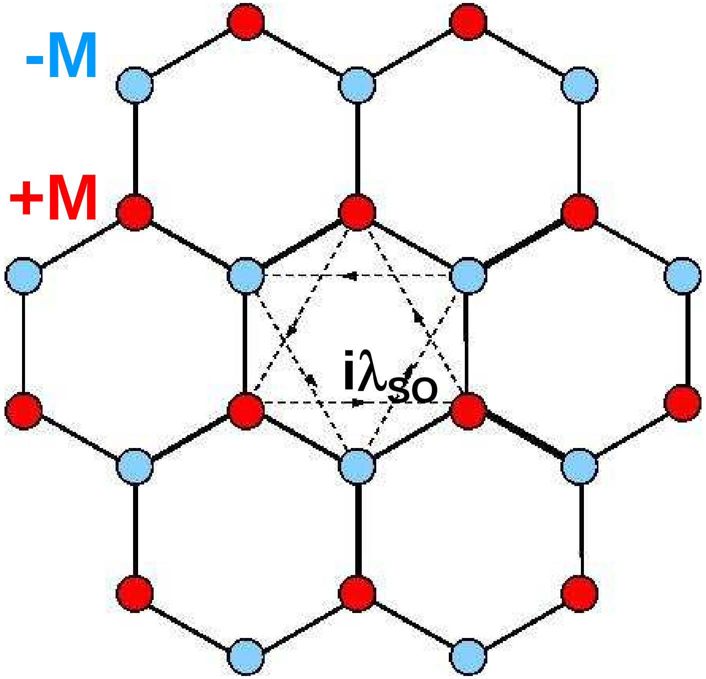
Kane and Mele: Quantum Spin Hall effect.

Charles Kane



Spinful fermions in a Graphene-like lattice model: 4-band model.

Inversion symmetry breaking (not really needed.

Spin orbit term connecting sites in the same sublattice!

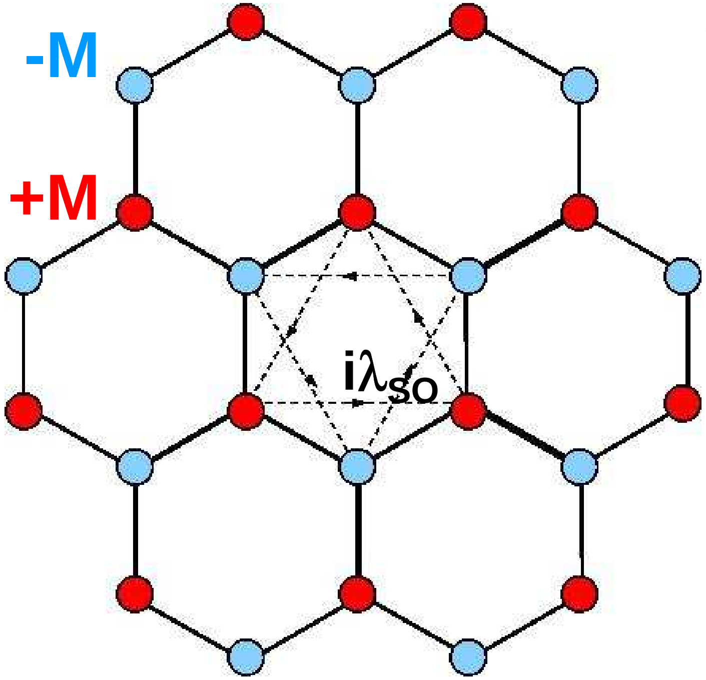
Hamiltonian obeys time-reversal symmetry.

A Rashba spin orbit coupling term can be added (results are qualitatively the same!)

