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# PG5295 – Muitos Corpos

1 – Electronic Transport in  
Quantum dots

2 – Kondo effect: Intro/theory.

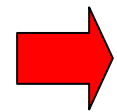
3 – Kondo effect in nanostructures

Prof. Luis Gregório Dias – DFMT

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# PG5295 – Muitos Corpos



1 – Electronic Transport in  
Quantum dots

2 – Kondo effect: Intro/theory.

3 – Kondo effect in nanostructures

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# Lecture 1: Outline

- Introduction: From atoms to “artificial atoms”.
  - What are Quantum Dots?
  - Confinement regimes.
  - Transport in QDs: General aspects.
  - Transport in QDs: Coulomb blockade regime.
  - Transport in QDs: Peak Spacing.
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# “More is Different”



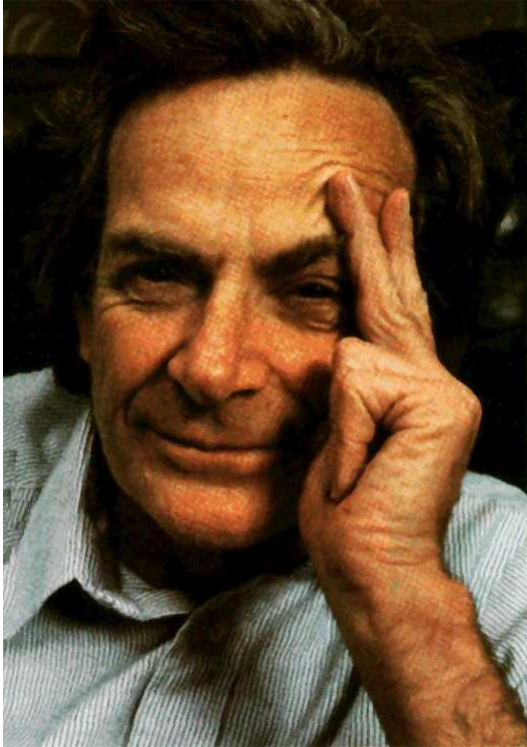
“ The behavior of large and complex aggregates of elementary particles, it turns out, is not to be understood in terms of simple extrapolation of the properties of a few particles.

Instead, at each level of complexity entirely new properties appear and the understanding of the new behaviors requires research which I think is as fundamental in its nature as any other.“

Phillip W. Anderson, “More is Different”,  
*Science* **177** 393 (1972)

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# “More is Different?”



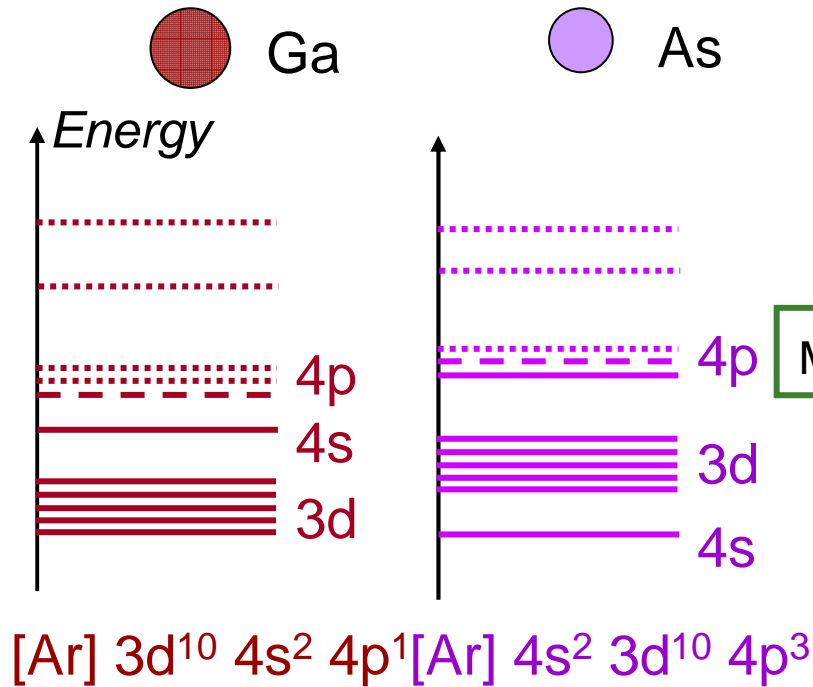
“If, in some cataclysm, all of scientific knowledge were to be destroyed, **and only one sentence passed on to the next generation** of creatures, what statement would contain the most information in the fewest words?

I believe it is the *atomic hypothesis* that ***All things are made of atoms—little particles that that move around in perpetual motion, attracting each other when they are a little distance apart, but repelling upon being squeezed into one another.***“

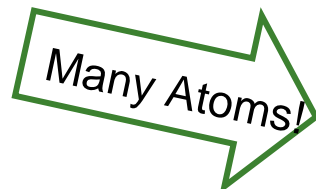
In that one sentence, there is an enormous amount of information about the world, **if just a little imagination and thinking are applied.**

R. P. Feynman – *The Feynman Lectures*

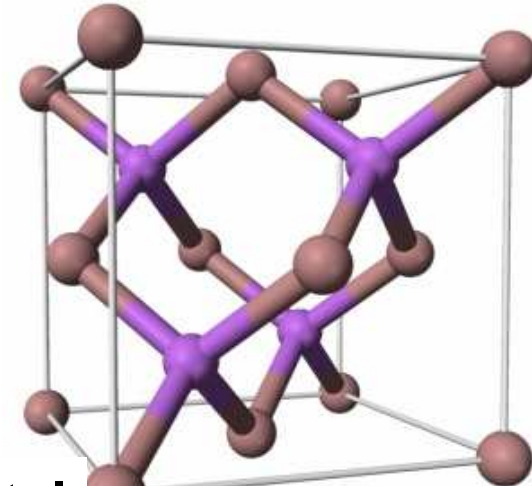
# Can you make “atoms” out of atoms?



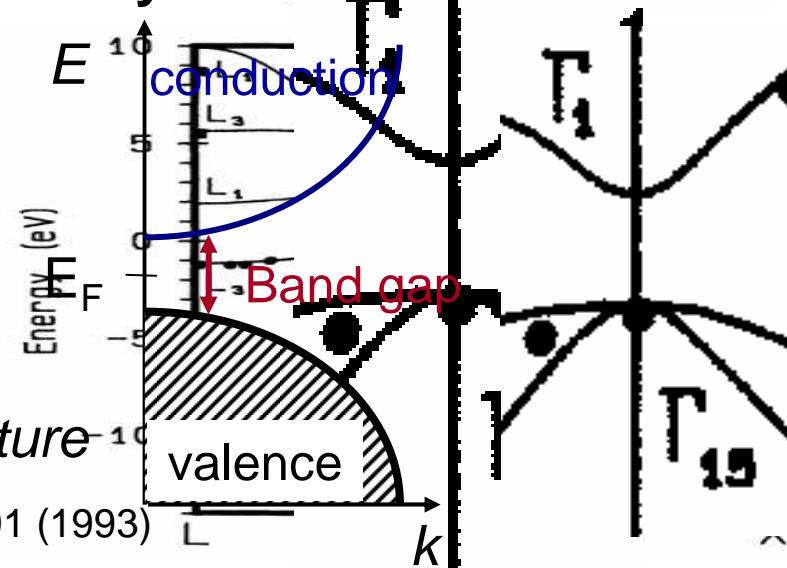
Atomic Energy levels



Band structure

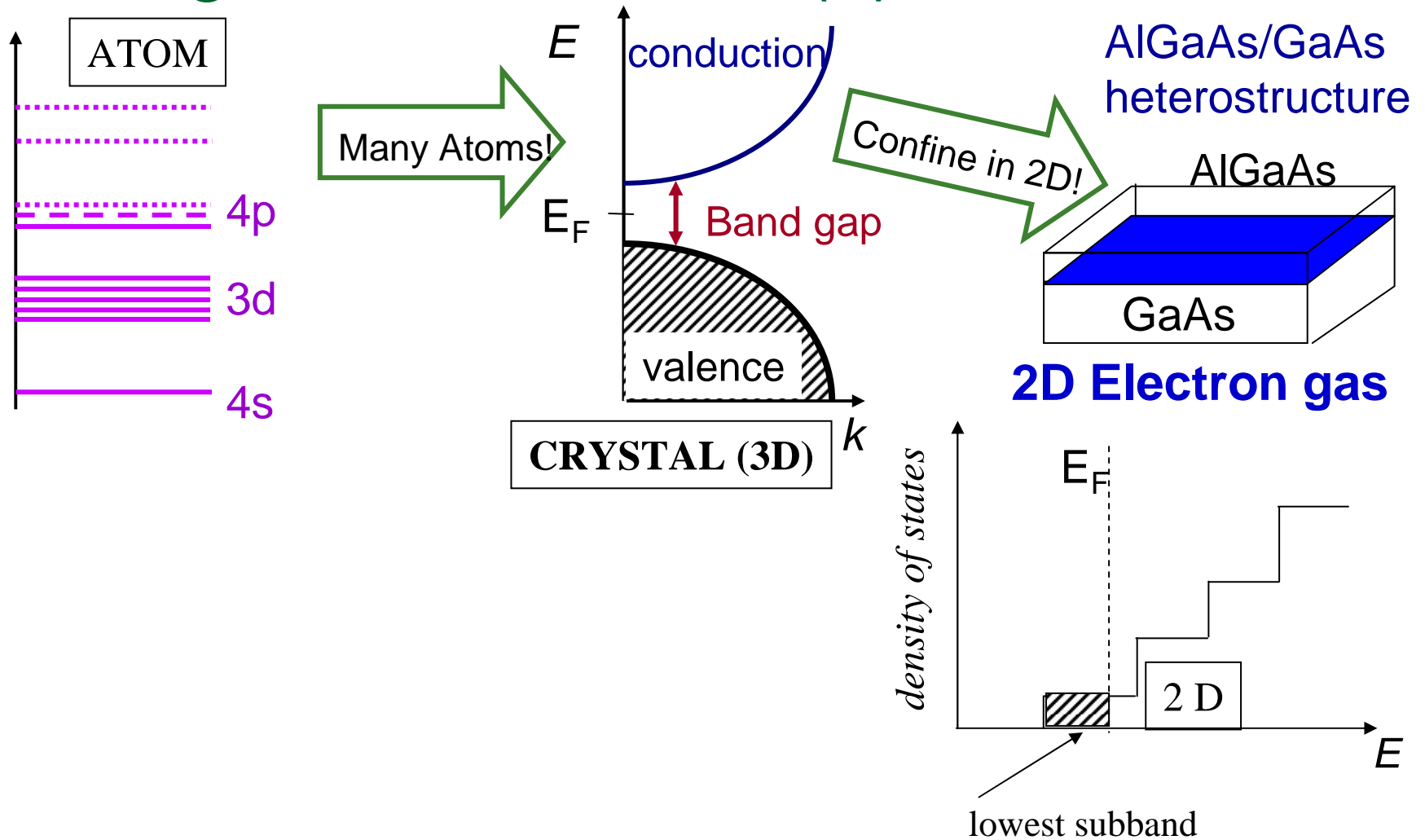


GaAs crystal



M. Rohlfing et al. PRB **48** 17791 (1993)

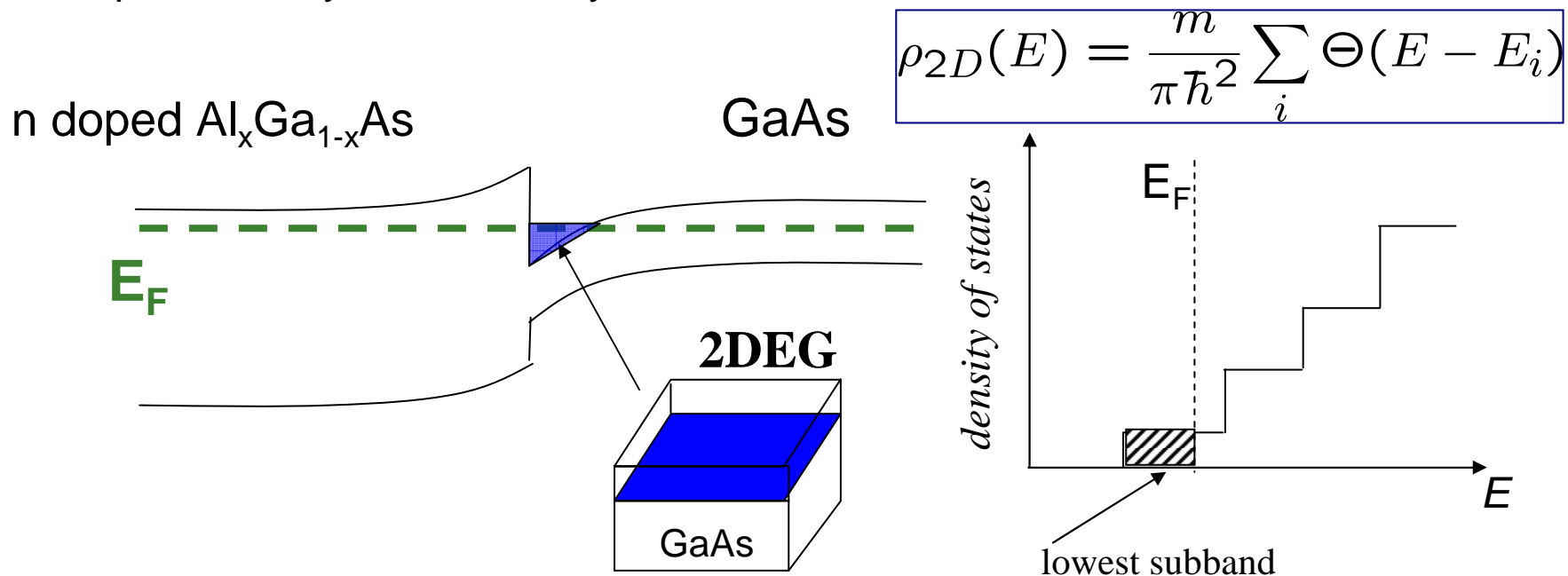
# Making “artificial atoms”(?) out of atoms



# Confined in 1 direction: 2D system

If a thin enough 2D plane of material (containing free electrons) is formed the electrons can be confined to be two dimensional in nature.

