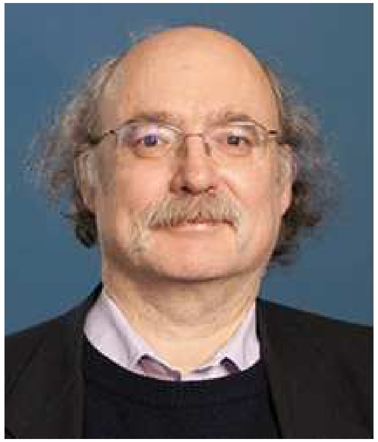
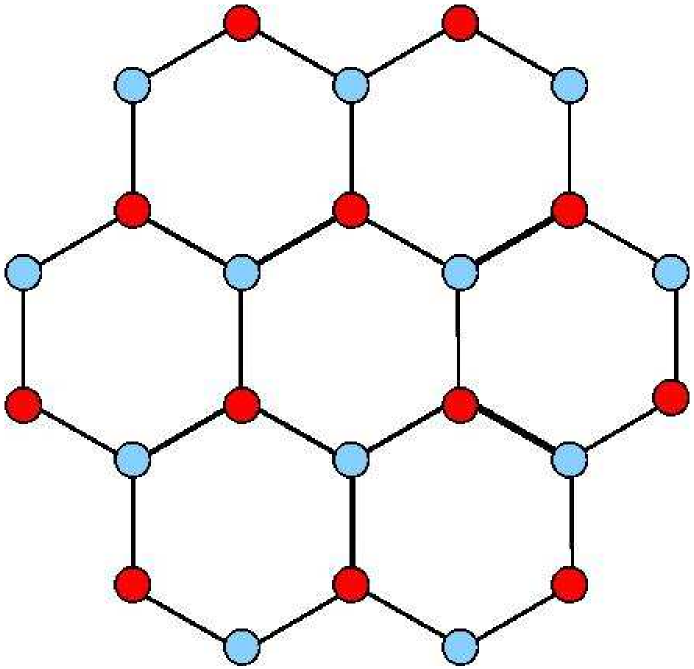
The road to the quantum spin Hall effect.

Haldane: Hall conductance with zero flux.



Duncan Haldane

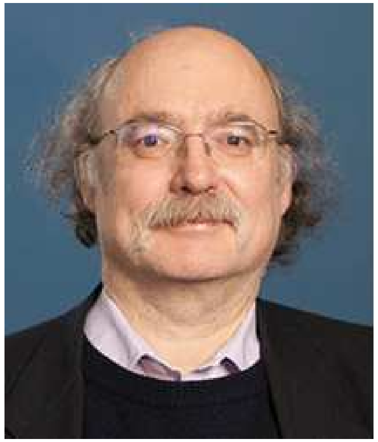
2016



Spinless fermions in a Graphene-like lattice model (two triangular sublattices)

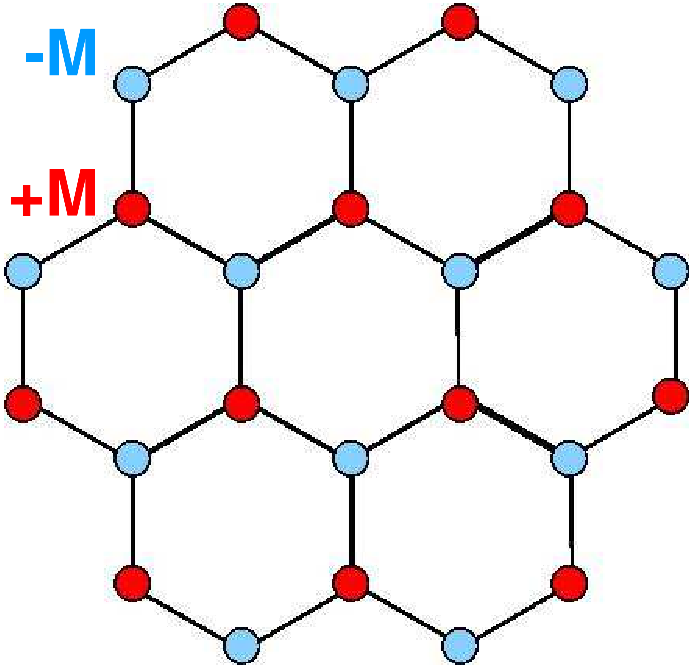
F.D.M. Haldane,
***Phys. Rev. Lett*. 61, 2015 (1988)**

Haldane: Hall conductance with zero flux.



Duncan Haldane

2016

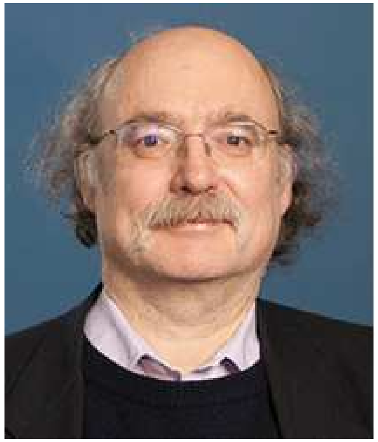


Spinless fermions in a Graphene-like lattice model (two triangular sublattices)

Each sublatice has a different "mass term": Inversion symmetry breaking.

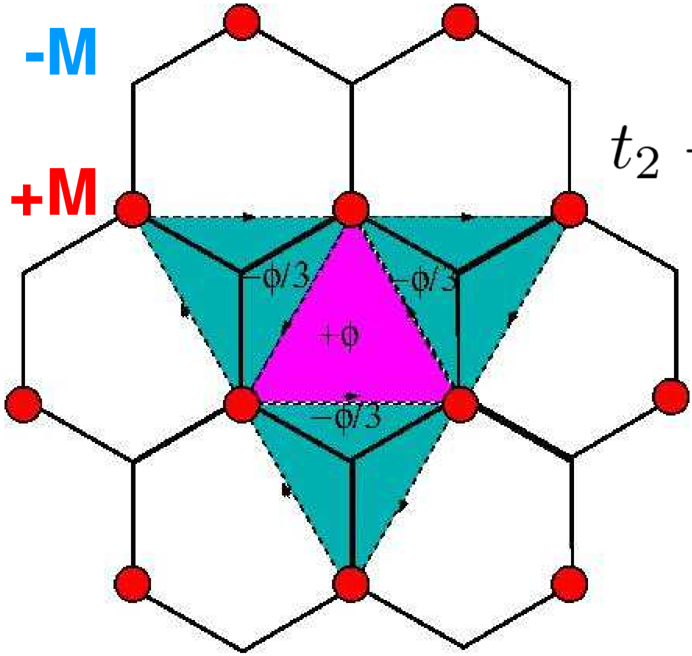
F.D.M. Haldane,
***Phys. Rev. Lett*. 61, 2015 (1988)**

Haldane: Hall conductance with zero flux.