C. L. Kane, E. J. Mele
Phys. Rev. Lett. 95, 146802 (2005)
Phys. Rev. Lett. 95, 226801 (2005).

https://topocondmat.org/w5_qshe/fermion_parity_pump.html

Kane and Mele model (no Rashba SOC)



Spin ↑ : essentially the Haldane model with $\phi=\pi/2$

$$\frac{\mathcal{H}^{\uparrow}_{\mathbf{q}}}{N} = \begin{pmatrix} M + 2\lambda_{SO} f(\mathbf{q}, \frac{\pi}{2}) & t_1\gamma_{\mathbf{q}} \\ t_1\gamma^*_{\mathbf{q}} & -M + 2\lambda_{SO} f(\mathbf{q}, -\frac{\pi}{2}) \end{pmatrix}$$

$$\gamma_{\mathbf{q}} = 1 + e^{i\mathbf{q}\cdot\mathbf{a}_2} + e^{i\mathbf{q}\cdot(\mathbf{a}_2-\mathbf{a}_1)}$$

$$\begin{aligned} f(\mathbf{q},\phi) \quad &= \quad \cos\left(\mathbf{q}\cdot\mathbf{a}_1+\phi\right) + \cos\left(\mathbf{q}\cdot\mathbf{a}_2-\phi\right) + \\ &\quad\quad \cos\left(\mathbf{q}\cdot(\mathbf{a}_2-\mathbf{a}_1)+\phi\right) \end{aligned}$$

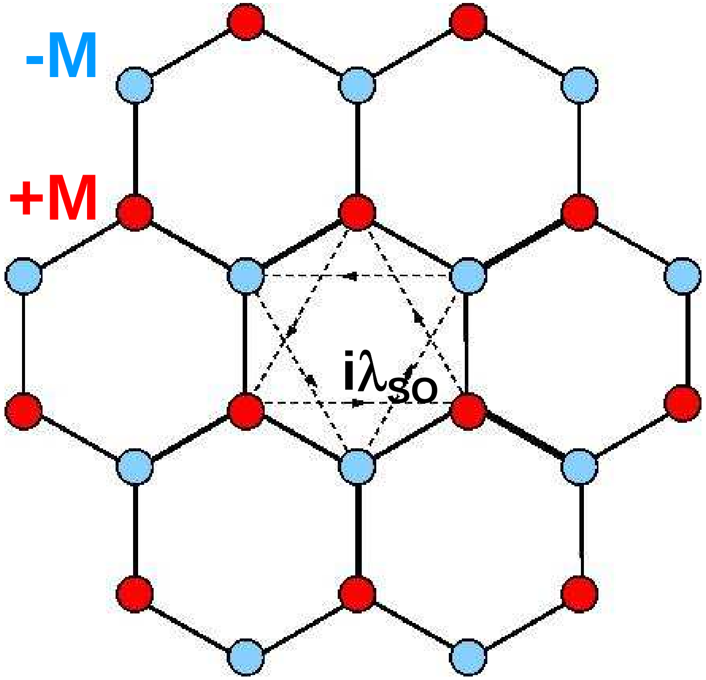
Time-reversal symmetry: $\boxed{\mathcal{H}^{\downarrow}_{\mathbf{q}} = \left(\mathcal{H}^{\uparrow}_{-\mathbf{q}}\right)^*}$

$$\begin{cases} \left(\gamma_{-\mathbf{q}}\right)^* = \gamma_{\mathbf{q}} \\ f(-\mathbf{q},\phi) = f(\mathbf{q},-\phi) \end{cases}$$

Spin ↓ : Haldane model with $\phi=-\pi/2$

$$\frac{\mathcal{H}^{\downarrow}_{\mathbf{q}}}{N} = \begin{pmatrix} M + 2\lambda_{SO} f(\mathbf{q}, -\frac{\pi}{2}) & t_1\gamma_{\mathbf{q}} \\ t_1\gamma^*_{\mathbf{q}} & -M + 2\lambda_{SO} f(\mathbf{q}, \frac{\pi}{2}) \end{pmatrix}$$

Kane and Mele: Quantum Spin Hall effect.