Experimentally this is usually done in semiconductors.



e.g. by growing a large band gap material with a smaller band gap material you can confine a region of electrons to the interface - **TWO DIMENSIONAL ELECTRON GAS (2DEG)**.

More details: Ando, Fowler, Stern *Rev. Mod. Phys.* **54** 437 (1982)



# Confined in 1 direction: 2D system

Provided the electrons are confined to the **lowest subband** the electrons behave exactly as if they are two-dimensional i.e. obey 2D Schrodinger equation etc.

2DEG: Rich source of Physics.

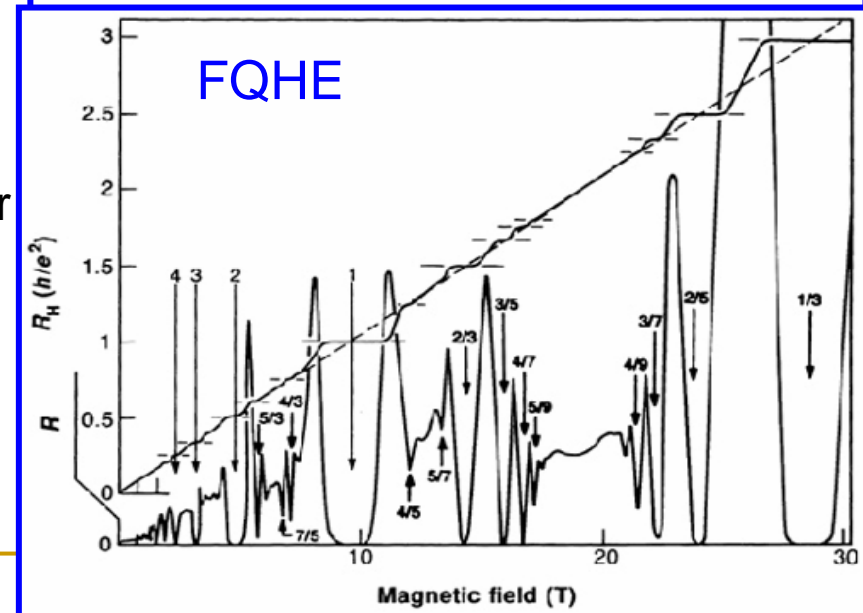
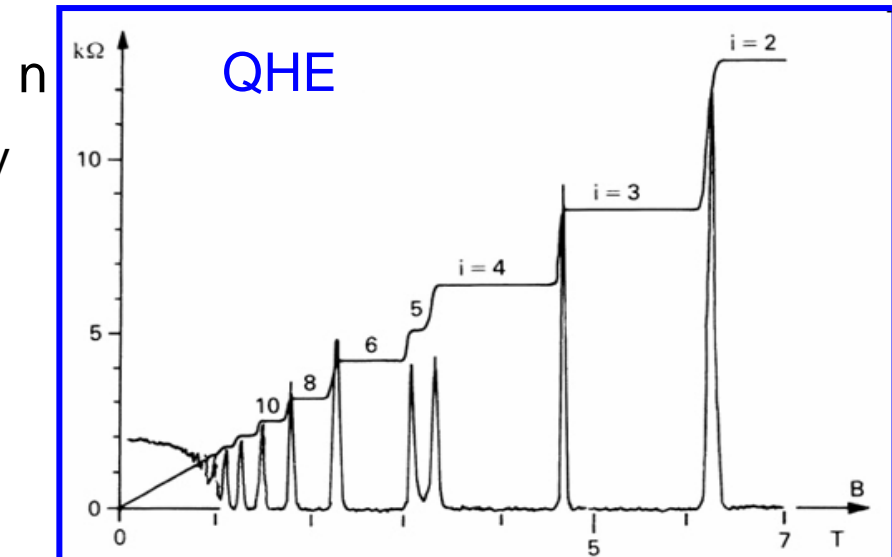
- Nobel Prizes in Physics:

- in 1985 to von Klitzing for the **Quantum Hall Effect (QHE)**,

- In 1998 to Tsui, Stormer and Laughlin for the **Fractional QHE**

- **Semiconductor heterostructures**, lithography

- Applications (lasers, QHE, etc.)



# Side example: electrons confined in 1D

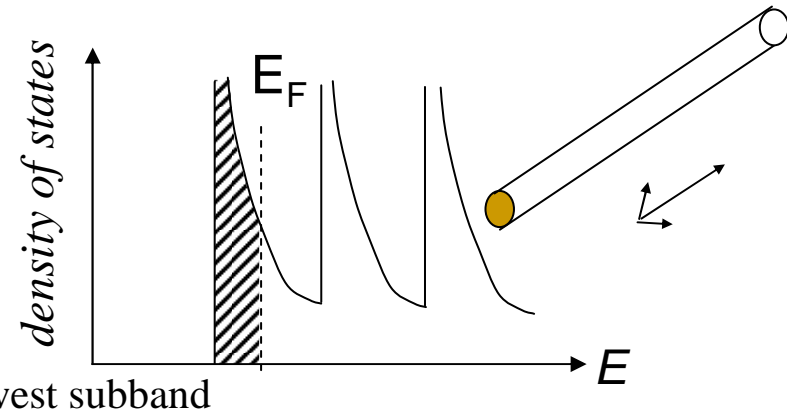
If the **confinement length is of the order of the Fermi wavelength ( $L \sim \lambda_F$  or  $E_1 < \sim E_F$ )** then electrons confined in one quantum mechanical state in two directions, but are free to move in the third: **1D confinement**.

A truly 1D DoS *diverges* at some energy values: these are **van Hove singularities**.

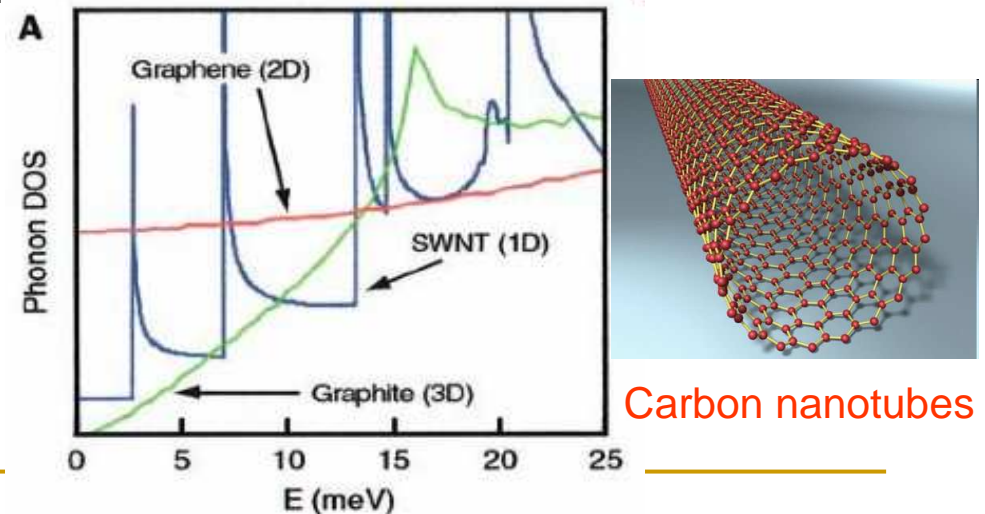
Electrons **interact differently** in 1D compared to 2D and 3D (e.g. Luttinger liquid physics).

Analogy: think of cars (electrons) moving along a single track lane. They “interact” differently compared to cars on dual lane roads, for instance.

Examples include **carbon nanotubes**, **nanowires**, **lithographically defined regions of 2DEGs** etc.



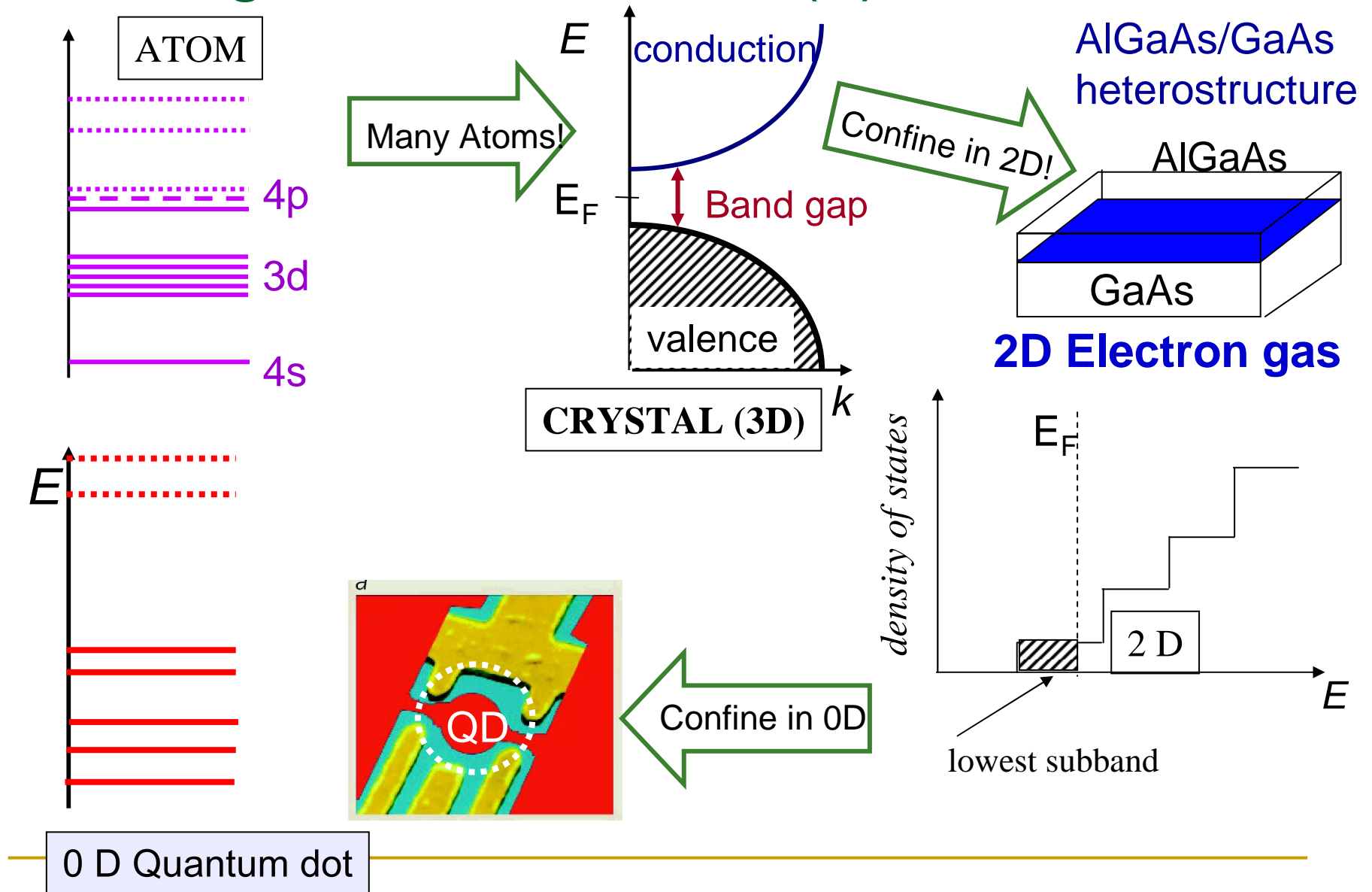
$$\rho_{1D}(E) = \left( \frac{2m}{\pi^2 \hbar^2} \right)^{1/2} \sum_i \frac{n_i \Theta(E - E_i)}{(E - E_i)^{1/2}}$$



Carbon nanotubes

Hone, J. et al., *Quantized Phonon Spectrum of Single-Wall Carbon Nanotubes*, Science, **289**, 1730, (2000).