Duncan Haldane

2016



$$t_2 \to t_2 e^{\pm\phi}$$

Spinless fermions in a Graphene-like lattice model (two triangular sublattices)

Each sublatice has a different "mass term": Inversion symmetry breaking.

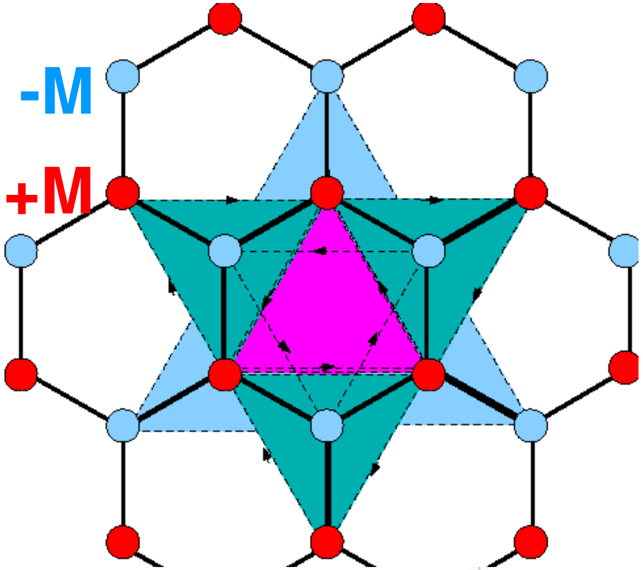
Space-varying B(r) with ZERO NET FLUX: Time reversal symmetry breaking.

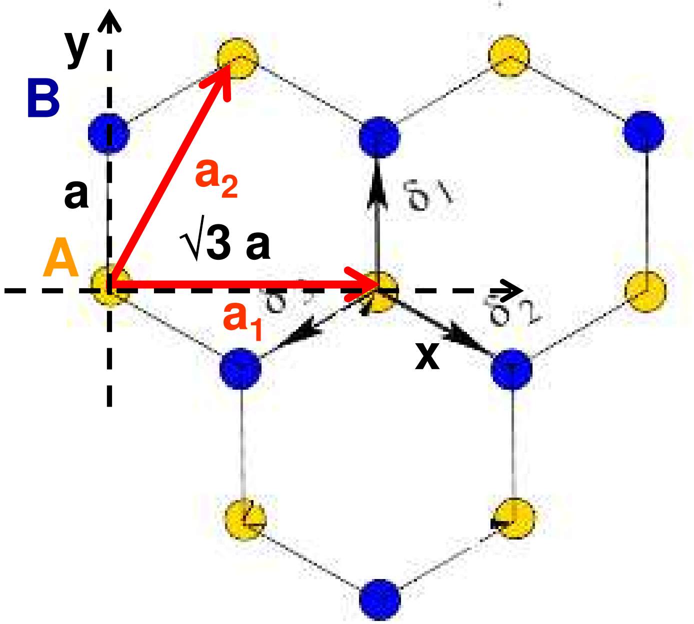
F.D.M. Haldane,
***Phys. Rev. Lett*. 61, 2015 (1988)**

$$\hat{H}_{\mathrm{Haldane}} = -t_1 \sum_{\langle \mathrm{i,j} \rangle} c_{\mathrm{i}}^{\dagger} c_{\mathrm{j}} - t_2 \sum_{\langle\langle \mathrm{i,j} \rangle\rangle} e^{i\phi_{ij}} c_{\mathrm{i}}^{\dagger} c_{\mathrm{j}} + M \sum_{\mathrm{i}} \varepsilon_i c_{\mathrm{i}}^{\dagger} c_{\mathrm{i}}$$

https://topocondmat.org/w4_haldane/haldane_model.html

Haldane model: eigenvalues



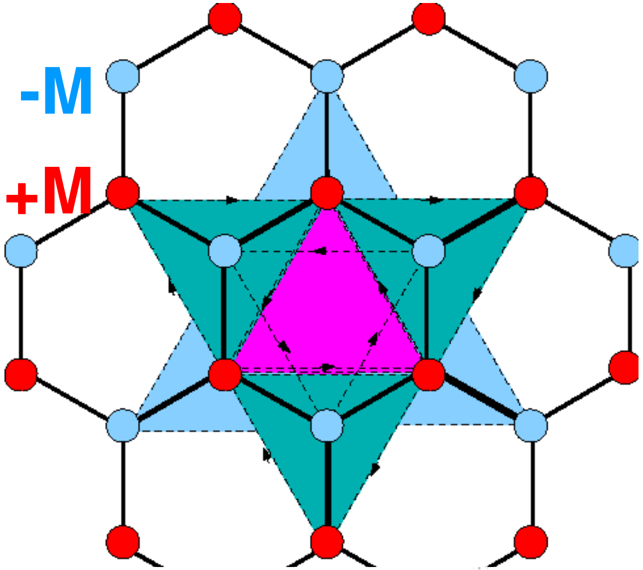


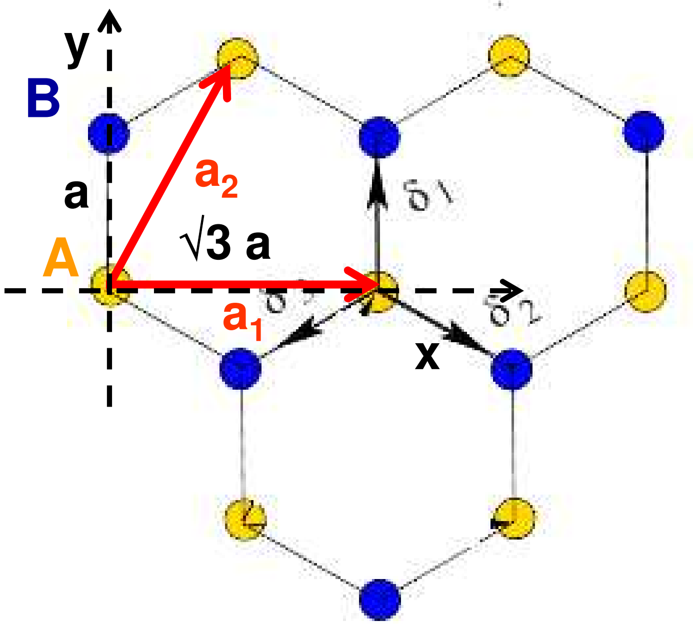
$$\frac{\mathcal{H}_{\mathbf{q}}}{N} = \begin{pmatrix} M + 2t_2 f(\mathbf{q}, \phi) & t_1 \gamma_{\mathbf{q}} \\ t_1 \gamma^*_{\mathbf{q}} & -M + 2t_2 f(\mathbf{q}, -\phi) \end{pmatrix}$$

$$\gamma_{\mathbf{q}} = 1 + e^{i\mathbf{q}\cdot\mathbf{a}_2} + e^{i\mathbf{q}\cdot(\mathbf{a}_2 - \mathbf{a}_1)}$$

$$\begin{aligned} f(\mathbf{q}, \phi) \quad = \quad & \cos\left(\mathbf{q}\cdot\mathbf{a}_1 + \phi\right) + \cos\left(\mathbf{q}\cdot\mathbf{a}_2 - \phi\right) + \\ & \cos\left(\mathbf{q}\cdot(\mathbf{a}_2 - \mathbf{a}_1) + \phi\right) \end{aligned}$$