Gap: $|6\sqrt{3}\lambda_{SO} - 2M|$

Topological phase : $M < 3\sqrt{3}\lambda_{SO}$

Chern number $n_s = \mathrm{sgn}\,(s\lambda_{SO})$

Z_2 invariant $\boxed{\nu = \frac{1}{2}(n_\uparrow - n_\downarrow) = \pm 1}$

New ingredients:

- **Particles with spin s.**
- **Spin-Orbit coupling λ_{SO} (TRS preserved)**
- **Assuming no Rashba SO.**

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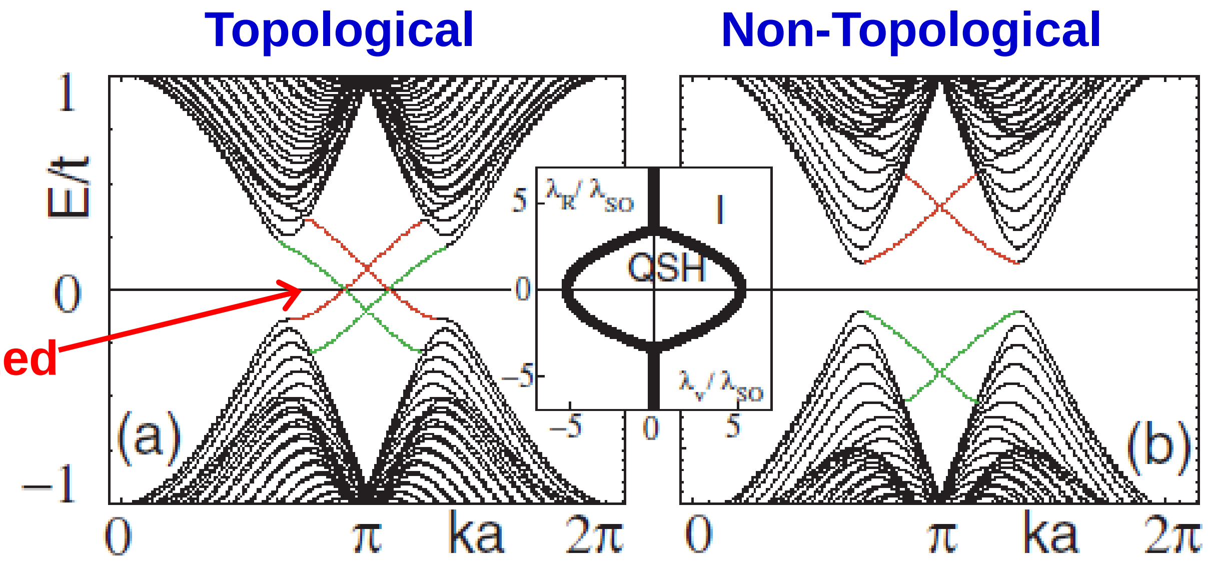
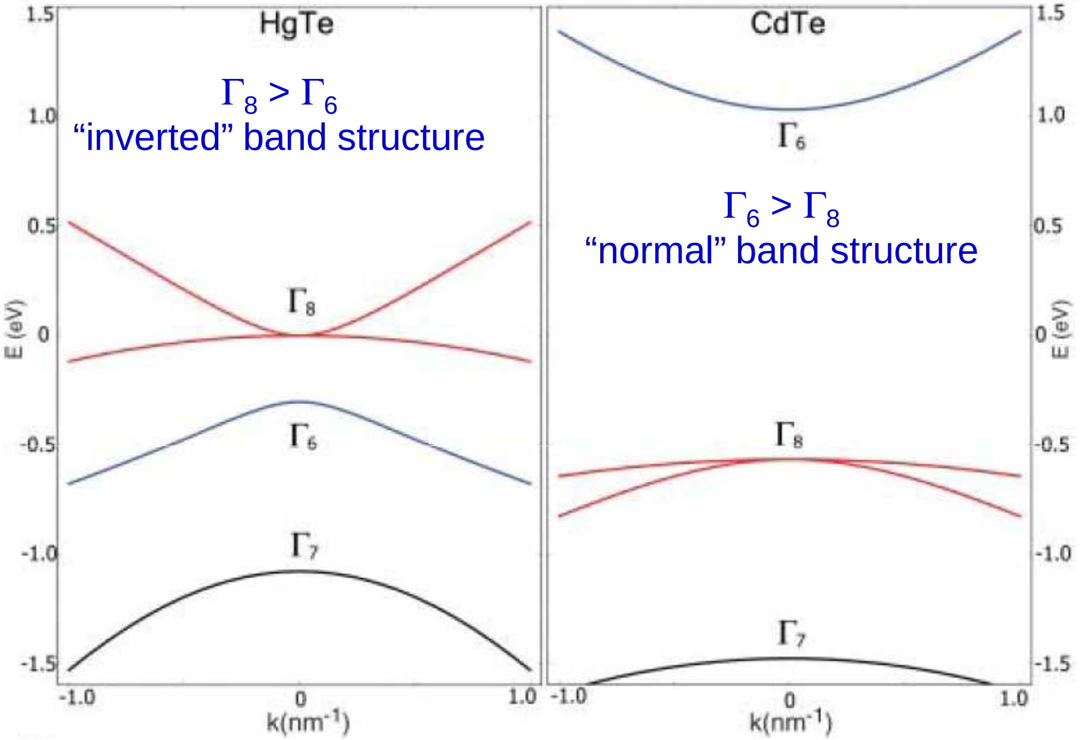

