



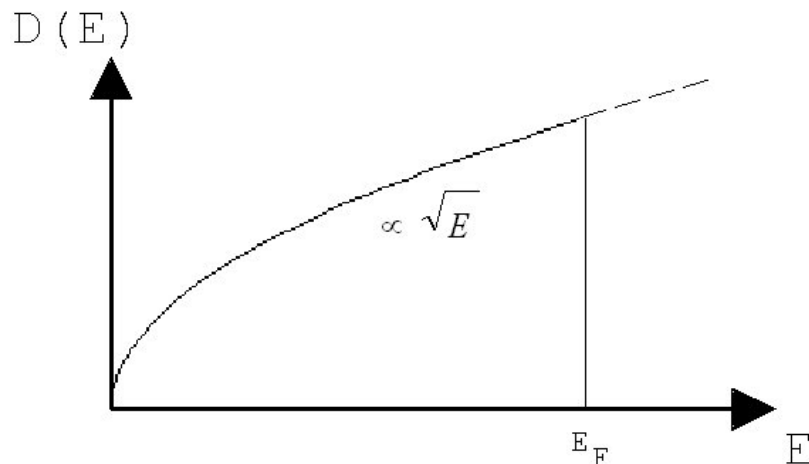
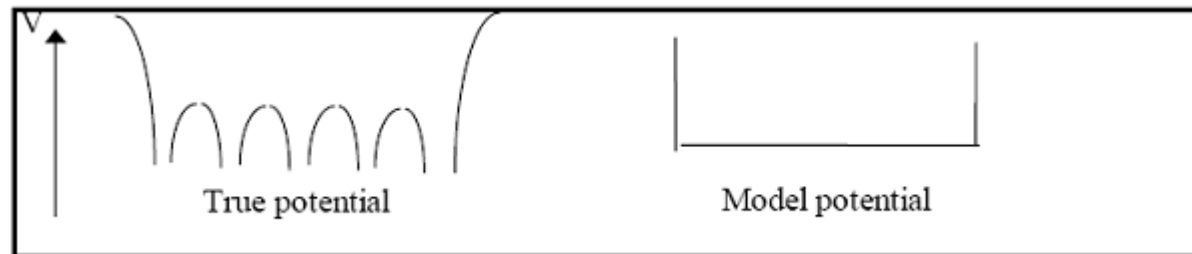
# Líquidos fermiônicos de baixa dimensionalidade

César Augusto Nieto Acuna

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Em 3D modelo de elétron livre funciona muito bem



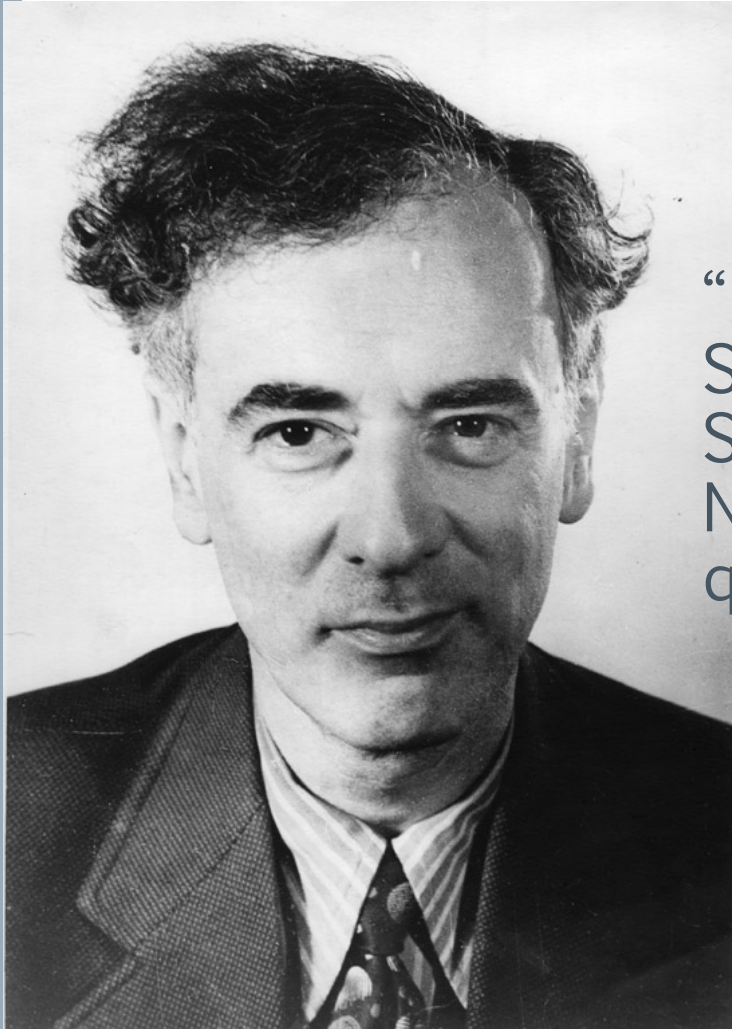
$$C_L^{\text{Pauli}} = \frac{\pi}{3} k_B \left( \frac{k_B T}{\hbar v_F} \right),$$