# Making “artificial atoms”(?) out of atoms



# Confined in *all* directions: 0D systems or Quantum Dots

Electron systems confined in all three directions. '0D'

man-made droplets of charge

e.g. nanocrystals (nanoparticles)  
molecules

degree of confinement does not have to be the same in all directions  
=> 2 D quantum dots and 1D quantum dots

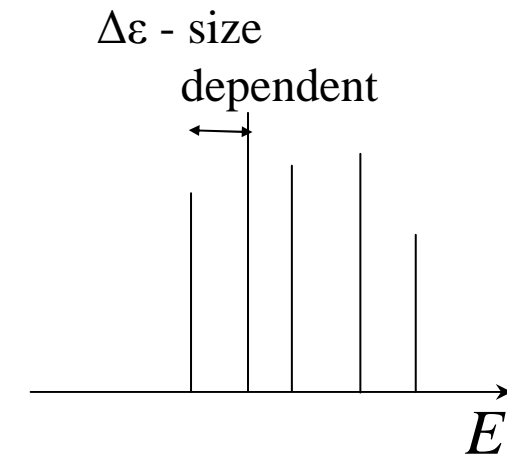
fabrication either "bottom up":

2DEG  
small metal islands

or "top down" fabrication:

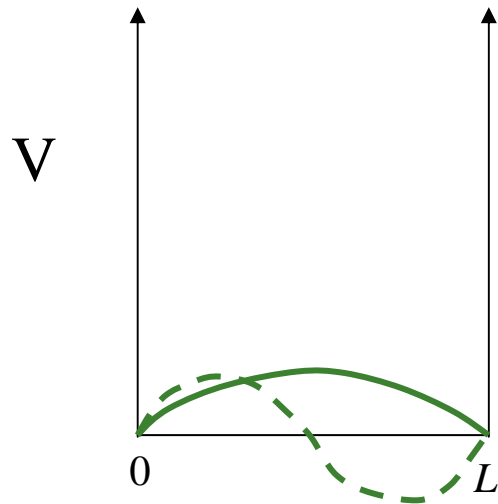
e.g. nanotubes  
nanowires

or combination of both.



$$\rho_{0D}(E) = \sum_i \delta(E - E_i)$$

# Confinement: Particle in a box



1-d box: wavefunction constrained so that

$$L = N \lambda / 2 \text{ or}$$
$$k = 2 \pi / \lambda = N \pi / L$$

Energy of states given by Schrodinger Equation:

$$\hat{H}\Psi = E\Psi$$

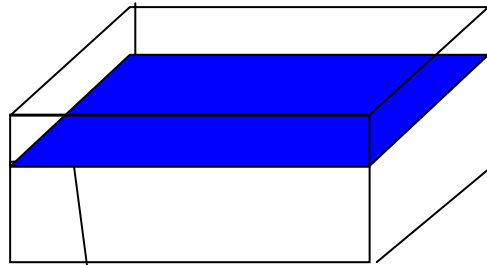
$$E_N = \frac{\hbar^2 k_N^2}{2m} = \frac{\hbar^2 \pi^2 N^2}{2mL^2}$$

Typical semiconductor dots:  
L in nm, E in meV range

As the length scale decreases the energy level spacing increases.

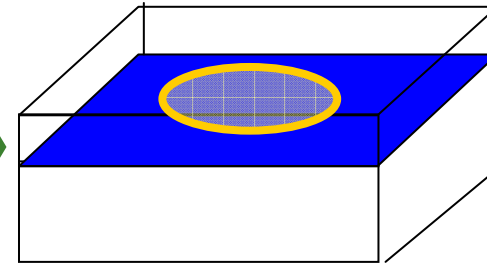
$$\Delta E = E_{N+1} - E_N \propto \frac{1}{L^2}$$

# Can you make “atoms” out of atoms?

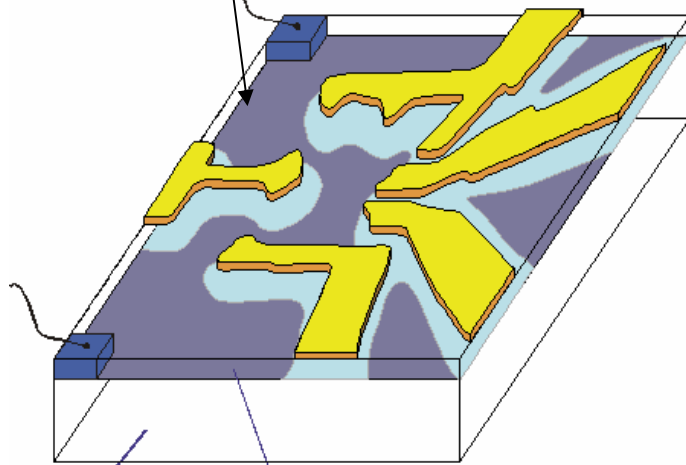


2D Electron gas

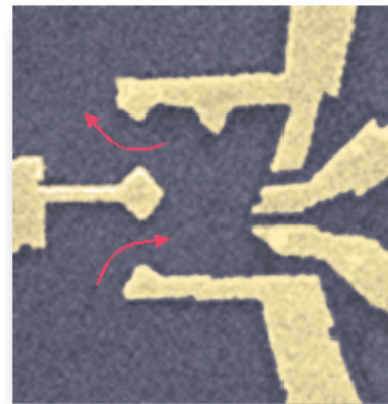
Electrostatically confine electrons within a small (nanometer-size) region.



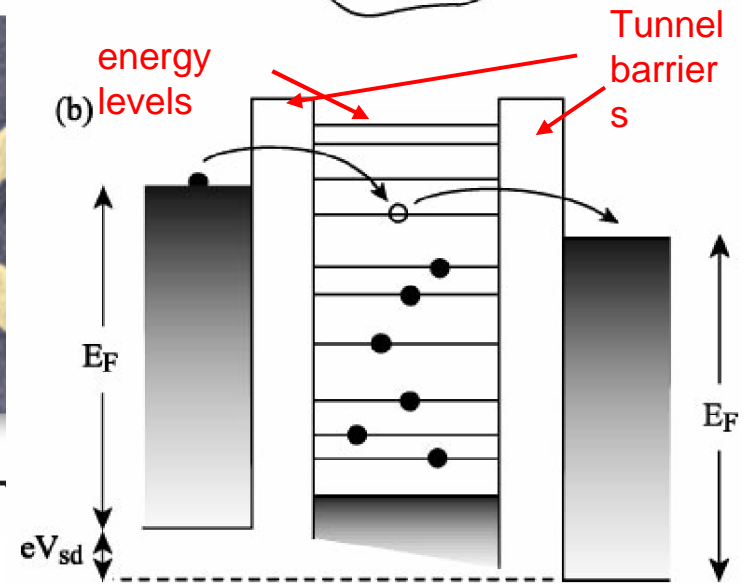
“Quantum dot”



GaAs  $\text{Al}_x\text{Ga}_{1-x}\text{As}$

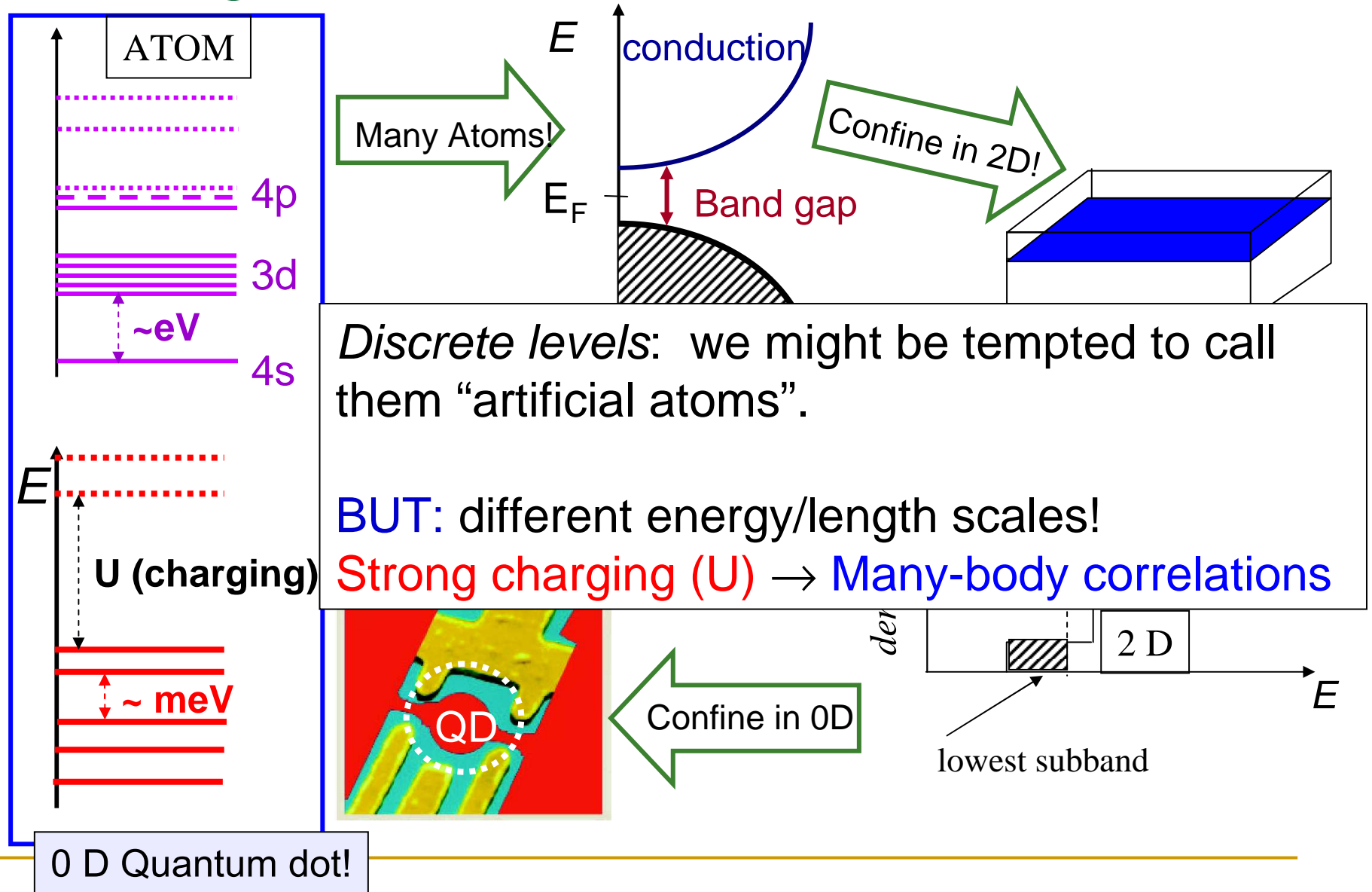


$1\mu\text{m}$

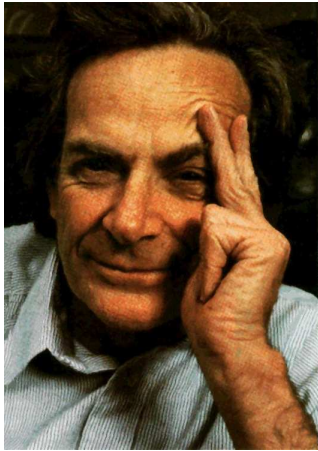


from Charlie Marcus' Lab website ([marcuslab.harvard.edu](http://marcuslab.harvard.edu))

# Making “artificial atoms”(?) out of atoms



# “Nano is Different”



“There is plenty of room at the bottom”.

“This field is not quite the same as the others in that it will not tell us much of the fundamental physics in the sense of, ‘What are the strange particles?’. But it is more like solid state physics in the sense that it might tell us much of the great interest about the strange phenomena that occur in complex situations.”