Topological **Non-Topological**

Spin-polarized Edge states

https://topocondmat.org/w5_qshe/fermion_parity_pump.html

HgTe Quantum Wells: “inverted” bands



$\Gamma_8 > \Gamma_6$
“inverted” band structure

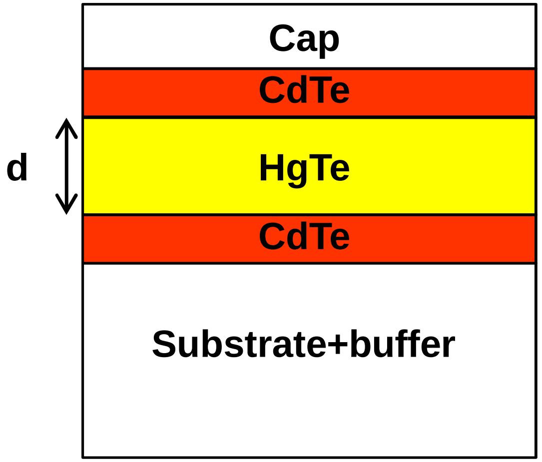
$\Gamma_6 > \Gamma_8$
“normal” band structure

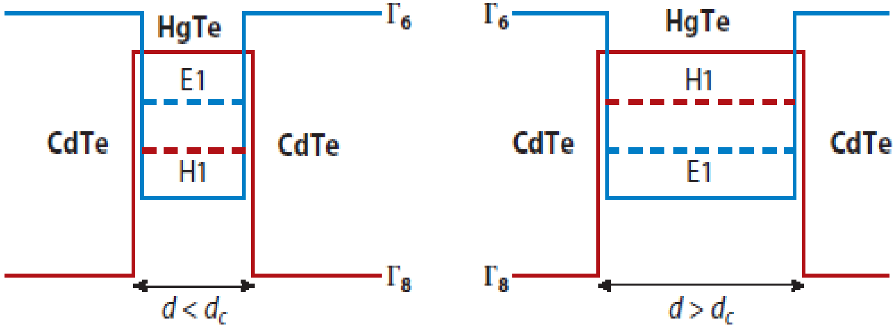
“Γ_6“: s-type (s orbitals) S=1/2

“Γ_8“: p-type (p orbitals) J=3/2
(“light and heavy holes”)

HgTe: “zero gap” semiconductor.

HgTe quantum wells





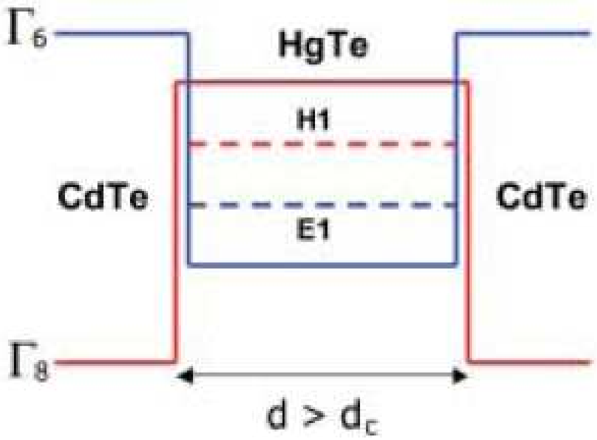
Gap: $M \equiv E_{E1} - E_{H1}$

$d < d_c \Rightarrow M > 0 \qquad d > d_c \Rightarrow M < 0$

d_c = 6.3 nm

Effective model for HgTe QWs (BHZ).