Tarefa 21: Haldane model





$$\frac{\mathcal{H}_{\mathbf{q}}}{N} = \begin{pmatrix} M + 2t_2 f(\mathbf{q},\phi) & t_1\gamma_{\mathbf{q}} \\ t_1\gamma^*_{\mathbf{q}} & -M + 2t_2 f(\mathbf{q},-\phi) \end{pmatrix}$$

$$\gamma_{\mathbf{q}} = 1 + e^{i\mathbf{q}\cdot\mathbf{a}_2} + e^{i\mathbf{q}\cdot(\mathbf{a}_2-\mathbf{a}_1)}$$

$$f(\mathbf{q},\phi) = \cos(\mathbf{q}\cdot\mathbf{a}_1+\phi) + \cos(\mathbf{q}\cdot\mathbf{a}_2-\phi) + \cos(\mathbf{q}\cdot(\mathbf{a}_2-\mathbf{a}_1)+\phi)$$

Consider: t_1=1 , $\phi=\pi/2$, and $\mathbf{a}_1$ and $\mathbf{a}_2$ as in the left.

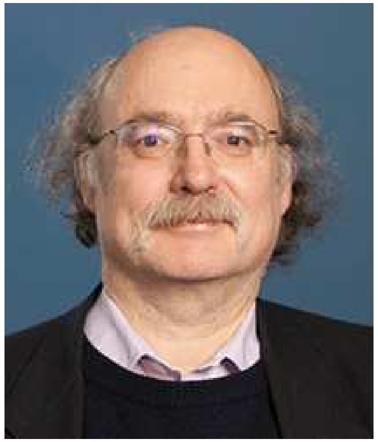
1) Calculate the Hamiltonian matrix for the Brillouin zone vertices **q**=K and **q**=K'. (remember Lista 03!)

2) Show that the gap *vanishes* for

$$t_2 = \pm M/(3\sqrt{3})$$

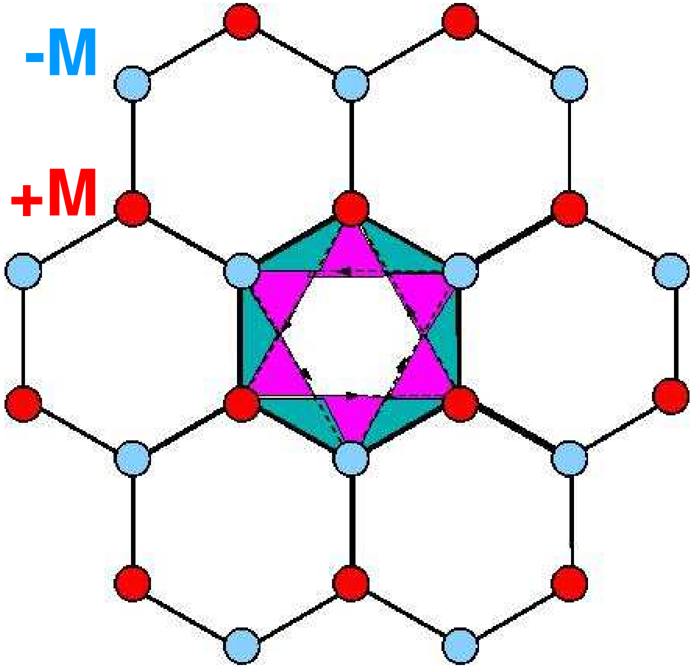
but not in K and K' at the same time!

Haldane: Hall conductance with zero flux.

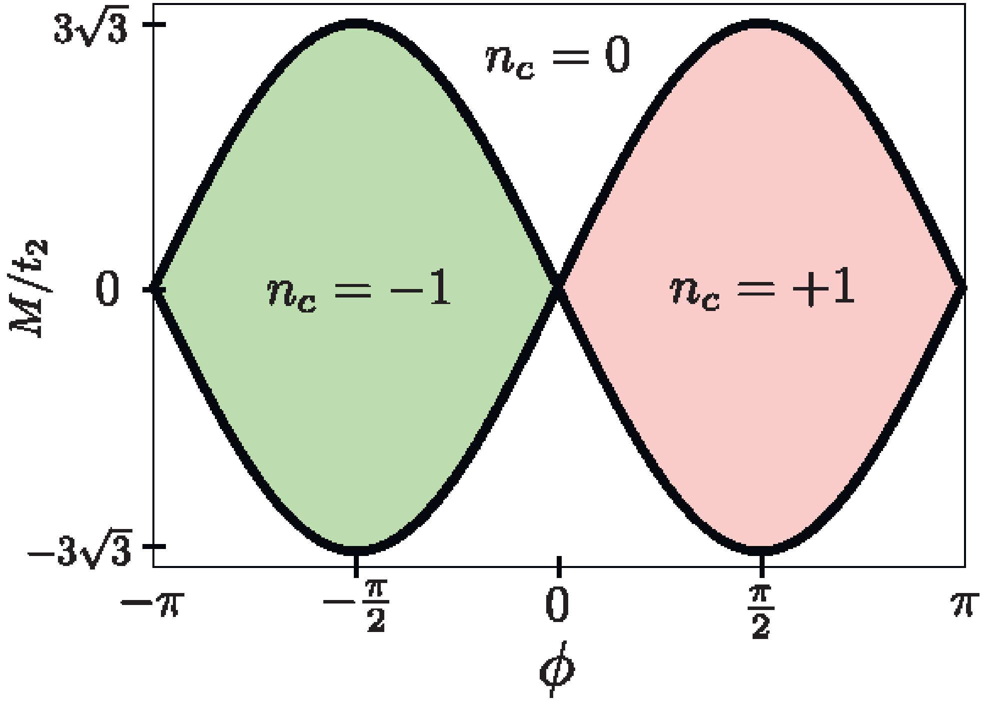


Duncan Haldane

2016



Gap: $\bar{M}_{\pm} = \pm M - 3\sqrt{3} t_2 \sin\phi$



F.D.M. Haldane,
***Phys. Rev. Lett.* 61, 2015 (1988)**

Hall conductance also given by a Chern number:

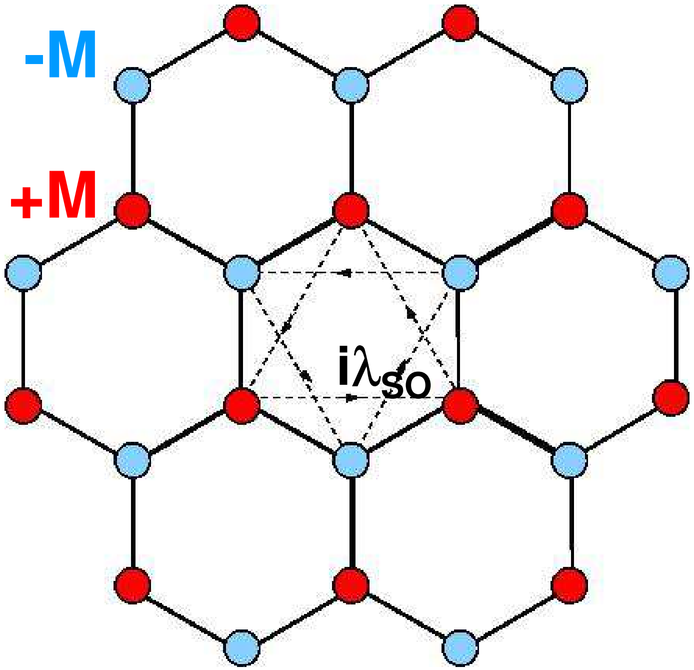
$$n_c = \frac{1}{2}\left[\mathrm{sgn}\left(\bar{M}_{+}\right) + \mathrm{sgn}\left(\bar{M}_{-}\right)\right]$$

$$\sigma_{xy} = n_c \frac{e^2}{h}$$

$n_c = \pm 1$: Topological phases

Kane and Mele: Quantum Spin Hall effect.

Charles Kane



Spinful fermions in a Graphene-like lattice model: 4-band model.

Inversion symmetry breaking (not really needed.

Spin orbit term connecting sites in the same sublattice!

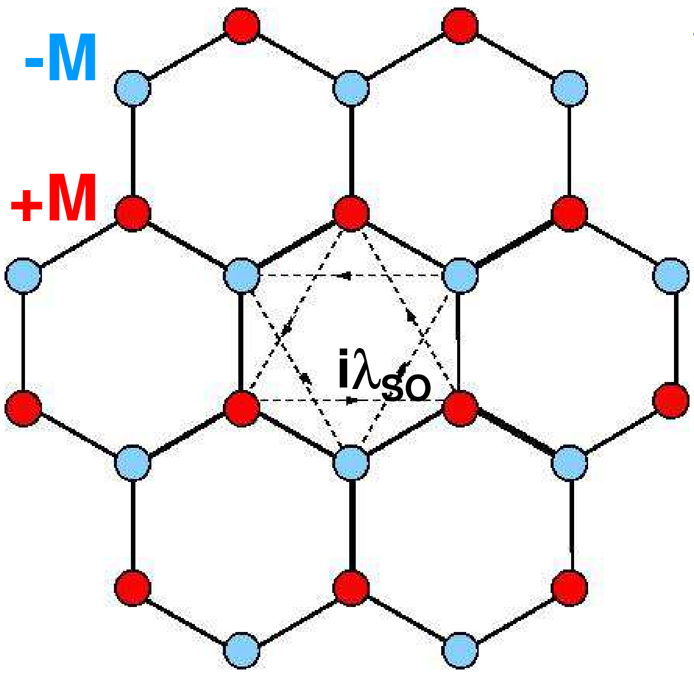
Hamiltonian obeys time-reversal symmetry.

A Rashba spin orbit coupling term can be added (results are qualitatively the same!)

C. L. Kane, E. J. Mele
Phys. Rev. Lett. 95, 146802 (2005)
Phys. Rev. Lett. 95, 226801 (2005).

https://topocondmat.org/w5_qshe/fermion_parity_pump.html

Kane and Mele model (no Rashba SOC)



Spin ↑ : essentially the Haldane model with $\phi=\pi/2$

$$\frac{\mathcal{H}^{\uparrow}_{\mathbf{q}}}{N} = \begin{pmatrix} M + 2\lambda_{SO} f(\mathbf{q}, \frac{\pi}{2}) & t_1\gamma_{\mathbf{q}} \\ t_1\gamma^*_{\mathbf{q}} & -M + 2\lambda_{SO} f(\mathbf{q}, -\frac{\pi}{2}) \end{pmatrix}$$

$$\gamma_{\mathbf{q}} = 1 + e^{i\mathbf{q}\cdot\mathbf{a}_2} + e^{i\mathbf{q}\cdot(\mathbf{a}_2-\mathbf{a}_1)}$$

$$f(\mathbf{q},\phi) = \cos(\mathbf{q}\cdot\mathbf{a}_1 + \phi) + \cos(\mathbf{q}\cdot\mathbf{a}_2 - \phi) + \cos(\mathbf{q}\cdot(\mathbf{a}_2 - \mathbf{a}_1) + \phi)$$

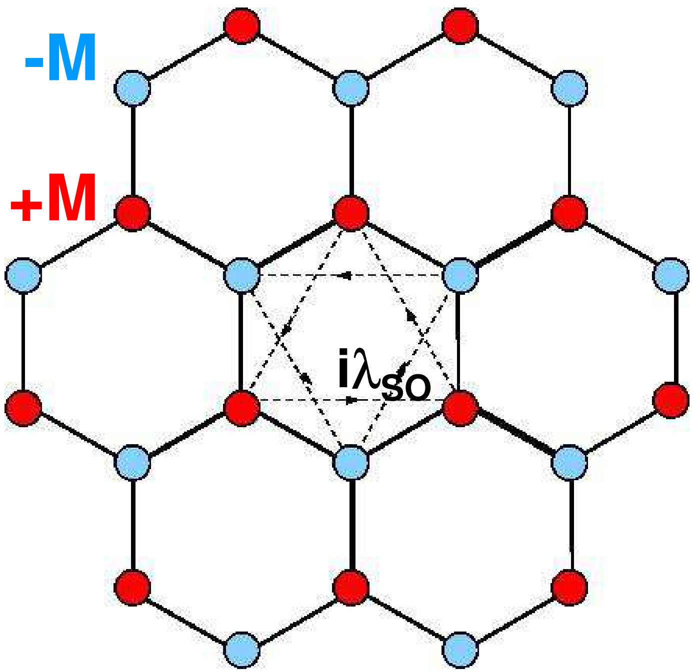
Time-reversal symmetry: $\mathcal{H}^{\downarrow}_{\mathbf{q}} = \left(\mathcal{H}^{\uparrow}_{-\mathbf{q}}\right)^*$

$$\begin{cases} (\gamma_{-\mathbf{q}})^* = \gamma_{\mathbf{q}} \\ f(-\mathbf{q},\phi) = f(\mathbf{q},-\phi) \end{cases}$$