– *Richard Feynman*



Or, as Anderson might put it:

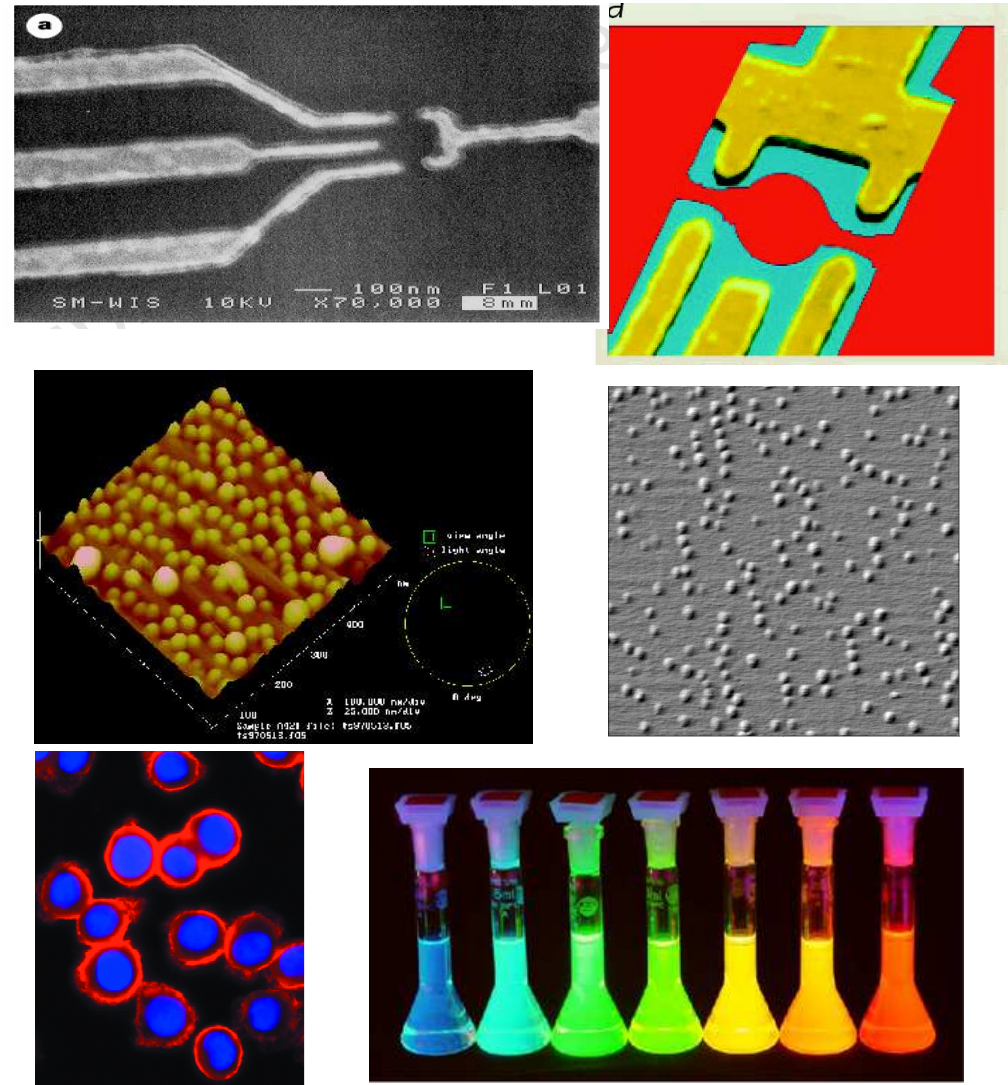
**The rules of the game are different at the bottom.**



# What are *Quantum Dots*?

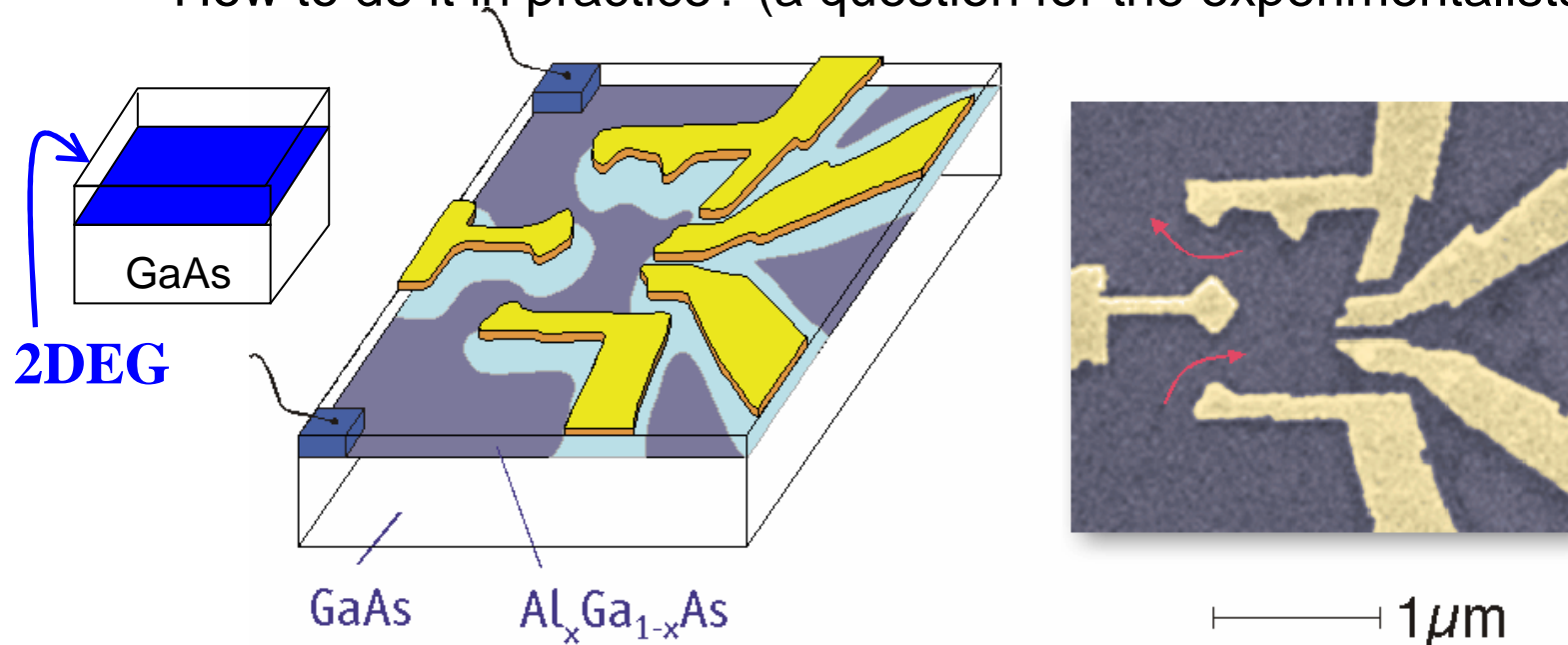
## Semiconductor Quantum Dots:

- Devices in which electrons are **confined** in nanometer size volumes.
- Sometimes referred to as “artificial atoms”.
- “Quantum dot” is a generic label: **lithographic QDs**, self-assembled QDs, colloidal QDs have different properties.



# Lithographic Quantum Dots

How to do it in practice? (a question for the experimentalists...)

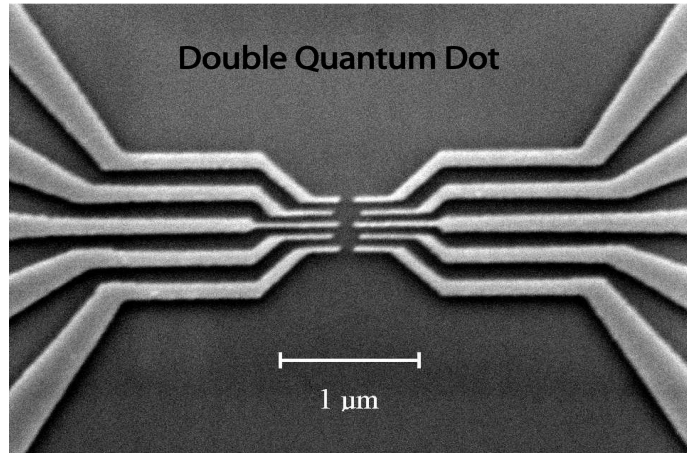


Ingredients:

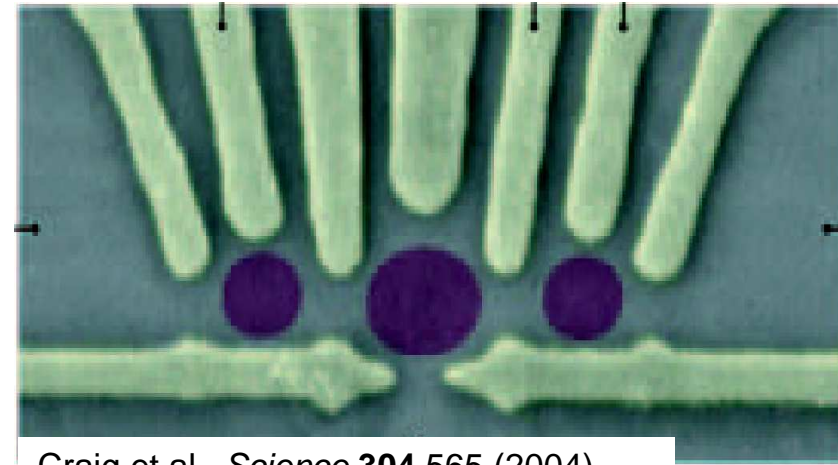
from Charlie Marcus' Lab website ([marcuslab.harvard.edu](http://marcuslab.harvard.edu))

- Growth of heterostructures to obtain the 2DEG
  - (good quality, large mean free-paths)
- Metallic electrodes electrostatically deplete charge: confinement
- Sets of electrodes to apply bias etc.
- **LOW TEMPERATURE! (~100 mK)**

# Lithographic Quantum Dots

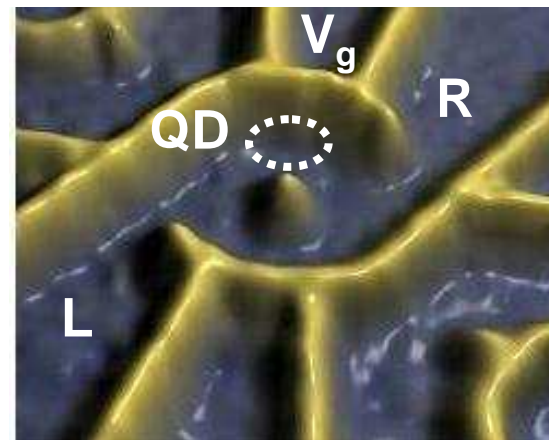


Jeong, Chang, Melloch *Science* **293** 2222 (2001)



Craig et al., *Science* **304** 565 (2004)

Lithography evolved quite a bit in the last decade or so. Allow different patterns: double dots, rings, etc.



From:  
K. Ensslin's group  
website

# Quantum Dots: transport

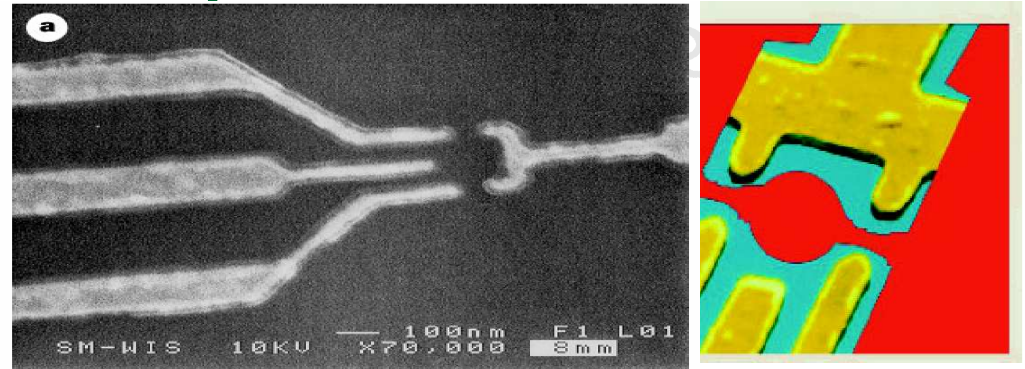
## Lithographic Quantum Dots:

- Behave like small capacitors:

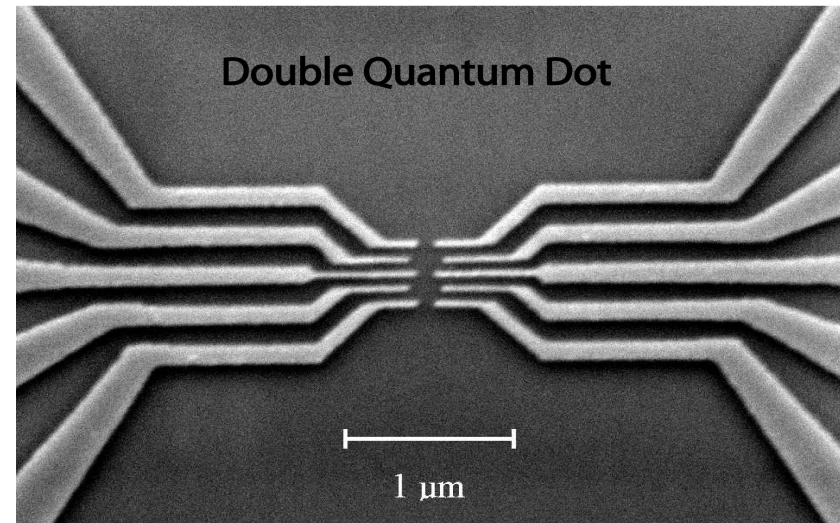
$$E_c = \frac{e^2}{C}$$

- Weakly connected to metallic leads.
- Energy scales: level spacing  $\Delta E$ ; level-broadening  $\Gamma$ .
- $E_c$  is usually largest energy scale:

$$E_c \gg \Delta E, \Gamma$$

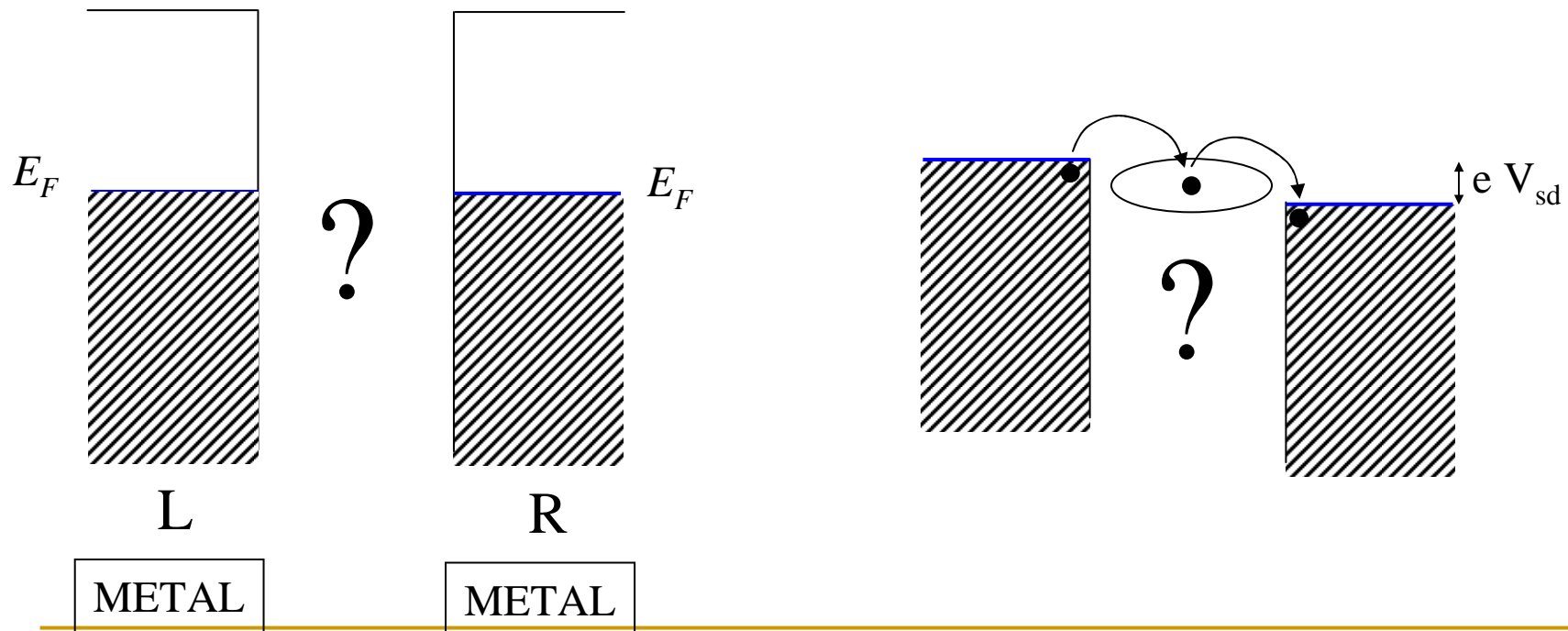
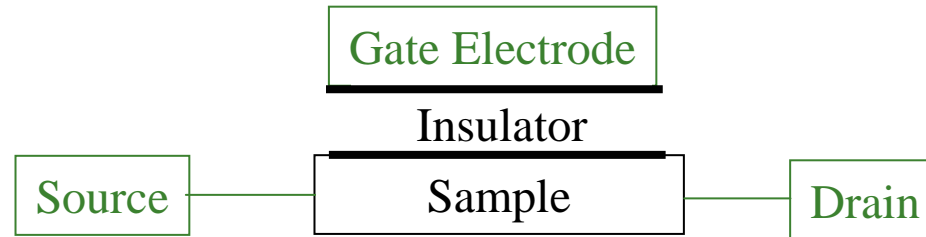


Goldhaber-Gordon *et al.* *Nature* **391** 156 (1998)

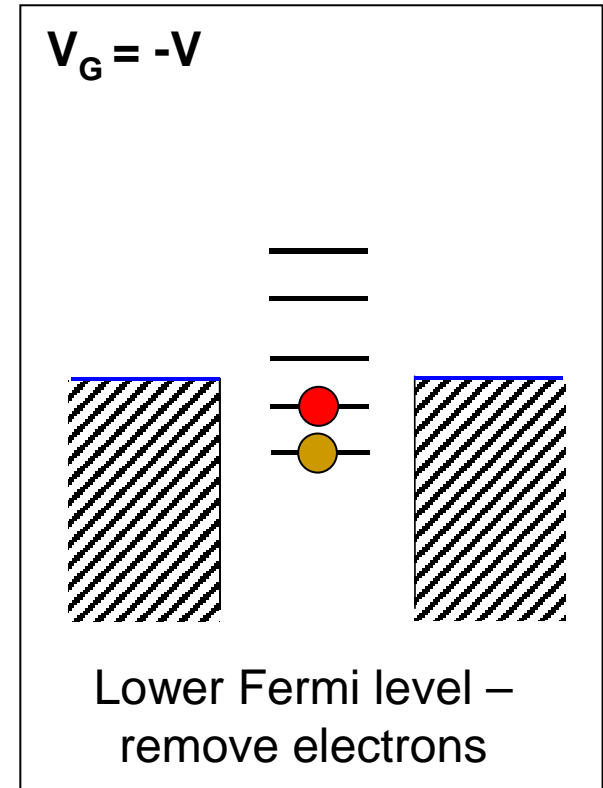
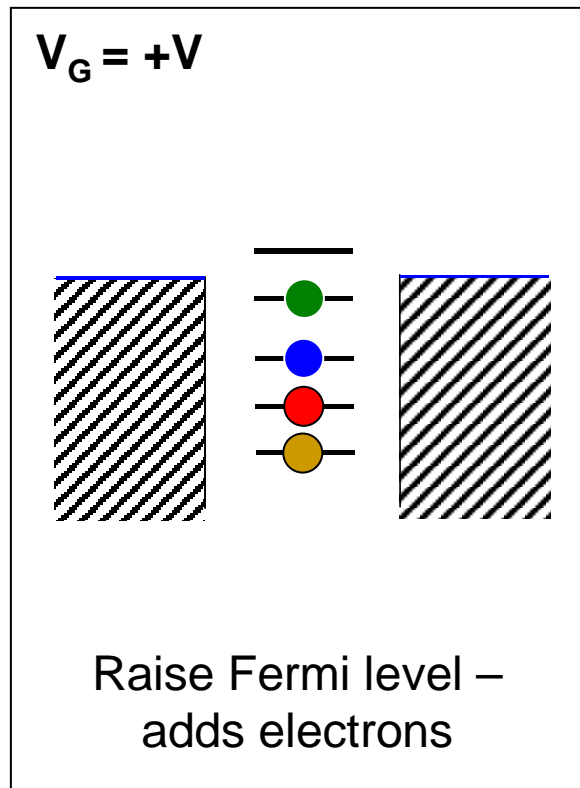
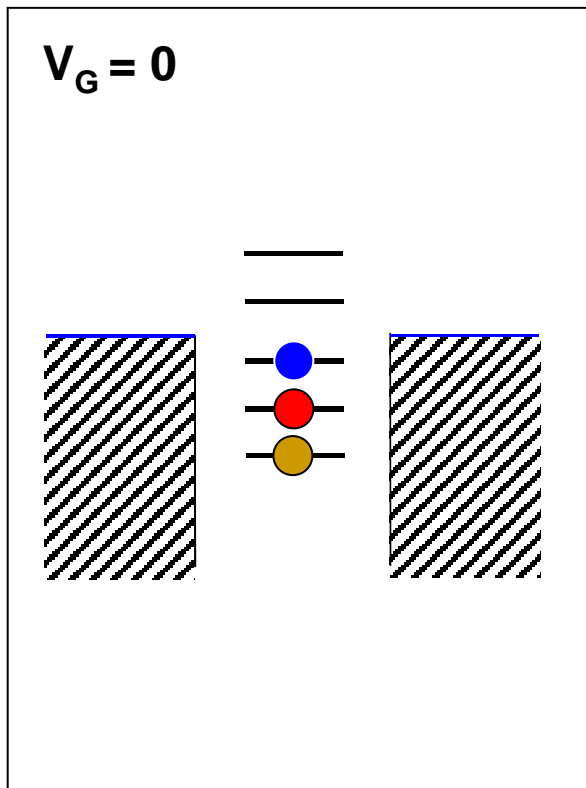
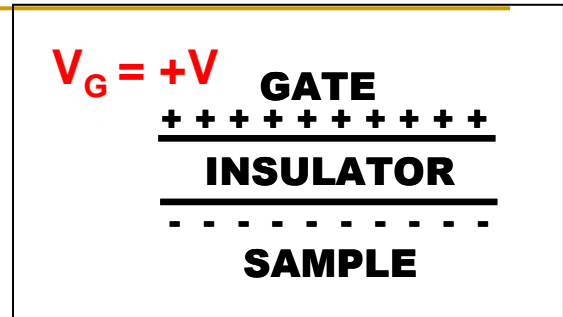
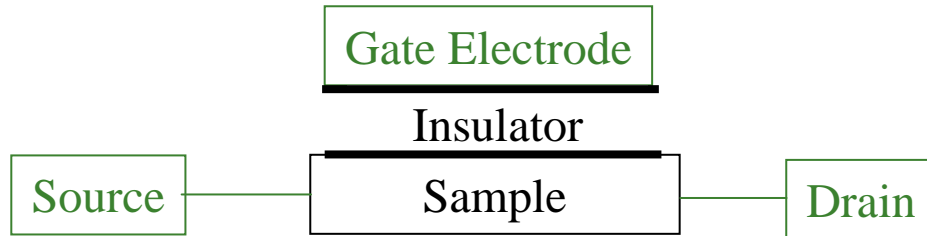


Jeong, Chang, Melloch *Science* **293** 2222 (2001)

# Electrical Transport



# Role of the Gate Electrode



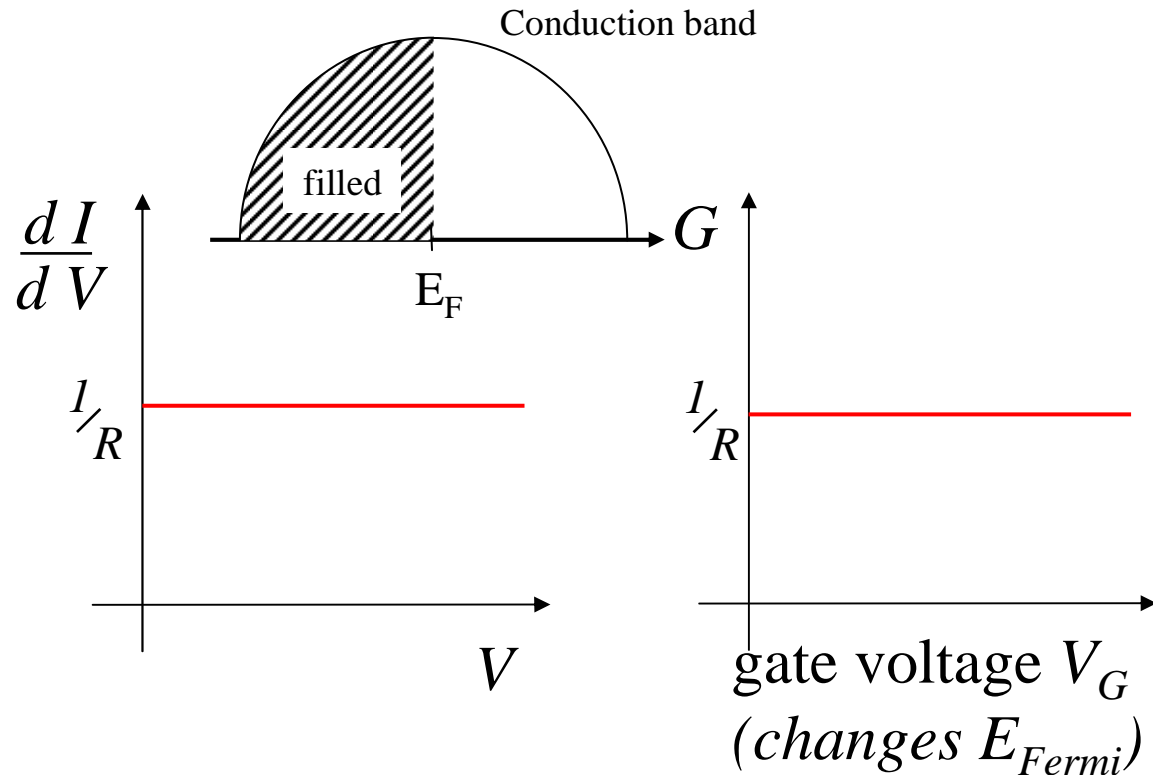
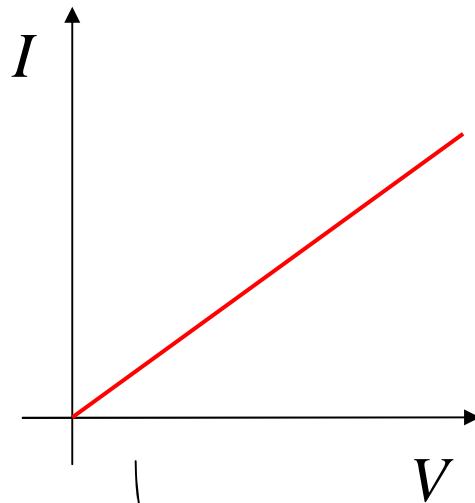
# Electrical Transport: Ohm's Law

