(m^*/m_0)}\left(\frac{B}{1\ \mathrm{Tesla}}\right)\ \mathrm{s}^{-1}$$

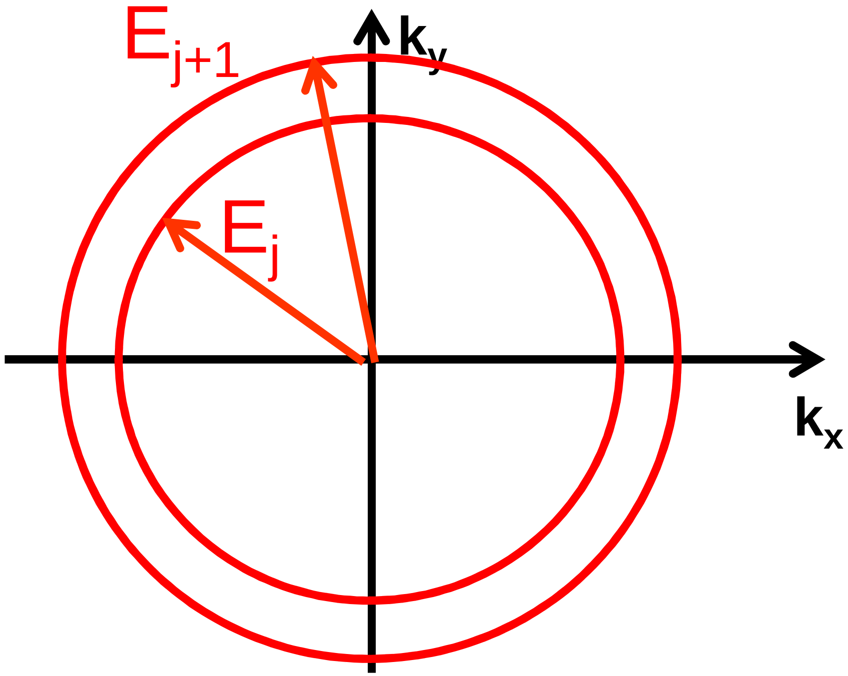
$$\hbar\omega_c = \frac{1.157\times 10^{-4}}{(m^*/m_0)}\left(\frac{B}{1\ \mathrm{Tesla}}\right)\ \mathrm{eV}$$

e = 1,6 x 10^{-19} Coulomb

m_0 = 9,11 x 10^{-31} kg

$\hbar$ = 6,58212 x 10^{-16} eV.s

Landau level density of states.



B=0: Number of states between **k** and **k**+Δ**k**:

$$\Delta N_{\mathrm{st}}^{\Delta\mathbf{k}} = 2\frac{\Delta A_{\mathbf{k}}}{A_{\mathbf{k}}^{1\mathrm{st}}} = 2A_{\mathbf{r}}\frac{2\pi k dk}{(2\pi)^2}$$

B≠0: Number of states between LL j and j+1:

$$E_j = \left(j + \frac{1}{2}\right)\hbar\omega_c \equiv \frac{\hbar^2 k_j^2}{2m^*} \qquad \omega_c = \frac{eB}{m^*}$$

$$\Delta N_{\mathrm{st}}^{\Delta j=1} = 2\frac{\Delta A_{\mathbf{k}}}{A_{\mathbf{k}}^{1\mathrm{st}}} = 2A_{\mathbf{r}}\frac{\pi k_{j+1}^2 - \pi k_j^2}{(2\pi)^2}$$

- Number of states per area unit:

$$\boxed{\frac{\Delta N_{\mathrm{st}}^{\Delta j=1}}{A_{\mathbf{r}}} = \frac{2eB}{h}}$$

Landau level density of states.

- Number of states per area unit: $$\frac{\Delta N_{\mathrm{st}}^{\Delta j=1}}{A_{\mathbf{r}}} = \frac{2eB}{h}$$

- "Filled" jth Landau level (given density n):

$$n = j_{\max}\frac{2eB}{h} \Rightarrow j_{\max} = \frac{1}{2}\frac{nh}{eB}$$

- "Filling factor" ν (per spin): $$\boxed{\nu = \frac{nh}{eB}}$$

- "Half-filled" jth Landau level.

$$n = \left(j + \frac{1}{2}\right)\frac{2eB}{h}$$