Spin ↓ : Haldane model with $\phi=-\pi/2$

$$\frac{\mathcal{H}^{\downarrow}_{\mathbf{q}}}{N} = \begin{pmatrix} M + 2\lambda_{SO} f(\mathbf{q}, -\frac{\pi}{2}) & t_1\gamma_{\mathbf{q}} \\ t_1\gamma^*_{\mathbf{q}} & -M + 2\lambda_{SO} f(\mathbf{q}, \frac{\pi}{2}) \end{pmatrix}$$

Kane and Mele: Quantum Spin Hall effect.



Gap: $|6\sqrt{3}\lambda_{SO} - 2M|$

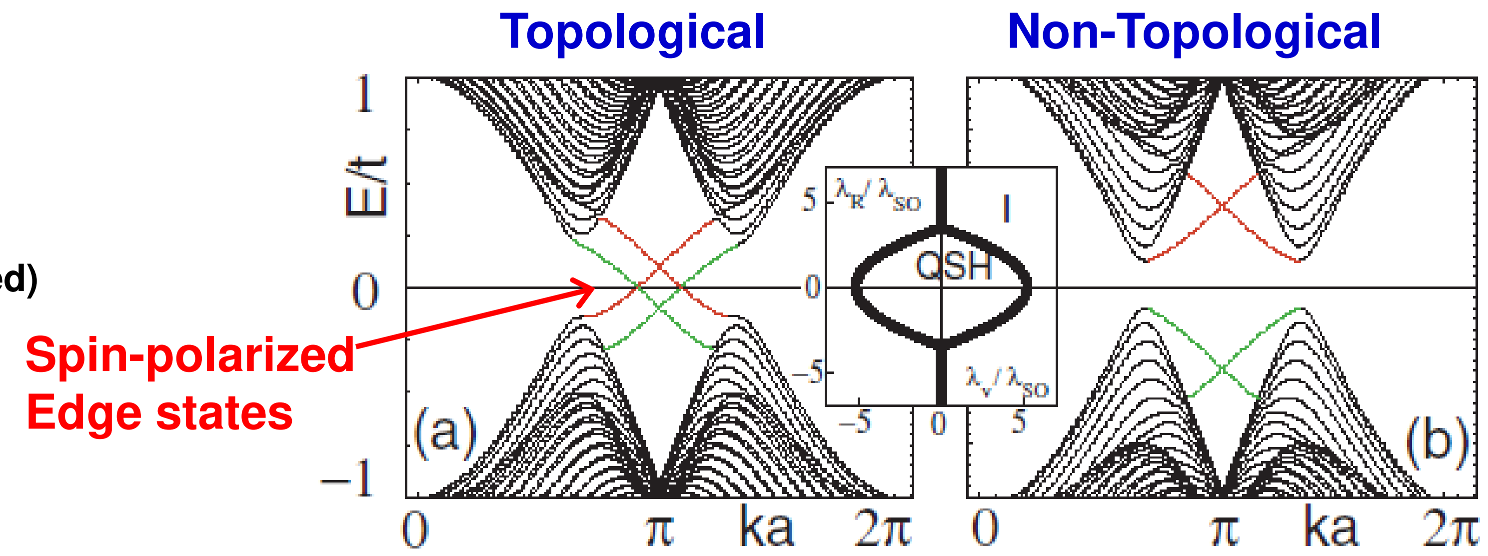
Topological phase : $M < 3\sqrt{3}\lambda_{SO}$

Chern number
(in the top. phase) $n_s = \mathrm{sgn}\,(s\lambda_{SO})$

Z_2 invariant $\nu = \frac{1}{2}(n_\uparrow - n_\downarrow) = \pm 1$

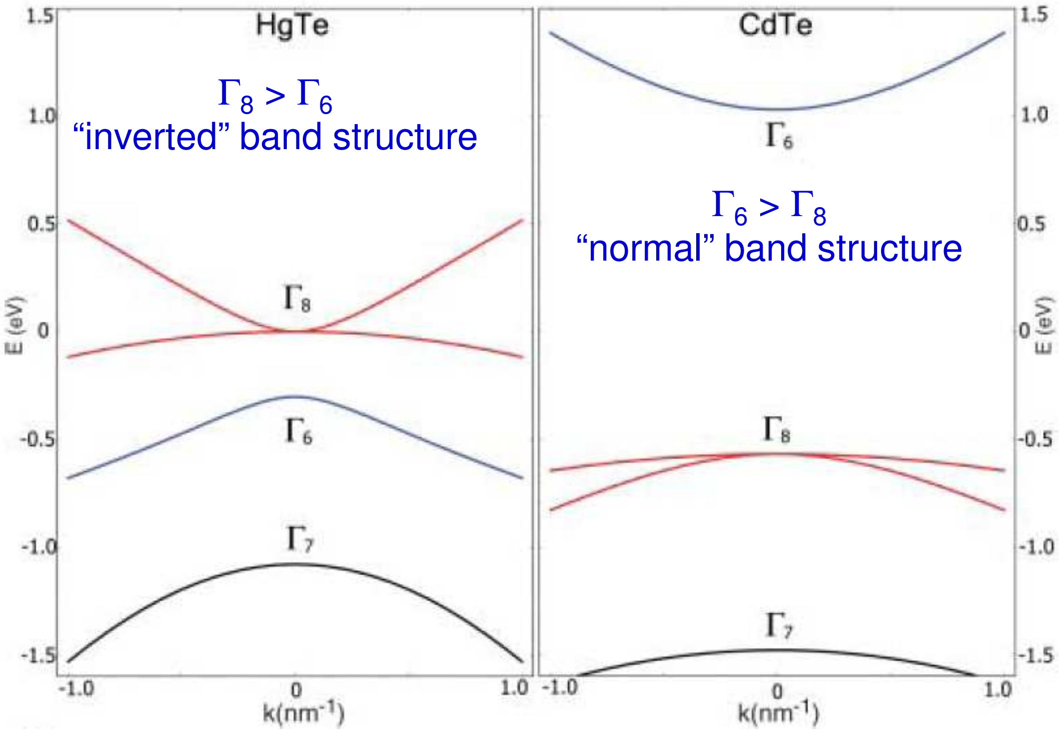
New ingredients:
- **Particles with spin s.**
- **Spin-Orbit coupling λ_{SO} (TRS preserved)**
- **Assuming no Rashba SO.**