Bernevig, Hughes, Zhang, *Science* 314, 1757 (2006)



Basis: $\left\{ \begin{array}{l} |E+\rangle \\ |H+\rangle \\ |E-\rangle \\ |H-\rangle \end{array} \right.$

Basis functions: $\left\{ \begin{array}{l} \Psi_{\mathbf{k}}^{E+}(\mathbf{r}) \\ \Psi_{\mathbf{k}}^{H+}(\mathbf{r}) \\ \Psi_{\mathbf{k}}^{E-}(\mathbf{r}) \\ \Psi_{\mathbf{k}}^{H-}(\mathbf{r}) \end{array} \right.$

Hamiltonian (low energy from **k.p** theory):

$$\mathcal{H}(\mathbf{k}) = \begin{pmatrix} h_+(\mathbf{k}) & 0 \\ 0 & h_+^*(-\mathbf{k}) \end{pmatrix}$$

$$h_+(k_x, k_y) = \begin{pmatrix} \epsilon(k) + \mathcal{M}(k) & Ak_- \\ Ak_+ & \epsilon(k) - \mathcal{M}(k) \end{pmatrix}$$

$$\left\{ \begin{array}{rcl} \epsilon(k) & = & C - Dk^2 \\ \mathcal{M}(k) & = & M - Bk^2 \\ k_\pm & = & k_x \pm ik_y \end{array} \right.$$

d (Å)	$A(eV)$	$B(eV)$	$C(eV)$	$D(eV)$	$M(eV)$	
58	-3.62	-18.0	-0.0180	-0.594	0.00922	**d<d_c**
70	-3.42	-16.9	-0.0263	0.514	-0.00686	**d>d_c**

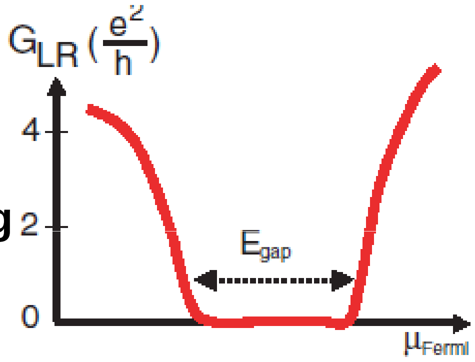
Table 1: Parameters for $Hg_{0.32}Cd_{0.68}Te$/HgTe quantum wells.

Quantum Spin Hall effect in HgTe QWs.

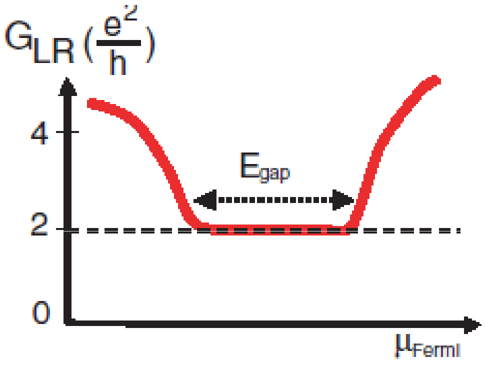
Shoucheng Zhang

Gap: $|E_{E1} - E_{H1}| \equiv |M|$

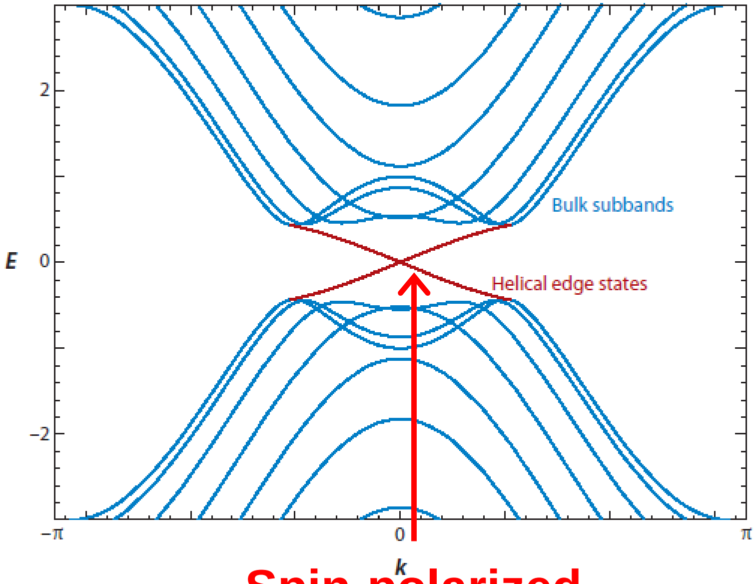
Chern number $n_s = \operatorname{sgn}(M)$



