

**The road to the  
quantum spin Hall effect.**

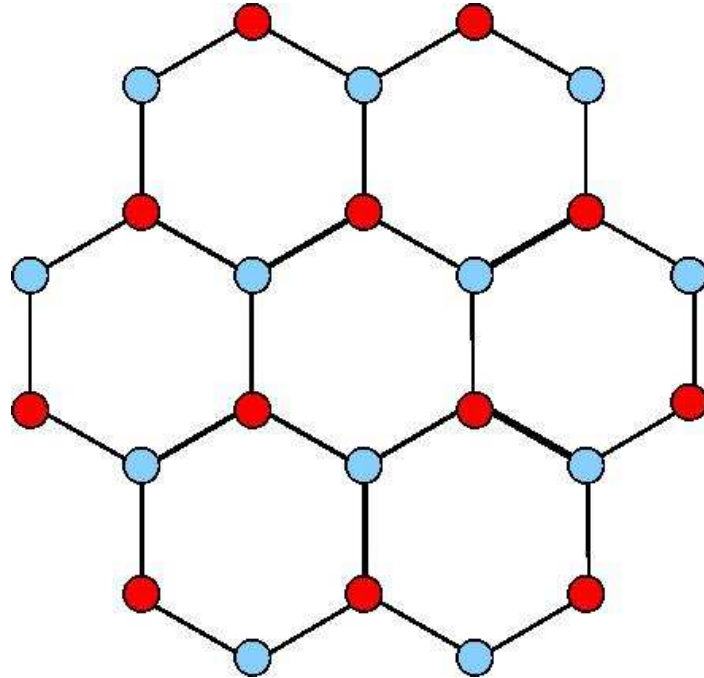
# Haldane: Hall conductance with zero flux.



Duncan Haldane



2016



Spinless fermions in a  
Graphene-like lattice  
model (two triangular  
sublattices)

F.D.M. Haldane,  
*Phys. Rev. Lett.* 61, 2055 (1988)

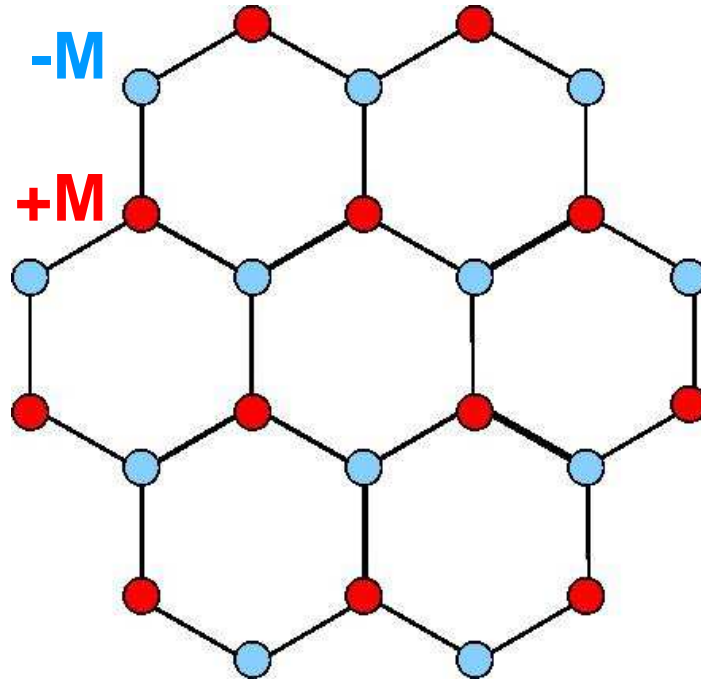
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Each sublattice has a different “mass term”: Inversion symmetry breaking.