C. L. Kane, E. J. Mele
Phys. Rev. Lett. 95, 146802 (2005)
Phys. Rev. Lett. 95, 226801 (2005).



https://topocondmat.org/w5_qshe/fermion_parity_pump.html

HgTe Quantum Wells: “inverted” bands

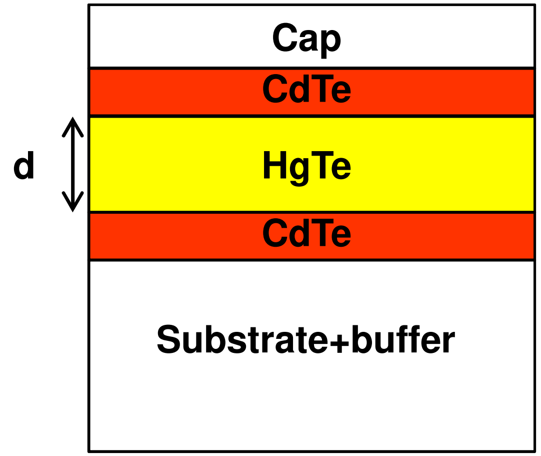


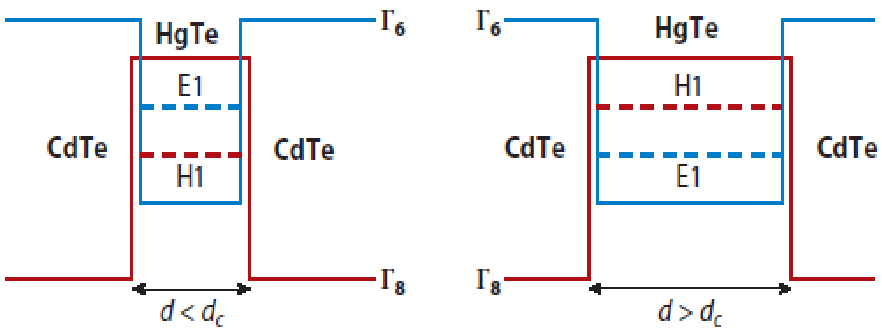
“Γ_6“: s-type (s orbitals) S=1/2

“Γ_8“: p-type (p orbitals) J=3/2
(“light and heavy holes”)

HgTe: “zero gap” semiconductor.

HgTe quantum wells





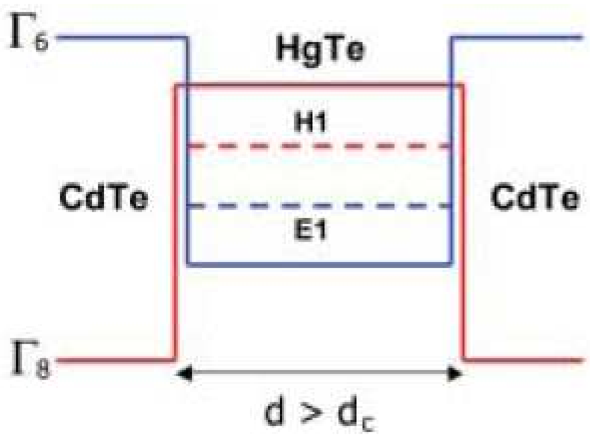
Gap: $M \equiv E_{E1} - E_{H1}$

$d < d_c \Rightarrow M > 0 \qquad d > d_c \Rightarrow M < 0$

d_c = 6.3 nm

Effective model for HgTe QWs (BHZ).

Bernevig, Hughes, Zhang, *Science* 314, 1757 (2006)



Basis: $\left\{ \begin{array}{l} |E+\rangle \\ |H+\rangle \\ |E-\rangle \\ |H-\rangle \end{array} \right.$

Basis functions: $\left\{ \begin{array}{l} \Psi_{\mathbf{k}}^{E+}(\mathbf{r}) \\ \Psi_{\mathbf{k}}^{H+}(\mathbf{r}) \\ \Psi_{\mathbf{k}}^{E-}(\mathbf{r}) \\ \Psi_{\mathbf{k}}^{H-}(\mathbf{r}) \end{array} \right.$

Hamiltonian (low energy from **k.p** theory):

$$\mathcal{H}(\mathbf{k}) = \begin{pmatrix} h_+(\mathbf{k}) & 0 \\ 0 & h_+^*(-\mathbf{k}) \end{pmatrix}$$

$$h_+(k_x, k_y) = \begin{pmatrix} \epsilon(k) + \mathcal{M}(k) & Ak_- \\ Ak_+ & \epsilon(k) - \mathcal{M}(k) \end{pmatrix}$$

$$\left\{ \begin{array}{rcl} \epsilon(k) & = & C - Dk^2 \\ \mathcal{M}(k) & = & M - Bk^2 \\ k_\pm & = & k_x \pm ik_y \end{array} \right.$$

d (Å)	$A(eV)$	$B(eV)$	$C(eV)$	$D(eV)$	$M(eV)$	
58	-3.62	-18.0	-0.0180	-0.594	0.00922	**$d<d_c$**
70	-3.42	-16.9	-0.0263	0.514	-0.00686	**$d>d_c$**

Table 1: Parameters for $Hg_{0.32}Cd_{0.68}Te$/HgTe quantum wells.

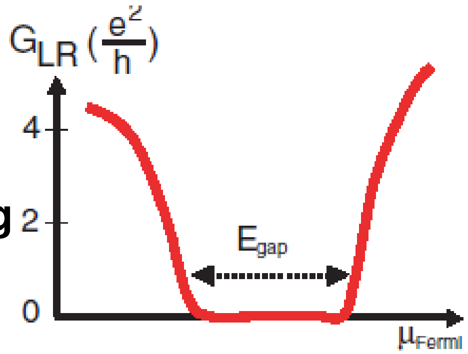
Quantum Spin Hall effect in HgTe QWs.