As interações elétron-elétron são importantes.  
Por quê, se são depreciadas, o modelo ainda funciona?

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## Lev Landau (1957)



“Um sistema de interações  
Suficientemente fracas. Pode  
Ser modelado como um sistema  
Não interagente mas de  
quase-partículas.”

# Aproximação RPA. Criação par elétron-buraco



$$\begin{aligned}
 -W^{\text{RPA}}(\mathbf{q}, iq_n) &\equiv \text{wavy line} \equiv \text{wavy line} + \text{bubble} + \text{chain} + \text{chain} + \dots \\
 &= \text{wavy line} + \text{bubble} \times \left[ \text{wavy line} + \text{bubble} + \text{chain} + \dots \right] \\
 &= \text{wavy line} + \text{bubble} \times \text{wavy line} \tag{14.14}
 \end{aligned}$$

In  $(\mathbf{q}, iq_n)$ -space this is an algebraic equation with the solution

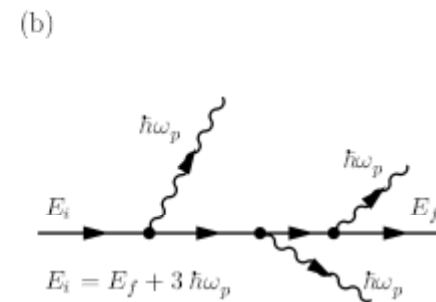
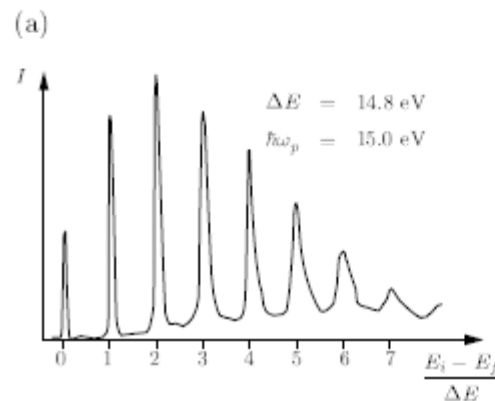
$$-W^{\text{RPA}}(\mathbf{q}, iq_n) = \text{wavy line} = \frac{\text{wavy line}}{1 - \text{bubble}} = \frac{-W(\mathbf{q})}{1 - W(\mathbf{q}) \chi_0(\mathbf{q}, iq_n)}. \tag{14.15}$$

$$\text{Re } \chi_0^R(\mathbf{q}, \omega) = \frac{n}{m} \frac{q^2}{\omega^2} \left[ 1 + \frac{3}{5} \left( \frac{qv_F}{\omega} \right)^2 \right], \quad \epsilon^{\text{RPA}}(\mathbf{q}, \omega) = 1 - \frac{\omega_p^2}{\omega^2} \left[ 1 + \frac{3}{5} \left( \frac{qv_F}{\omega} \right)^2 \right],$$



# Plasmons

$$\epsilon^{\text{RPA}}(\mathbf{q}, \omega) = 0 \Rightarrow \omega^2 \approx \omega_p^2 + \frac{3}{5} (qv_F)^2 \Rightarrow \omega(q) \approx \omega_p + \frac{3}{10} \frac{v_F^2}{\omega_p} q^2.$$

