Tarefa 20: identity for the velocity

Using:

$$\mathbf{v}(\mathbf{k}) = \frac{1}{\hbar}\nabla_{\mathbf{k}} H(\mathbf{k}) = \frac{1}{\hbar}\left(\frac{\partial H}{\partial k_x}\mathbf{i} + \frac{\partial H}{\partial k_y}\mathbf{j} + \frac{\partial H}{\partial k_z}\mathbf{k}\right)$$

$$H|n, \mathbf{k}(t)\rangle = E_n[\mathbf{k}(t)]|n, \mathbf{k}(t)\rangle$$

$$i\hbar\frac{d}{dt}|n, \mathbf{k}(t)\rangle = i\hbar\frac{d\mathbf{k}(t)}{dt}\cdot\nabla_{\mathbf{k}}|n, \mathbf{k}(t)\rangle$$

$$H|\Psi(t)\rangle = i\hbar\frac{d}{dt}|\Psi(t)\rangle$$

Show that:

1) $$\nabla_{\mathbf{k}}(H|n, \mathbf{k}\rangle) = (\nabla_{\mathbf{k}} H)|n\mathbf{k}(t)\rangle + H(\nabla_{\mathbf{k}}|n, \mathbf{k}\rangle)$$

2) $$H(\nabla_{\mathbf{k}}|n, \mathbf{k}\rangle) = i\hbar\nabla_{\mathbf{k}}\left(\frac{d\mathbf{k}(t)}{dt}\cdot\nabla_{\mathbf{k}}|n, \mathbf{k}\rangle\right)$$

3) $$\hbar\mathbf{v}|n\mathbf{k}(t)\rangle = \nabla_{\mathbf{k}}(E_n[\mathbf{k}]|n, \mathbf{k}\rangle) - i\hbar\nabla_{\mathbf{k}}\left(\frac{d\mathbf{k}(t)}{dt}\cdot\nabla_{\mathbf{k}}|n, \mathbf{k}\rangle\right)$$

Tip: Do it by components so you don’t get confused!!