Shoucheng Zhang

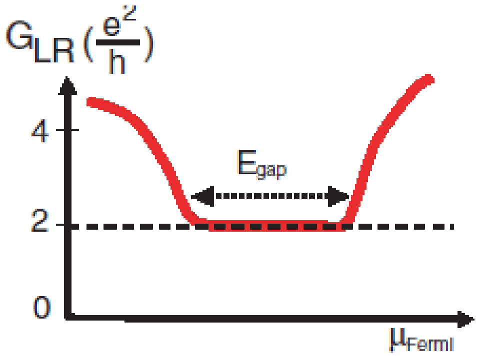
Andrei Bernevig

Gap: $|E_{E1} - E_{H1}| \equiv |M|$

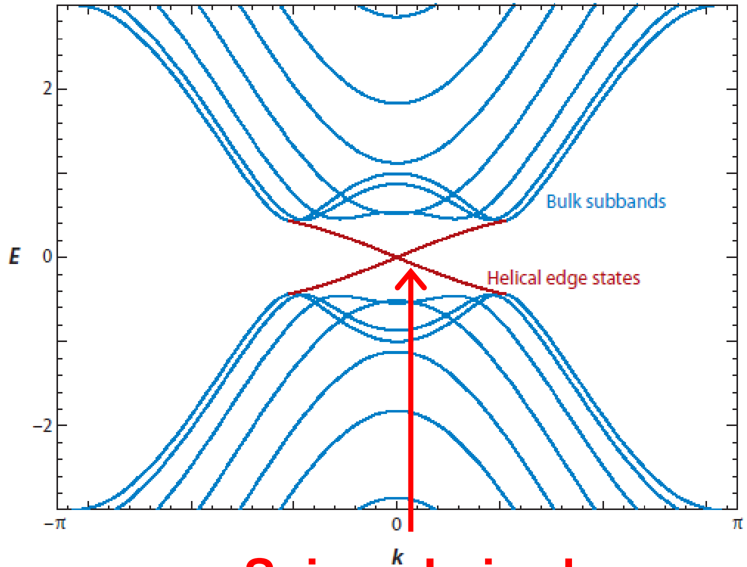
Chern number $n_s = \mathrm{sgn}\,(M)$



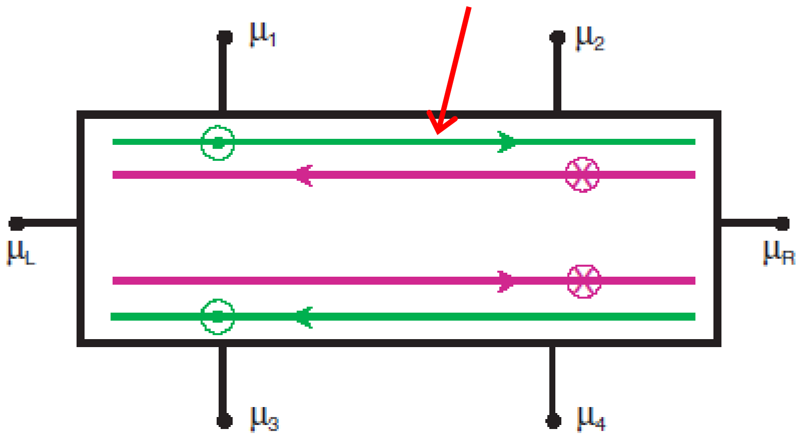
$d < d_c$ normal regime



$d > d_c$ inverted regin



Spin-polarized Edge states

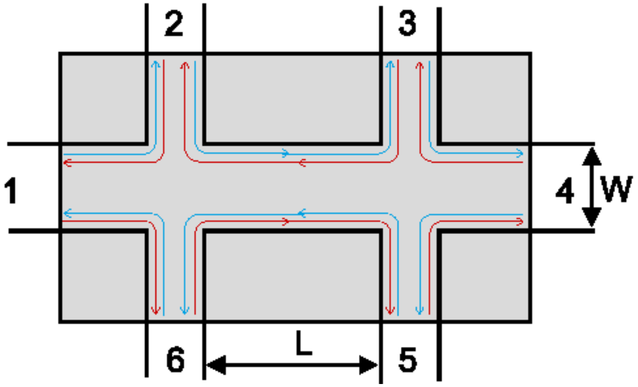


$$\Delta\sigma_{xy} = 2\frac{e^2}{h}$$

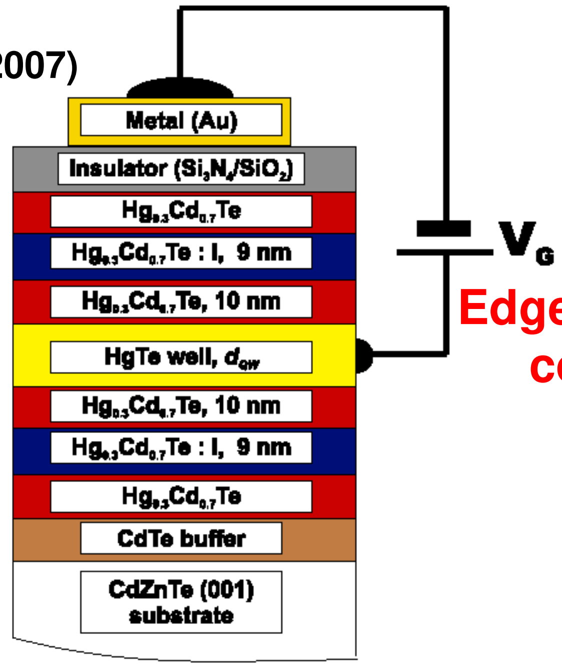
Bernevig, Hughes, Zhang, *Science* 314, 1757 (2006)