Ohm's Law holds for metallic conductors =>  $V = I R$

We can also define a conductance which can be bias dependent  
 The **zero bias conductance**,  $G$ , is conventionally quoted.

$$\frac{dI}{dV}$$

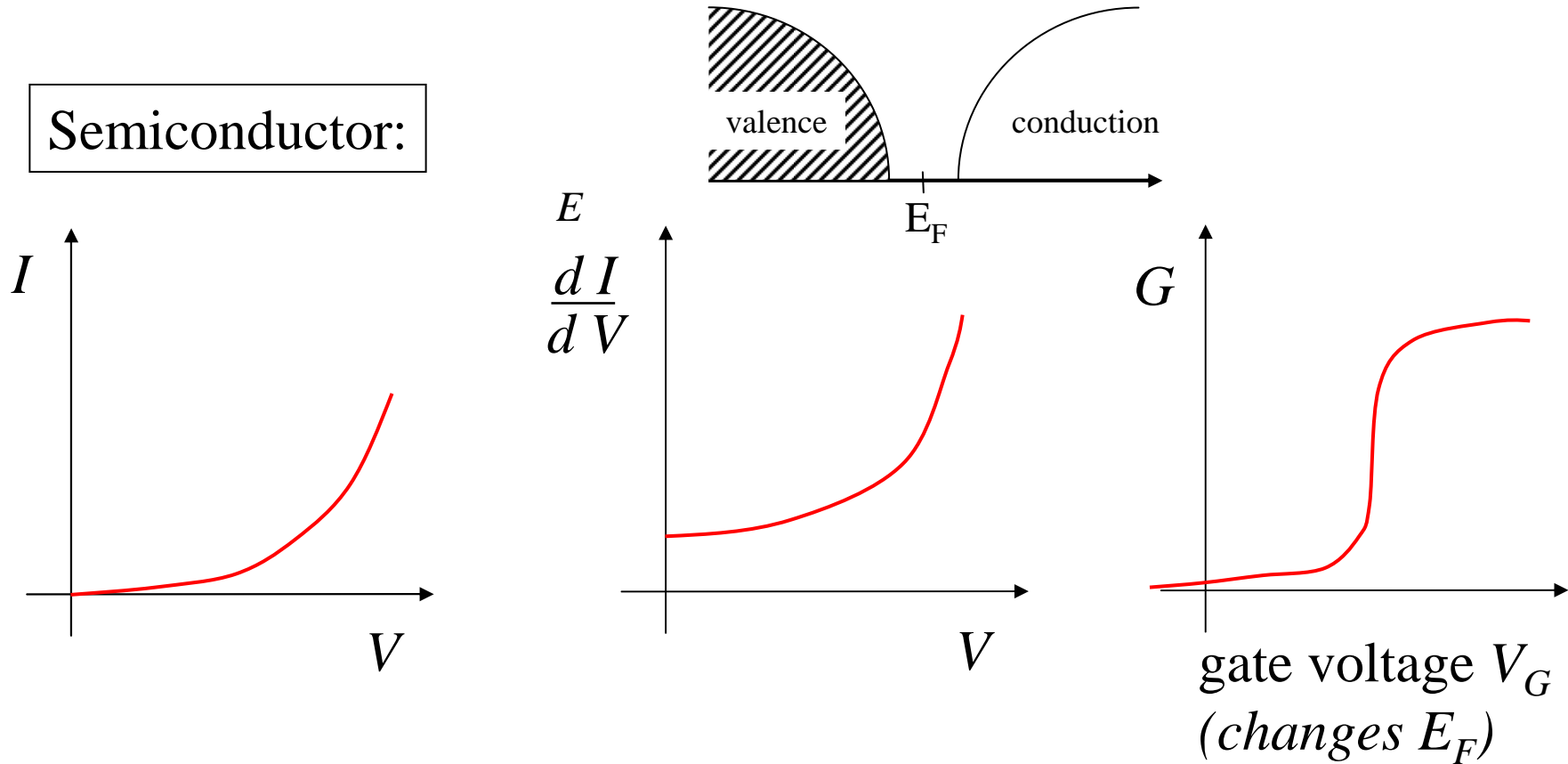
Metallic conductor:



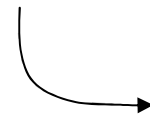
Resistance due to scattering off impurities, mfp ~ 10 nm

# Electrical Transport: semiconductors

Semiconductor:



Semiconductor - nonlinear  $I - V$  response



tunneling through Schottky barrier  
or out of band gap.



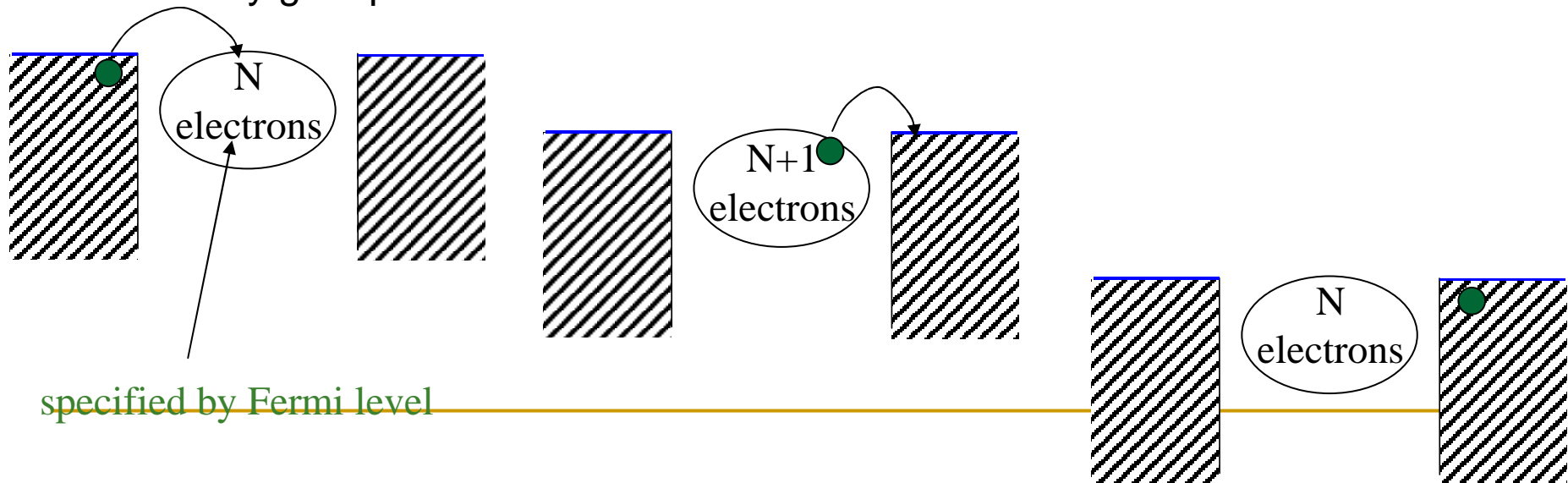
# Conductance through quantum dot

Quantum dots contain an integer number of electrons.

Adding an electron to the QD changes its energy => electrostatic charging energy  $\frac{Q^2}{2C}$

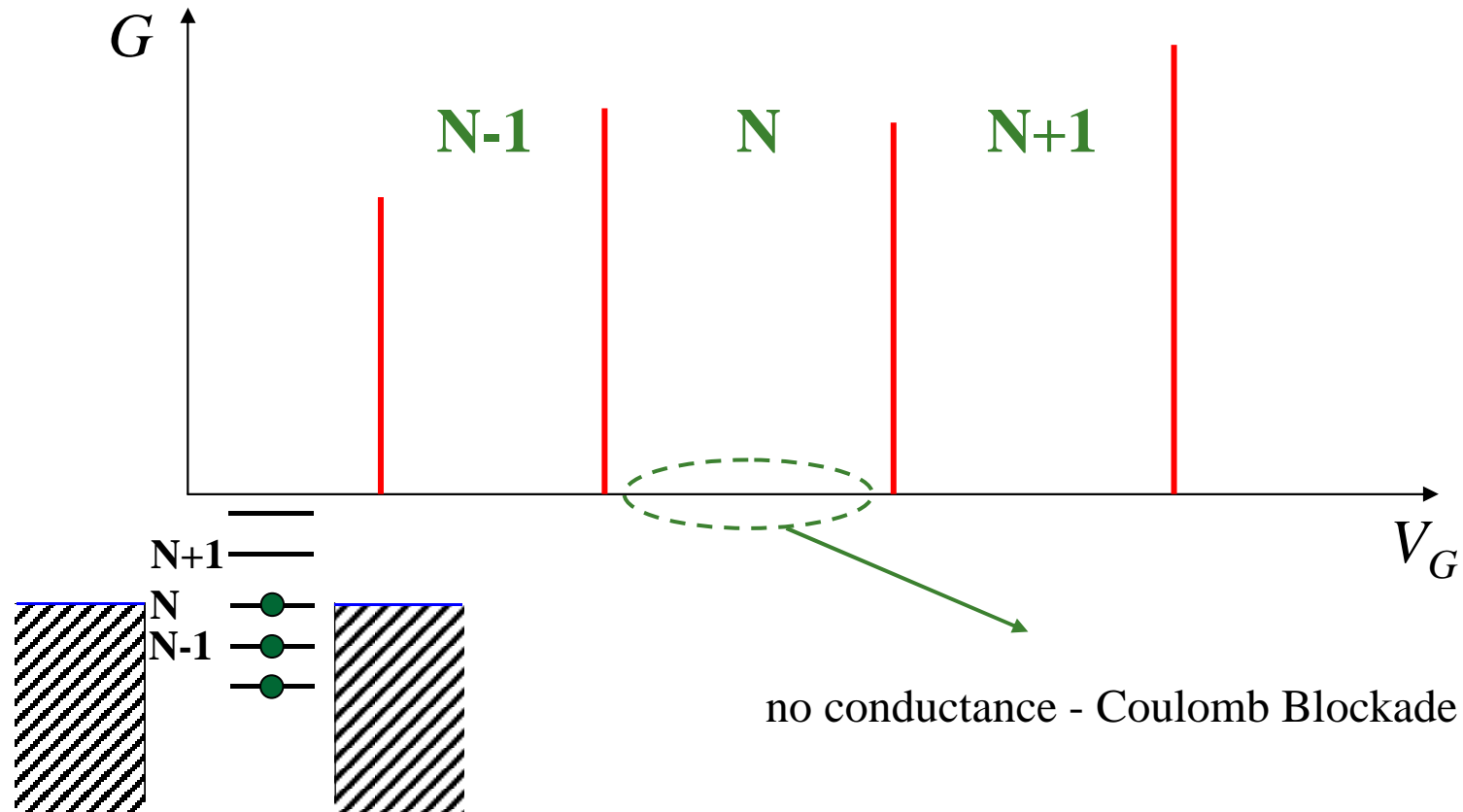
In order for a current to pass an electron must tunnel onto the dot, and an electron must tunnel off the dot.

For **conduction at zero bias** this requires the energy of the dot with  $N$  electrons must equal the energy with  $N+1$  electrons. i.e. charging energy balanced by gate potential.