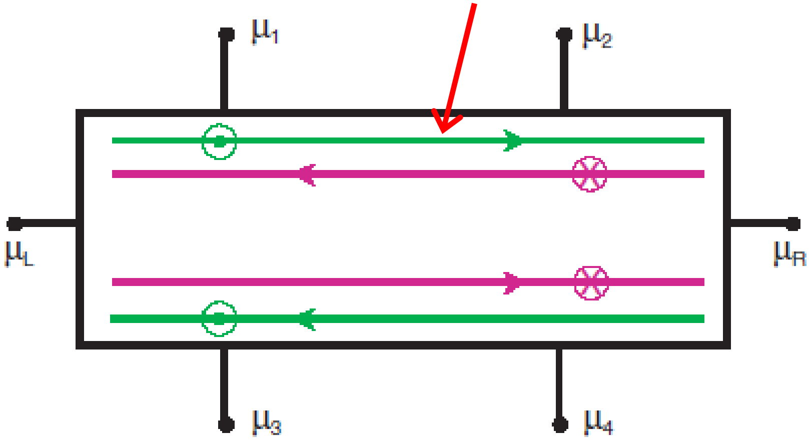
d < d$_c$ normal regime



d > d$_c$ inverted regin



Spin-polarized Edge states



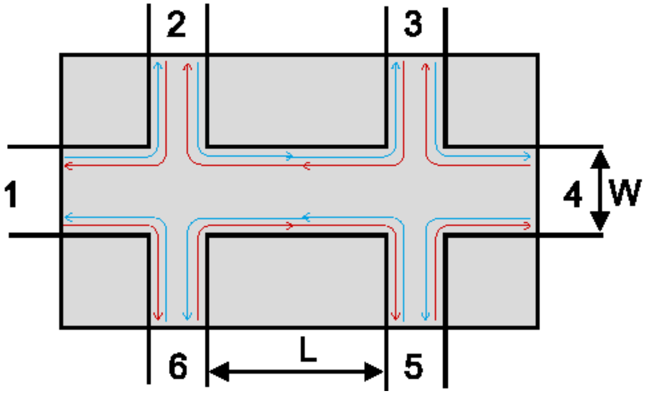
Andrei Bernevig

$$\boxed{\Delta\sigma_{xy} = 2\frac{e^2}{h}}$$

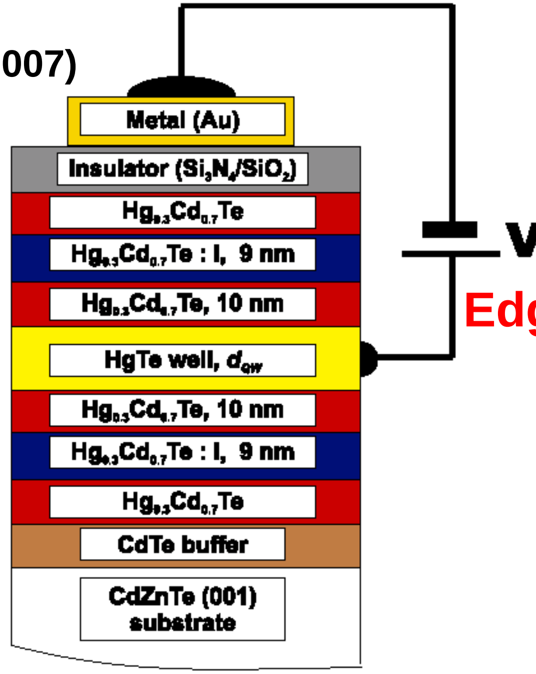
Bernevig, Hughes, Zhang, *Science* 314, 1757 (2006)