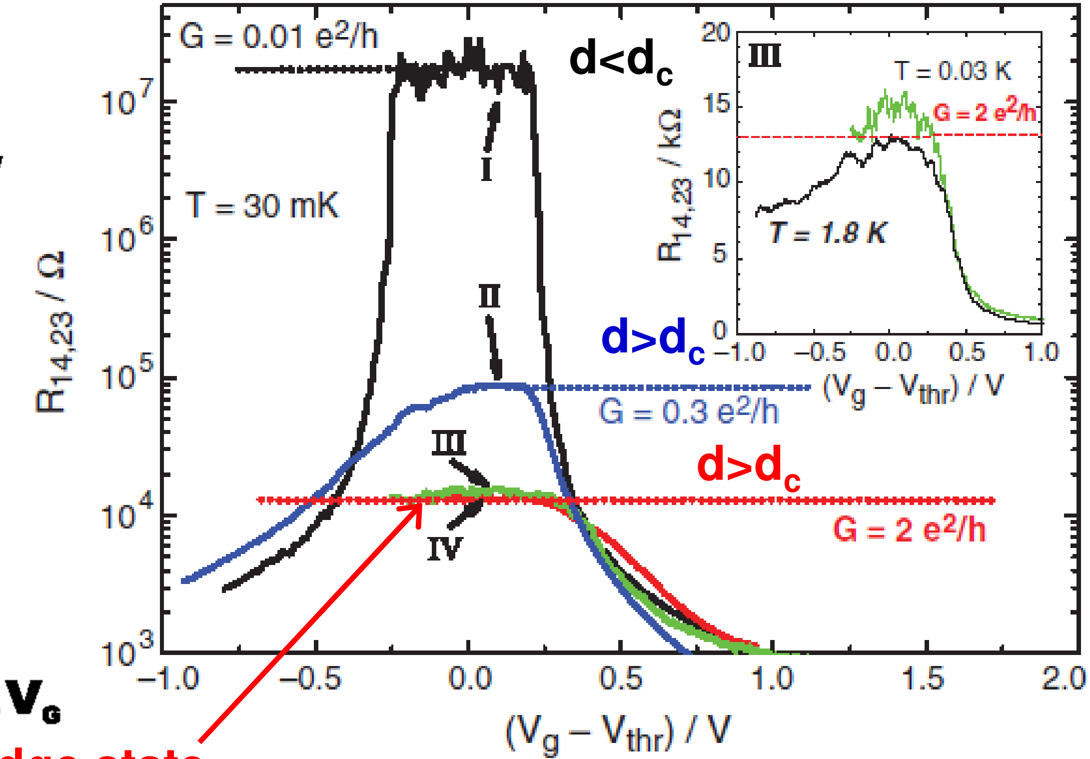
QSH effect in HgTe QWs: Experiment

Laurens Molenkamp



Konig et al, *Science* 318, 766 (2007)





Edge state condutance

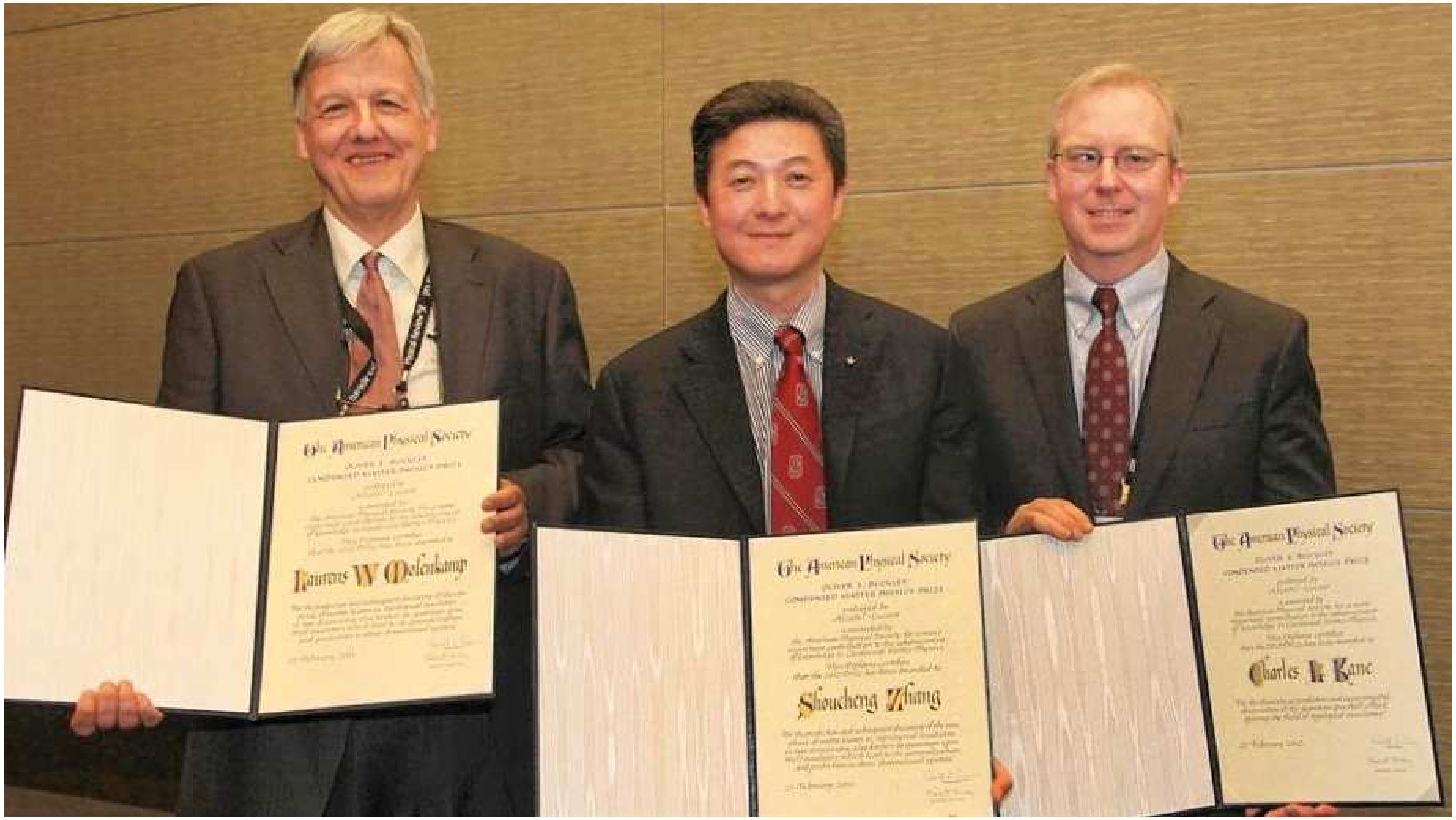
I-L=20 μm W=13 μm ($d< d_c$)

II-L=20 μm W=13 μm ($d>d_c$)

III-L=1 μm W=1 μm ($d>d_c$)

IV-L=1 μm W=0.5 μm ($d>d_c$)

A future Nobel Prize?



Physics Frontiers Prize 2013

also: APS Buckley Prize 2012

Tarefa 22: BHZ model

$$\mathcal{H}(k_x, k_y) = \begin{pmatrix} \epsilon(k) + \mathcal{M}(k) & Ak_- & 0 & 0 \\ Ak_+ & \epsilon(k) - \mathcal{M}(k) & 0 & 0 \\ 0 & 0 & \epsilon(k) + \mathcal{M}(k) & -Ak_+ \\ 0 & 0 & -Ak_- & \epsilon(k) - \mathcal{M}(k) \end{pmatrix}$$

$$\begin{cases} \epsilon(k) & = & C - Dk^2 \\ \mathcal{M}(k) & = & M - Bk^2 \\ k_\pm & = & k_x \pm ik_y \end{cases}$$

d (Å)	$A(eV)$	$B(eV)$	$C(eV)$	$D(eV)$	$M(eV)$	
58	-3.62	-18.0	-0.0180	-0.594	0.00922	**$d<d_c$**
70	-3.42	-16.9	-0.0263	0.514	-0.00686	**$d>d_c$**

Table 1: Parameters for $Hg_{0.32}Cd_{0.68}Te/HgTe$ quantum wells.

1) Show that 2 of the 4 bands are always degenerate independently of the parameters.

2) Calculate the value of M such that the energy gap *vanishes* at **k**=(0,0).