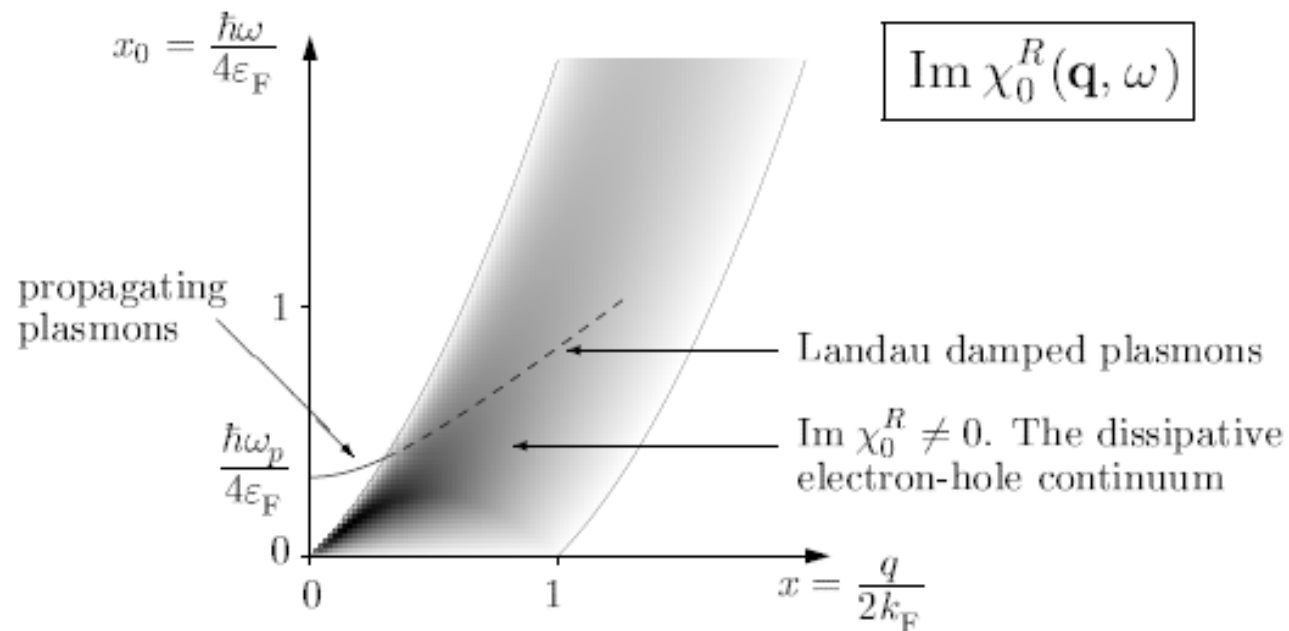


Ocorrem oscilações para  $\text{Re}(\epsilon)$  aprox zero

$$\text{Im } \chi_0^R(\mathbf{q}, \omega) = -d(\epsilon_F) \begin{cases} \frac{\pi}{8x} \left[ 1 - \left( \frac{x_0}{x} - x \right)^2 \right], & \text{for } |x - x^2| < x_0 < x + x^2 \\ \frac{\pi}{2} \frac{x_0}{x}, & \text{for } 0 < x_0 < x - x^2 \\ 0, & \text{for other } x_0 \geq 0. \end{cases}$$



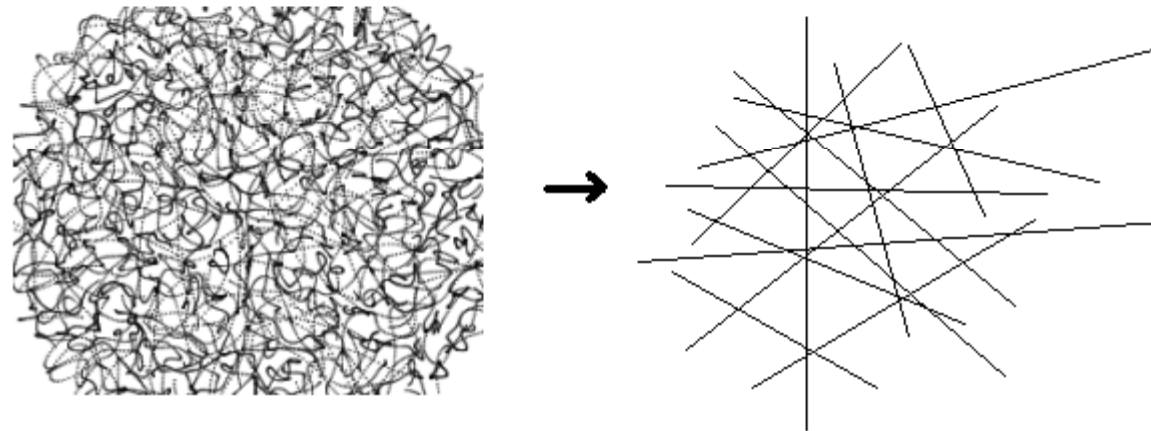
# Landau Damping





# Quase-particulas

$$H_\zeta = H_0 + H_{\text{int}} e^{-\zeta|t|}$$



$$\tau_{\text{life}}^{-1} \ll \zeta \ll k_B T.$$



# Método semi-clasico de Landau

Considerando uma distribuição de Fermi de não equilíbrio mas que conserva a quantidade total de partículas.