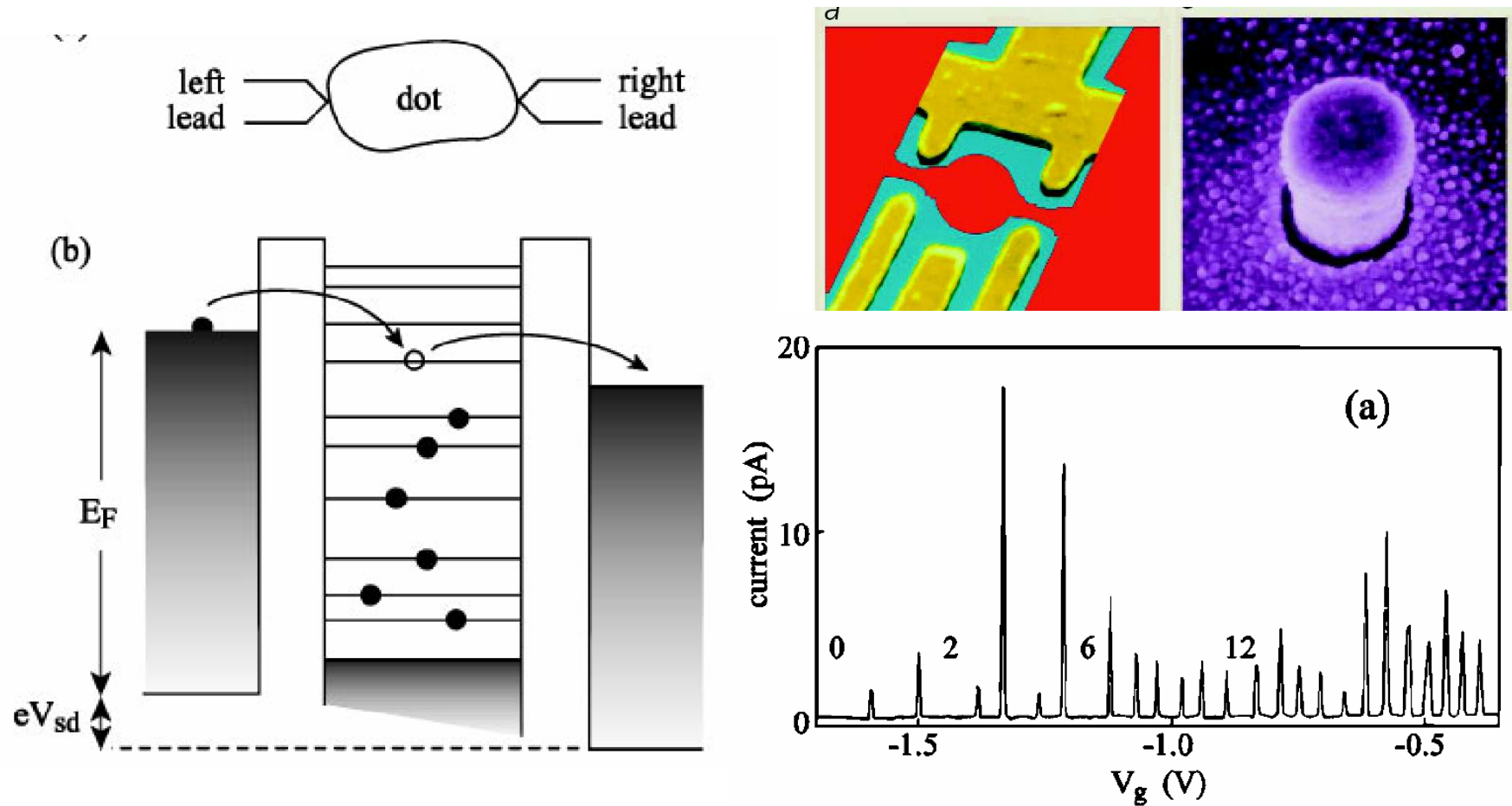
# Coulomb blockade

The addition of an electron to the dot is blocked by the charging energy as well as the level spacing => **Coulomb blockade**



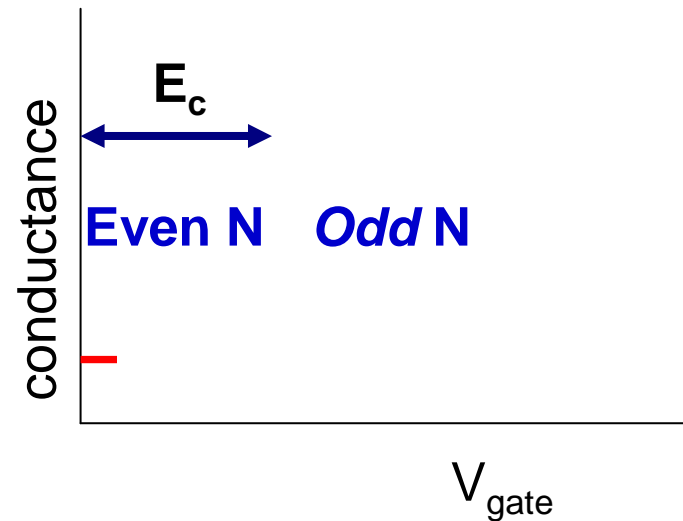
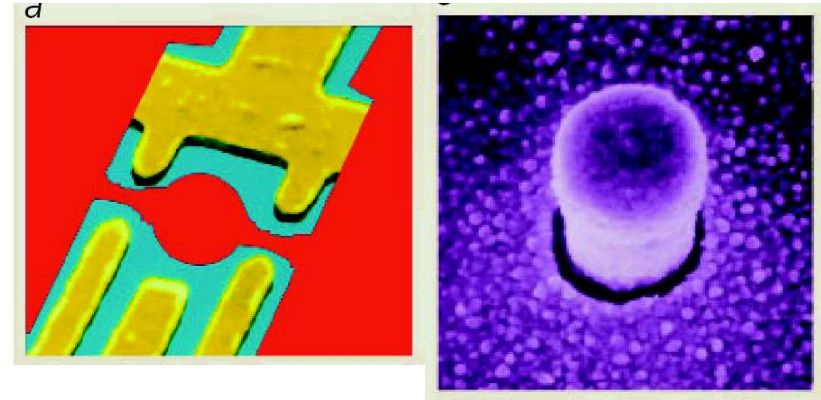
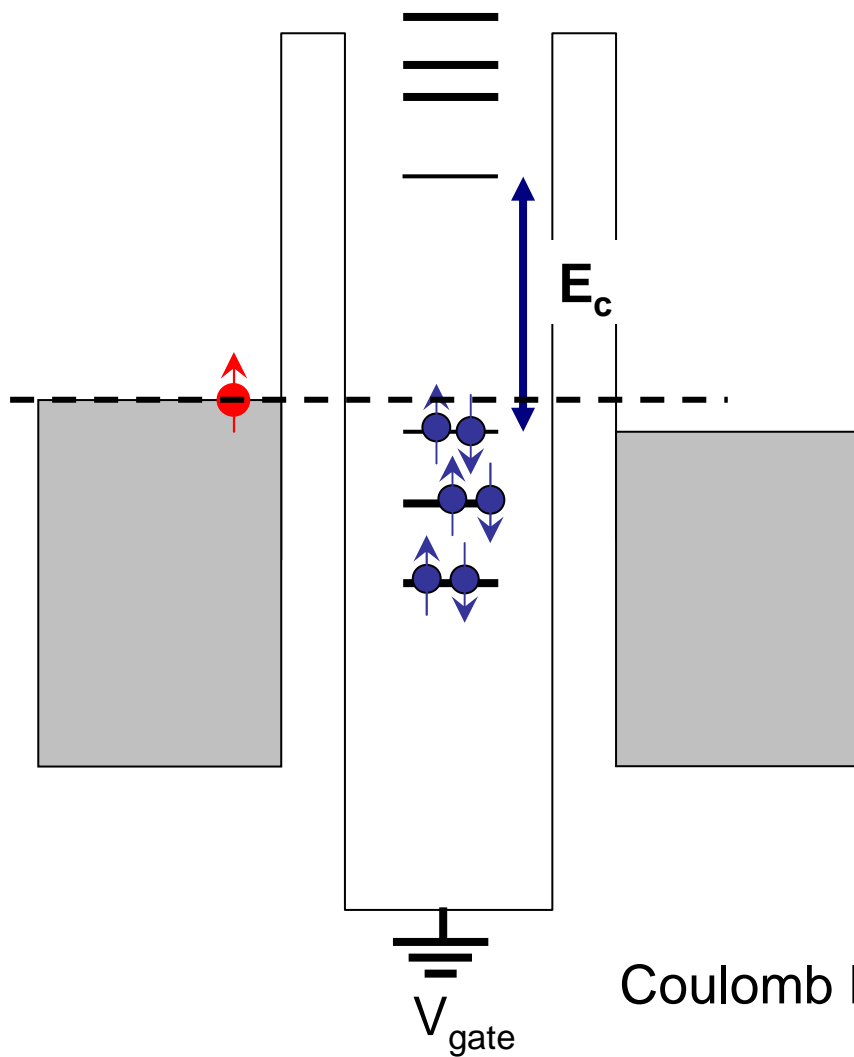
The number of electrons on the QD is adjusted by the gate potential.  
Conduction only occurs when  $E_N = E_{N+1}$

# Coulomb Blockade in Quantum Dots



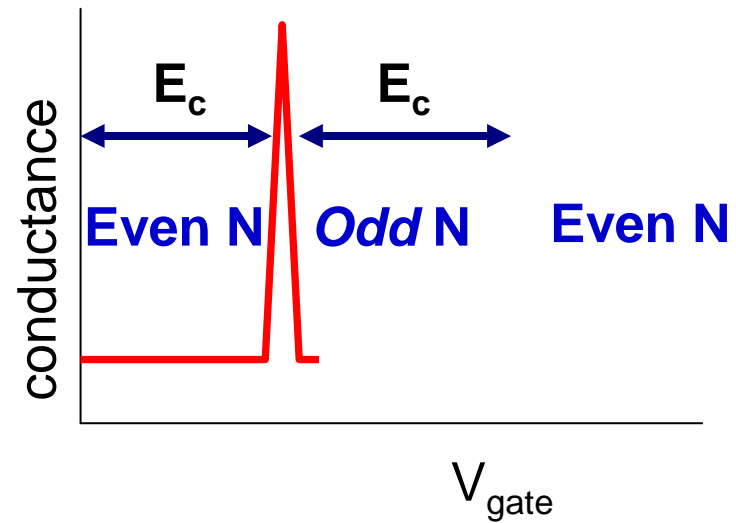
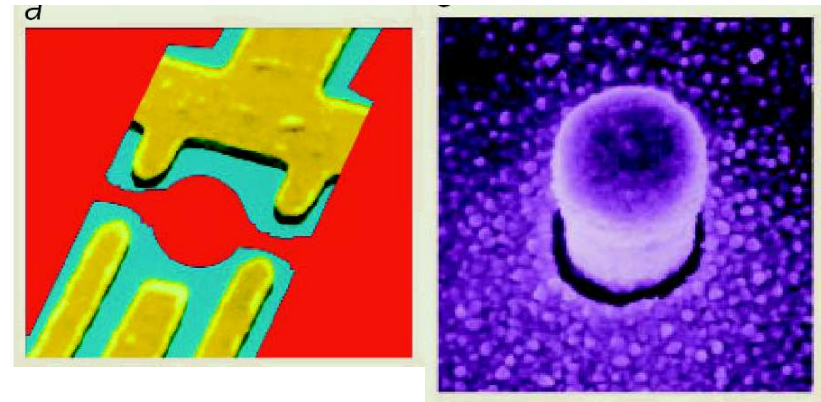
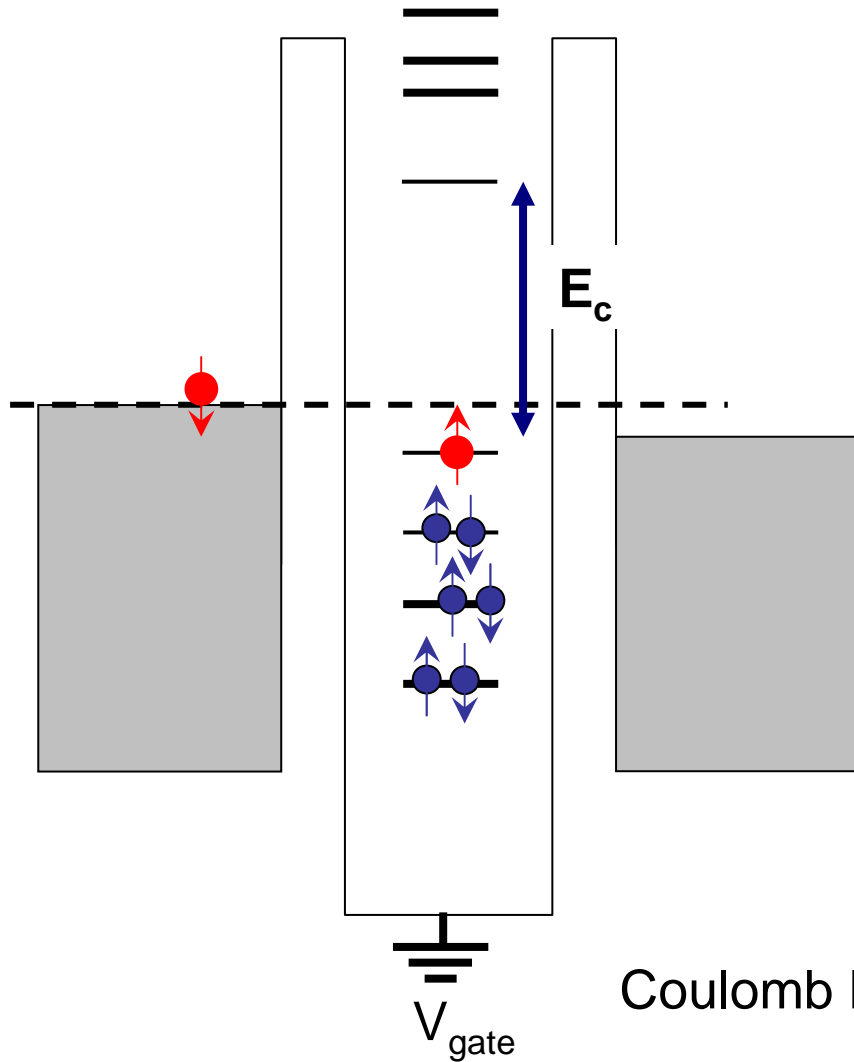
Coulomb Blockade in Quantum Dots