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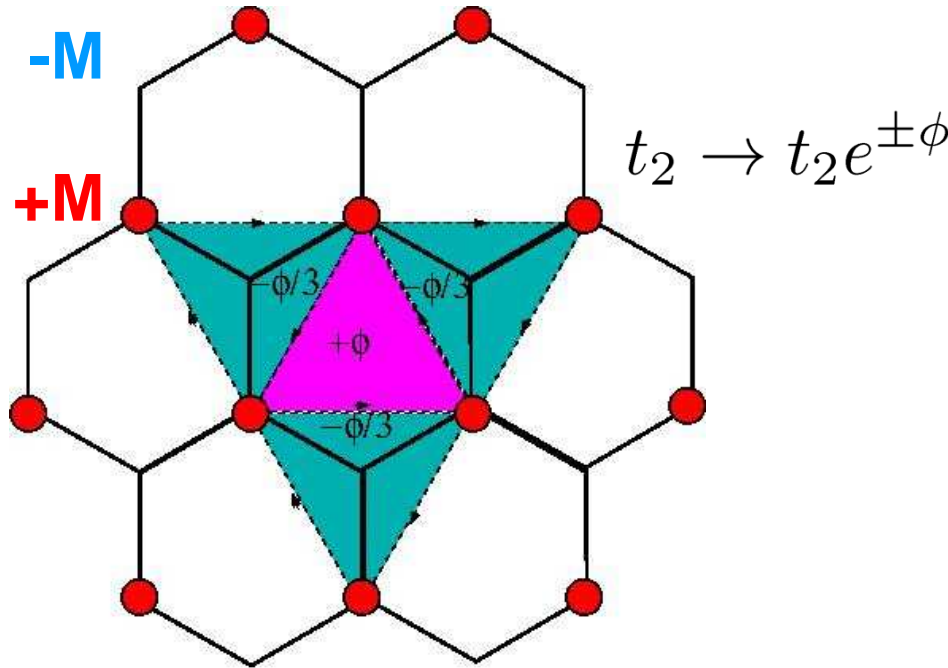
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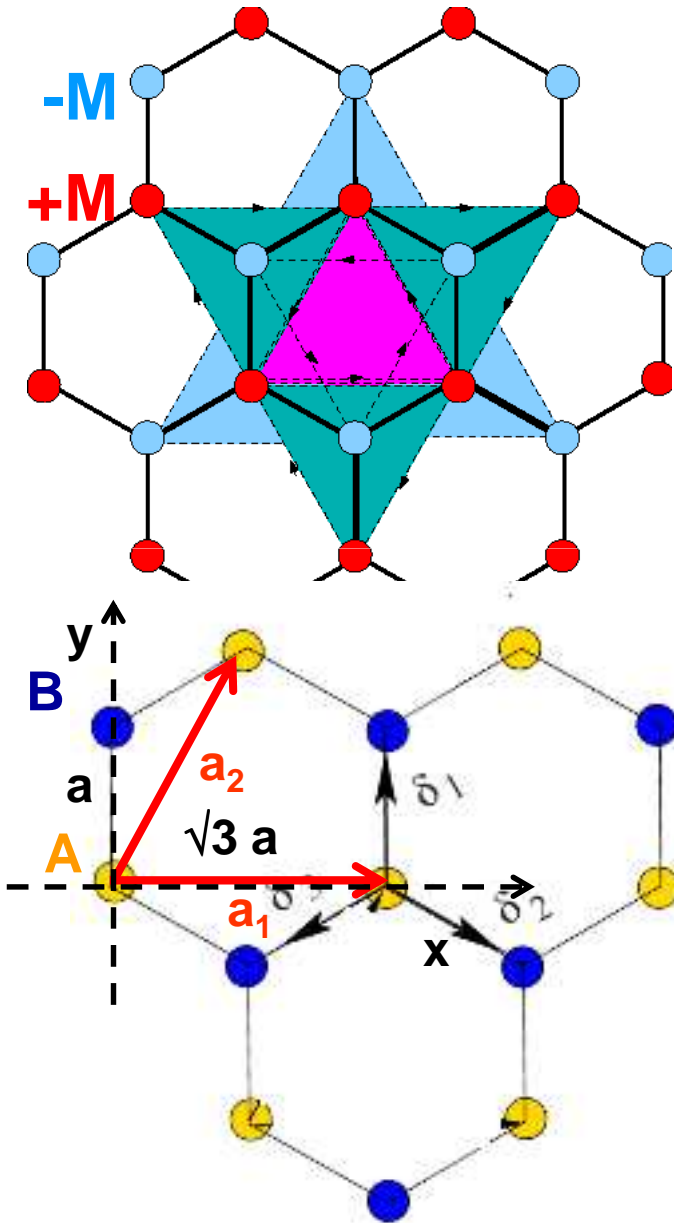
Space-varying  $B(r)$  with **ZERO NET FLUX**: Time reversal symmetry breaking.

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$$\hat{H}_{\text{Haldane}} = -t_1 \sum_{\langle i,j \rangle} c_i^\dagger c_j - t_2 \sum_{\langle\langle i,j \rangle\rangle} e^{i\phi_{ij}} c_i^\dagger c_j + M \sum_i \epsilon_i c_i^\dagger c_i$$

[https://topocondmat.org/w4\\_haldane/haldane\\_model.html](https://topocondmat.org/w4_haldane/haldane_model.html)

# Haldane model: eigenvalues



$$\frac{\mathcal{H}_{\mathbf{q}}}{N} = \begin{pmatrix} M + 2t_2 f(\mathbf{q}, \phi) & t_1 \gamma_{\mathbf{q}} \\ t_1 \gamma_{\mathbf{q}}^* & -M + 2t_2 f(\mathbf{q}, -\phi) \end{pmatrix}$$

$$\gamma_{\mathbf{q}} = 1 + e^{i\mathbf{q} \cdot \mathbf{a}_2} + e^{i\mathbf{q} \cdot (\mathbf{a}_2 - \mathbf{a}_1)}$$

$$f(\mathbf{q}, \phi) = \cos(\mathbf{q} \cdot \mathbf{a}_1 + \phi) + \cos(\mathbf{q} \cdot \mathbf{a}_2 - \phi) + \cos(\mathbf{q} \cdot (\mathbf{a}_2 - \mathbf{a}_1) + \phi)$$