$$\dot{n}_{\mathbf{k}} + \nabla_r \dot{\mathbf{j}}_{\mathbf{k}} = \partial_t(n_{\mathbf{k}}) + \dot{\mathbf{k}} \cdot \nabla_{\mathbf{k}} n_{\mathbf{k}} + v_{\mathbf{k}} \cdot \nabla_r n_{\mathbf{k}} = 0$$

$$\varepsilon(\mathbf{q}) \approx 1 - \frac{W(\mathbf{q})}{\omega^2} \frac{2}{V} \sum_{\mathbf{k}} (\mathbf{q} \cdot \mathbf{v}_{\mathbf{k}})^2 \left( -\frac{\partial n_F(\xi_{\mathbf{k}})}{\partial \xi_{\mathbf{k}}} \right) = 1 - \left( \frac{\omega_p}{\omega} \right)^2$$



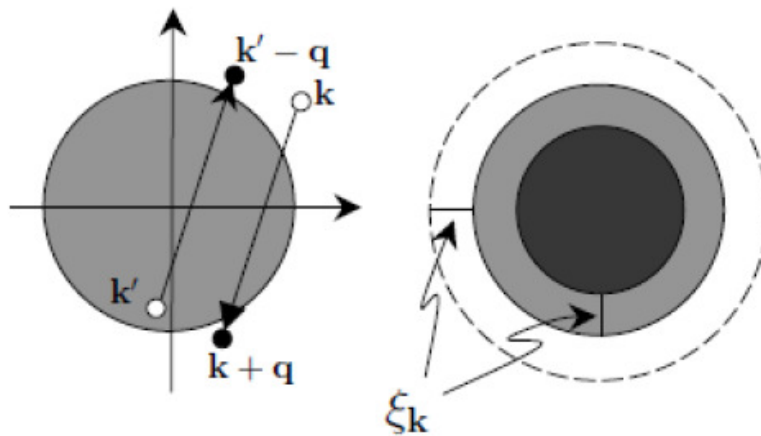


# Tempo de vida medio

A transição pode-se modelar por a regra de ouro de Fermi

$$\frac{1}{\tau_k} = \sum_{k', q} |\langle k + q, k' - q | W(q) | k', k \rangle|^2 \delta(\xi_k + \xi_{k'} - \xi_{k+q} - \xi_{k'-q})$$