# Coulomb Blockade in Quantum Dots



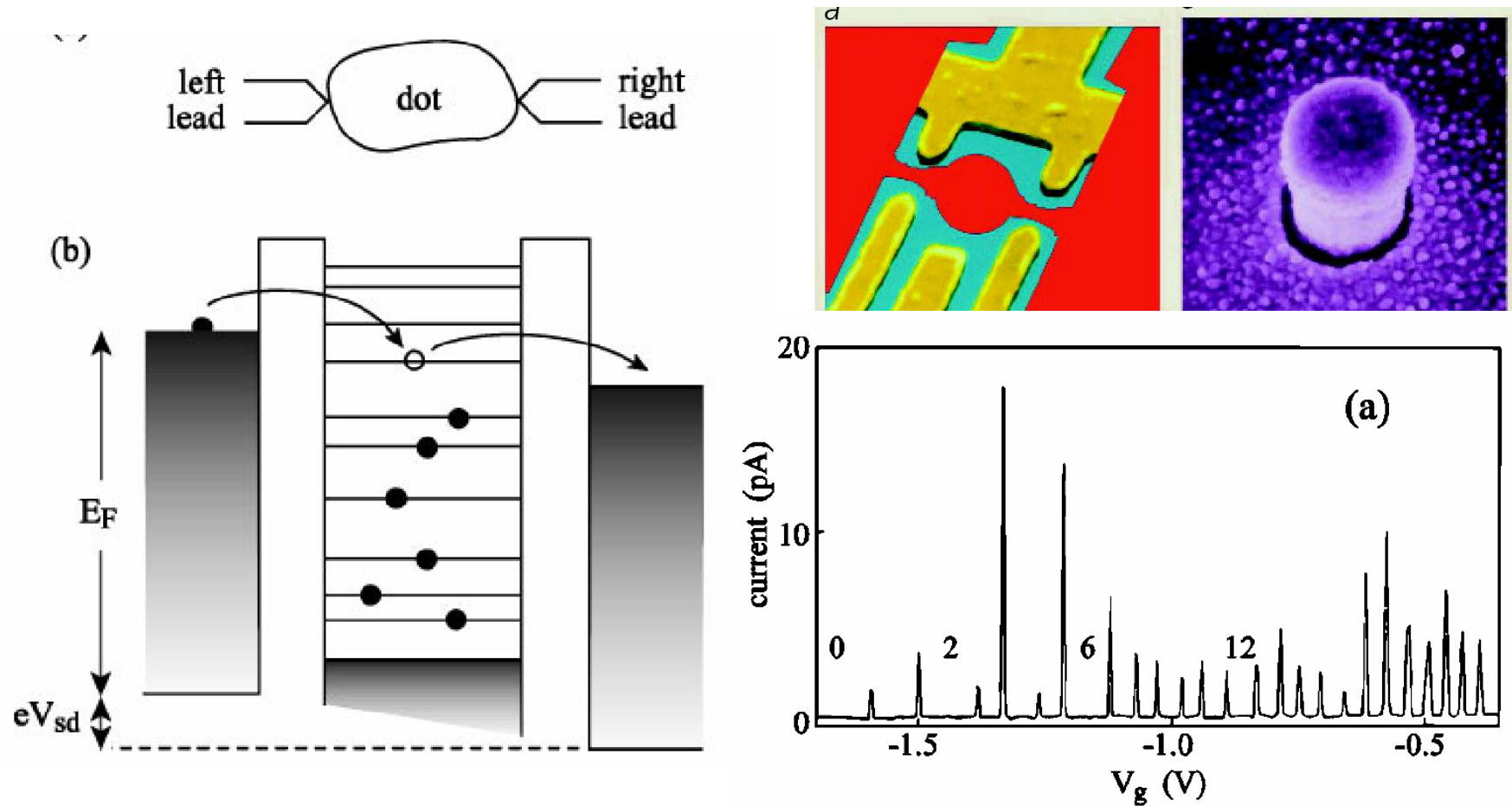
Coulomb Blockade in Quantum Dots

# Coulomb Blockade in Quantum Dots



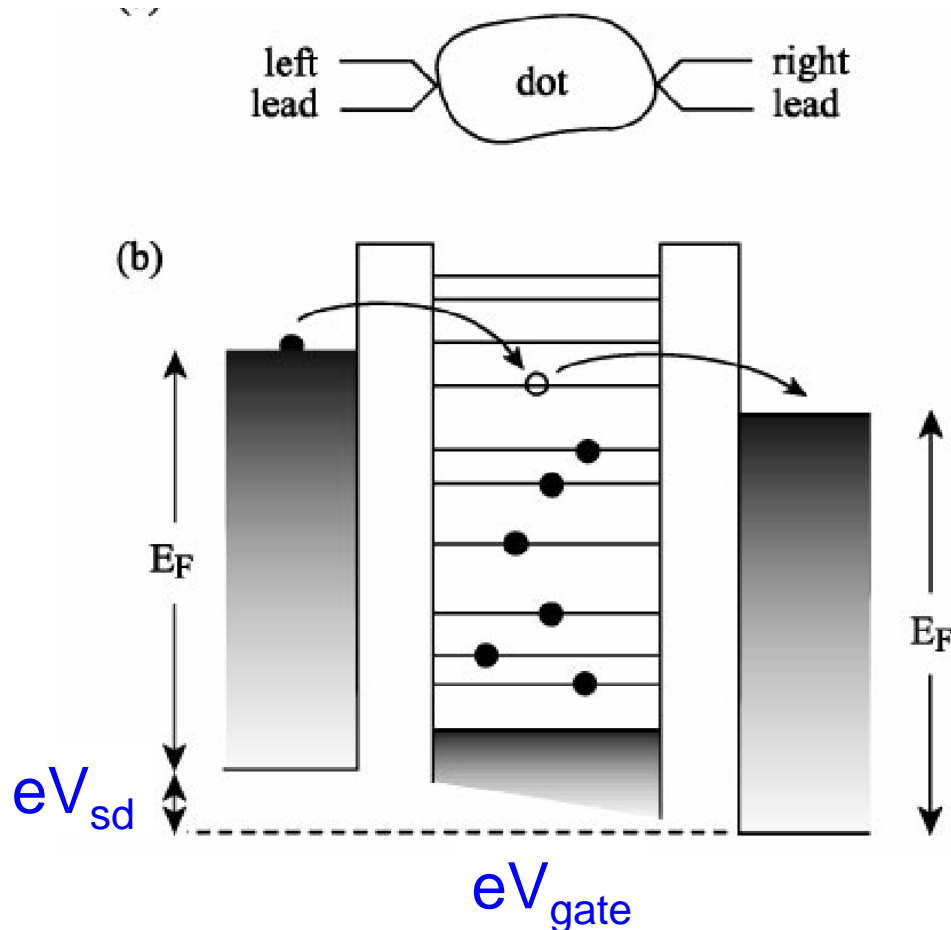
Coulomb Blockade in Quantum Dots

# Coulomb Blockade in Quantum Dots

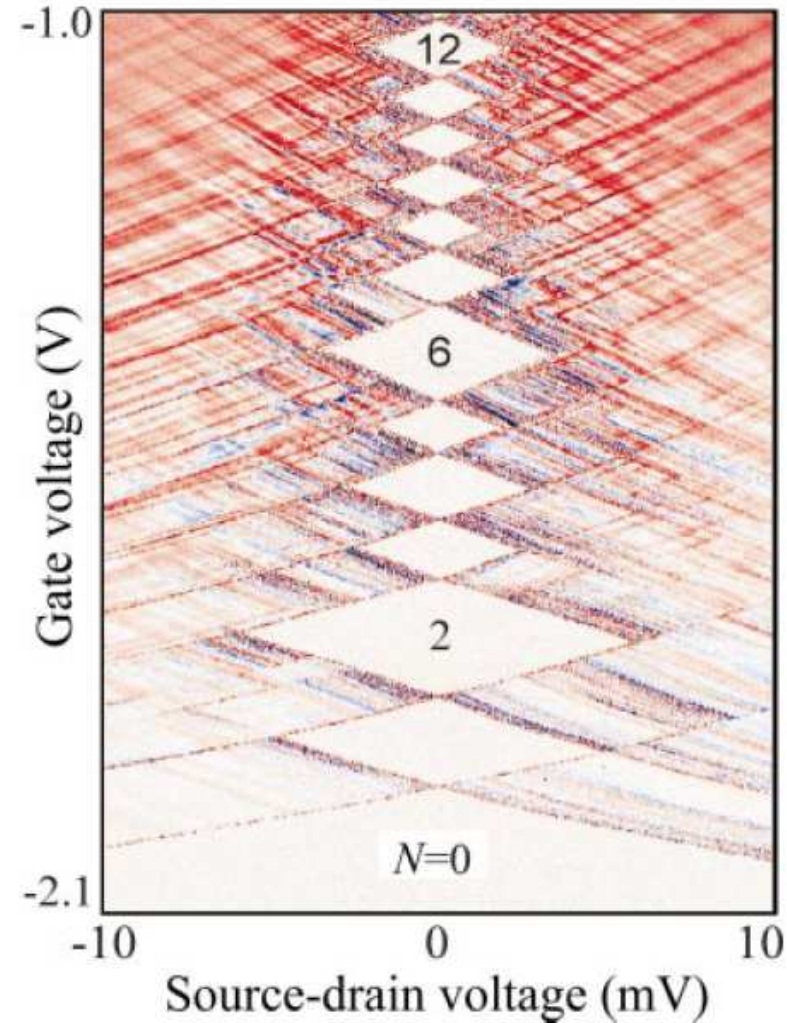


Coulomb Blockade in Quantum Dots: “dot spectroscopy”

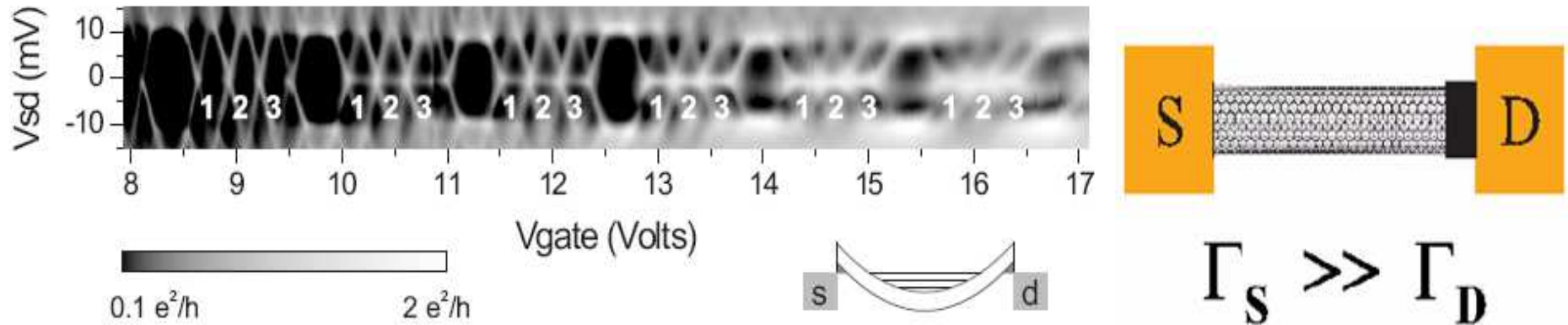
# “Coulomb Diamonds” (Stability Diagram)



Coulomb Blockade in Quantum Dots

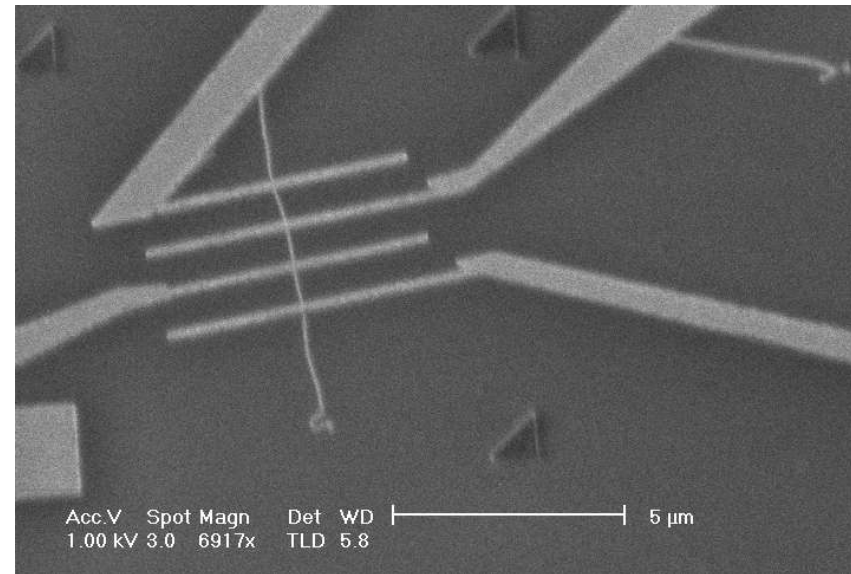


# “Carbon nanotube Quantum dots”.



Makarovski, Zhukov, Liu, Filkenstein *PRB* **75** 241407R (2007).

- Carbon nanotubes deposited on top of metallic electrodes.
- Quantum dots defined *within* the carbon nanotubes.
- More structure than in quantum dots: “shell structure” due to *orbital* degeneracy.



Gleb Filkenstein's webpage: <http://www.phy.duke.edu/~gleb/>



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# Lecture 2 (coming up...)

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