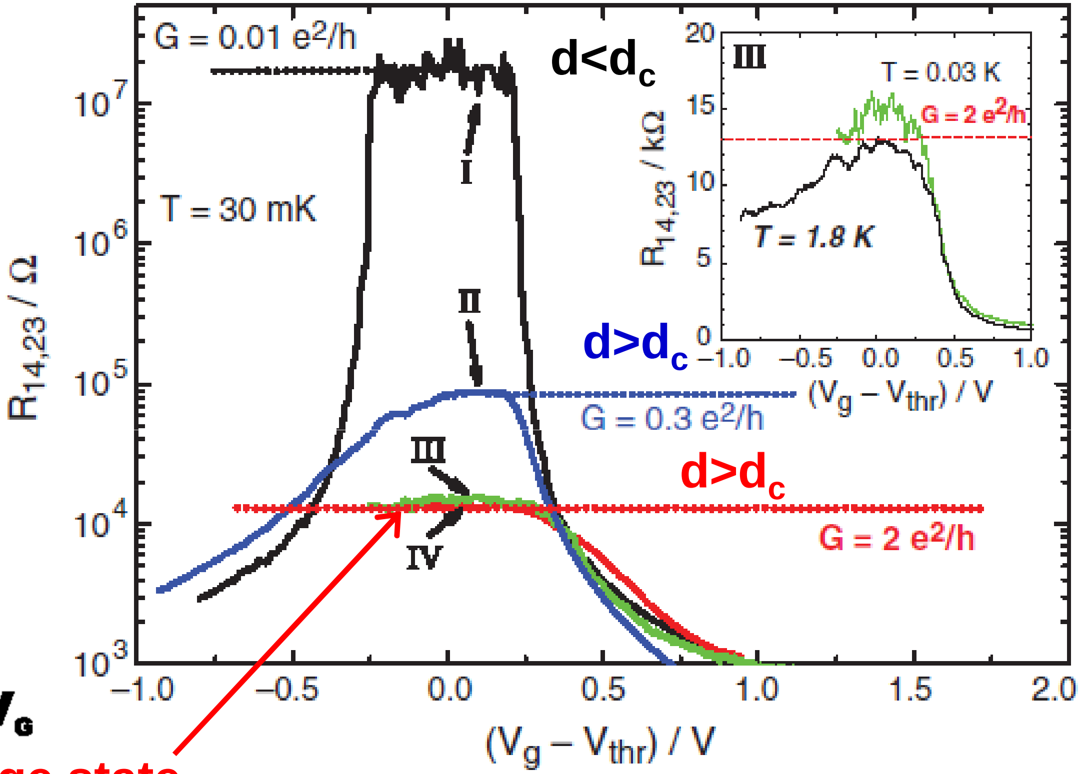
QSH effect in HgTe QWs: Experiment

Laurens Molenkamp



Konig et al, *Science* 318, 766 (2007)