Maximum phase space =  $(4\pi k_F^2 \xi_k)^2$



$$\frac{1}{\tau_k} \sim |W|^2 [D(\epsilon_F)]^3 \xi_k^2 / 2$$

$$\frac{1}{\tau} \propto T^2$$



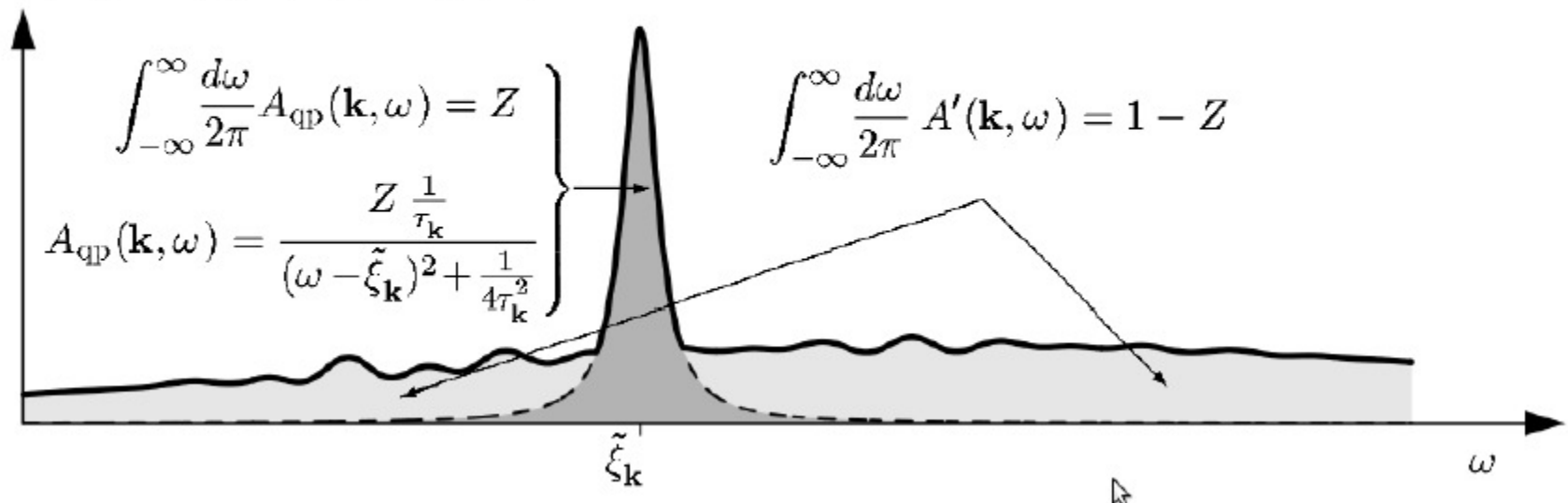
# Função de Green renormalizada

$$G^R(k, \omega) = \frac{1}{\omega - \xi_k - \Sigma^R(k, \omega)}$$

$$\frac{1}{\tilde{\tau}_k(\omega)} = -2Z \text{Im} \Sigma^R$$

$$A(k, \omega) = -2 \text{Im} G^R(k, \omega) \approx 2\pi Z \delta(\omega - \tilde{\xi}_k)$$

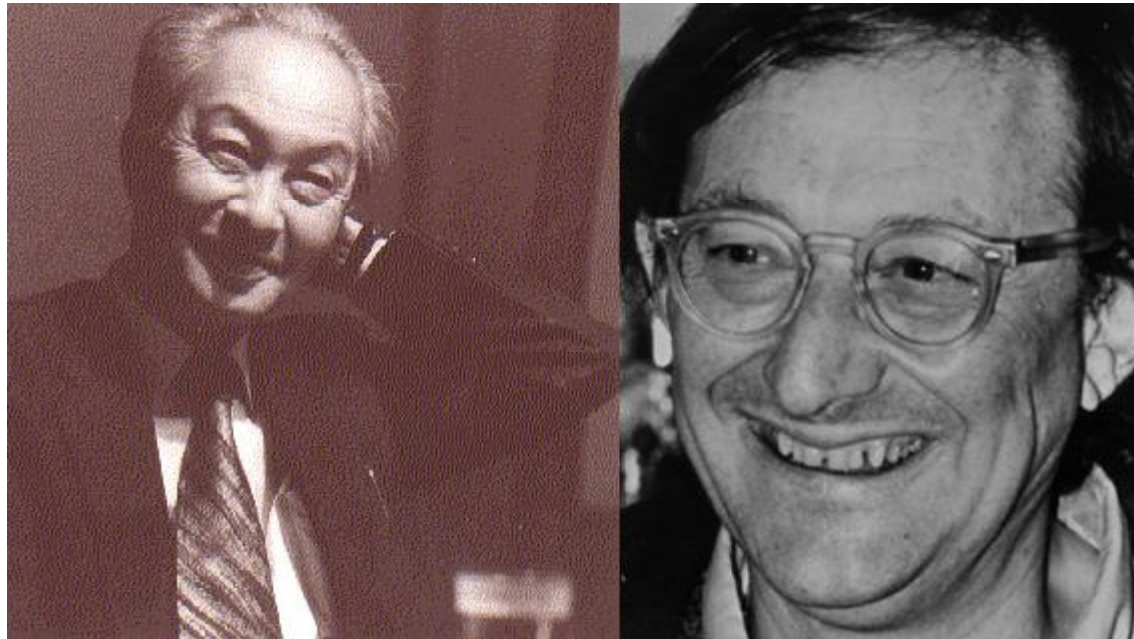
$$A(\mathbf{k}, \omega) = A_{\text{qp}}(\mathbf{k}, \omega) + A'(\mathbf{k}, \omega)$$



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# LIQUIDO EM 1 DIMENSÃO



1950

1963



# Possíveis candidatos

Figure 1: Atomic arrangement of self-organized gold chains on Ge(001).