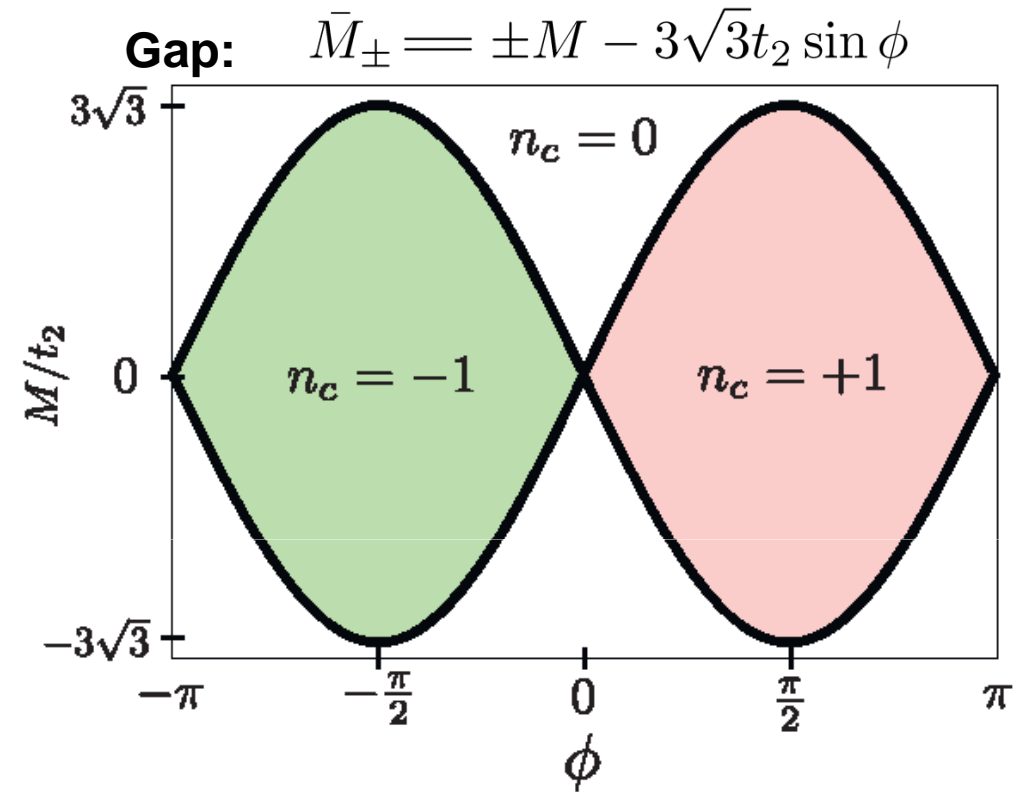
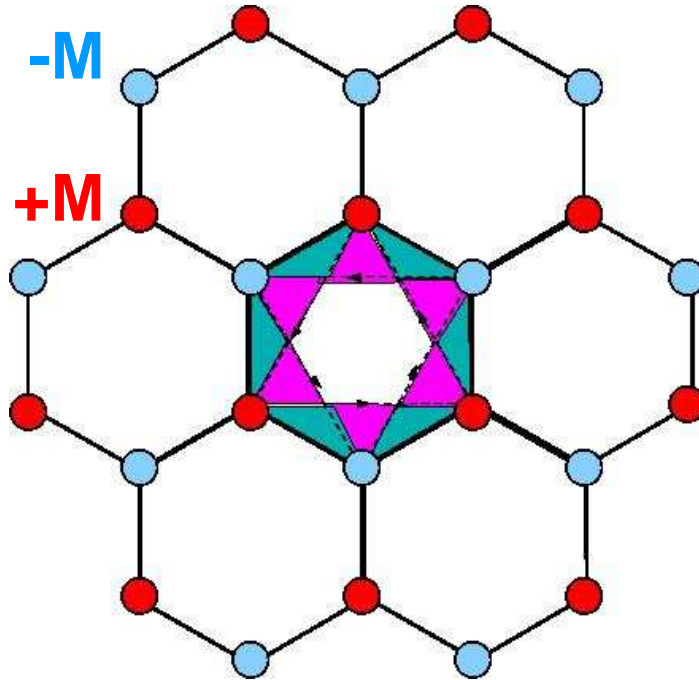
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Hall conductance also given by a  
**Chern number:**

$$n_c = \frac{1}{2} \text{sgn}(\bar{M}_+) + \text{sgn}(\bar{M}_-)$$

$$\sigma_{xy} = n_c \frac{e^2}{h}$$

$n_c = \pm 1$ : **Topological phases**