Edge state condutance

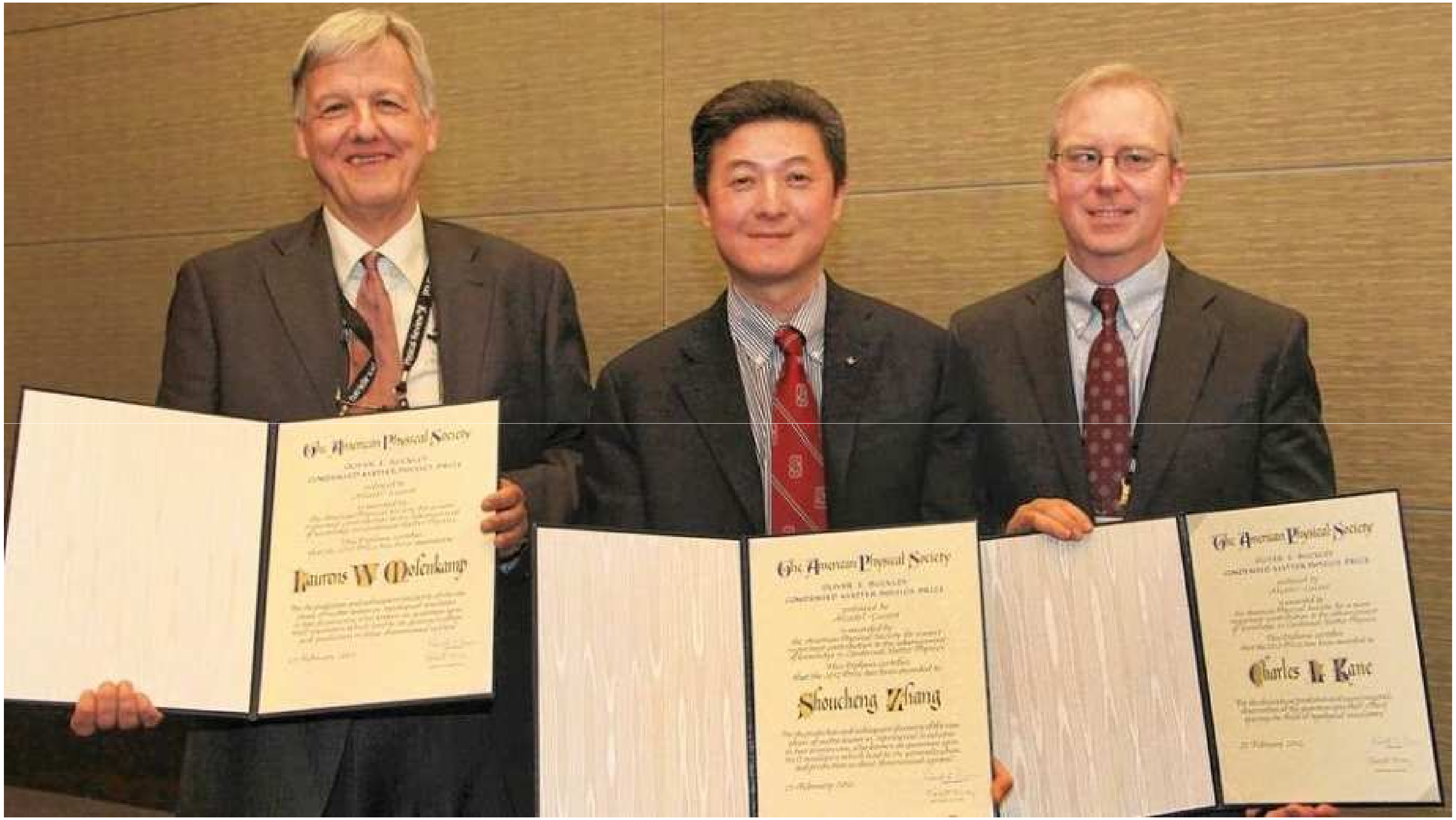
I-L=20 μm W=13 μm ($d< d_c$)

II-L=20 μm W=13 μm ($d>d_c$)

III-L=1 μm W=1 μm ($d>d_c$)

IV-L=1 μm W=0.5 μm ($d>d_c$)

A future Nobel Prize?



Physics Frontiers Prize 2013

also: APS Buckley Prize 2012

Curso de magnetismo: prof. Bilap Sanyal

Prof. Biplab Sanyal, da Universidade de Uppsala

28/11/2018, Sala 206 da Ala Central, entre as 10h e as 12h.

MINICURSO

MAGNETISM: FUNDAMENTALS TO APPLICATIONS (PGF5334)

Profs. Marcio Teixeira do Nascimento Varella e Biplab Sanyal (Uppsala - Suécia)

Datas e horários

De 23 a 29/11/2018, das 10 às 12h

Nos dias 22 e 30/11, das 10h às 12h e das 14h às 16h

Locais

Auditório Novo 2

Salas 206; 208/209 e 212 da Ala Central; Sala 210 da Ala 2

Matrículas

Até dia 21/11/2018, junto à Secretaria de Pós-Graduação

Link para mini-curso:

http://portal.if.usp.br/pg/pt-br/not%C3%ADcia/minicurso-2