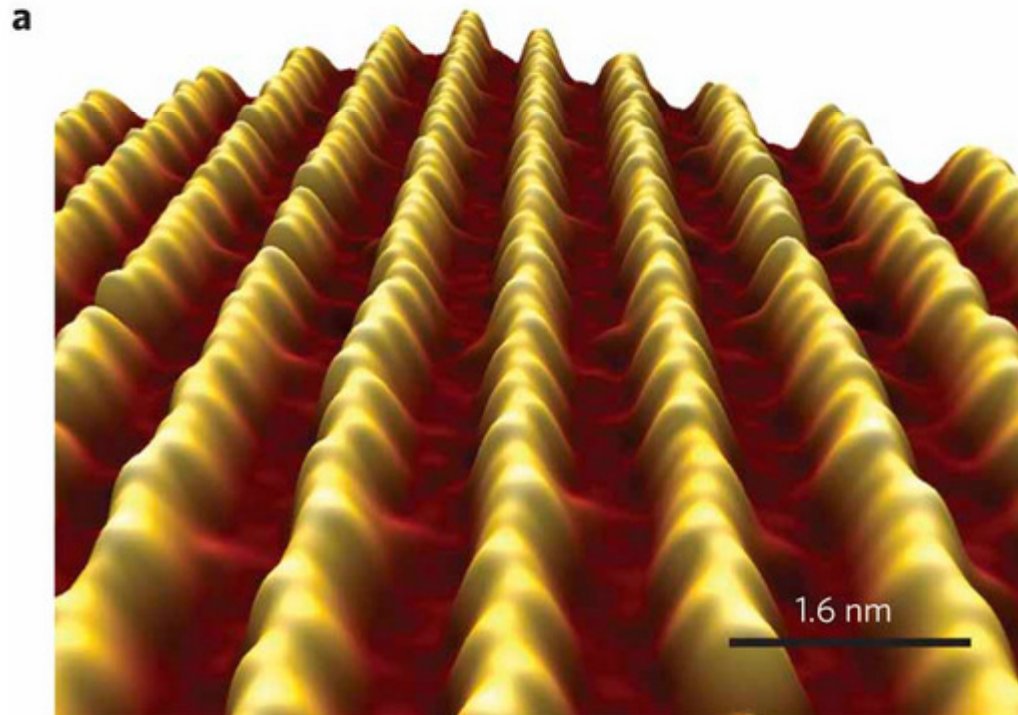
From

Atomically controlled quantum chains hosting a Tomonaga–Luttinger liquid

C. Blumenstein, J. Schäfer, S. Mietke, S. Meyer, A. Dollinger, M. Lochner, X. Y. Cui, L. Patthey, R. Matzdorf & R. Claessen

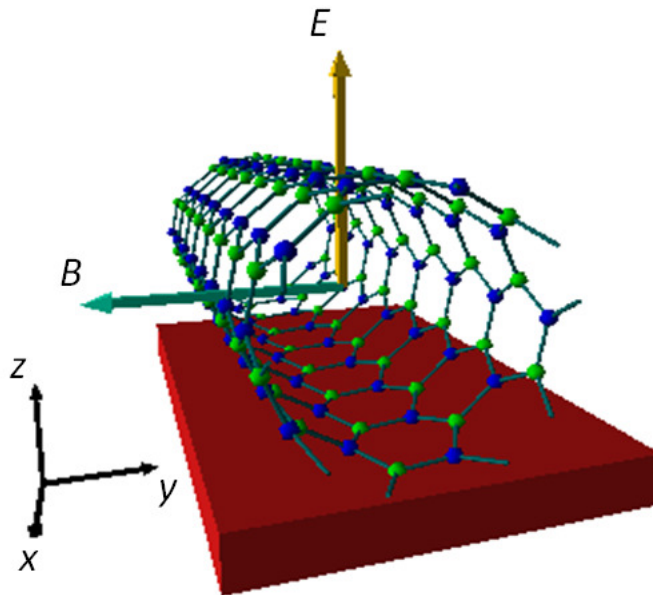
*Nature Physics* 7, 776–780 (2011) | doi:10.1038/nphys2051

Figure 1: Atomic arrangement of self-organized gold chains on Ge(001).





# Carbon nanotubes



## Luttinger-liquid behaviour in carbon nanotubes

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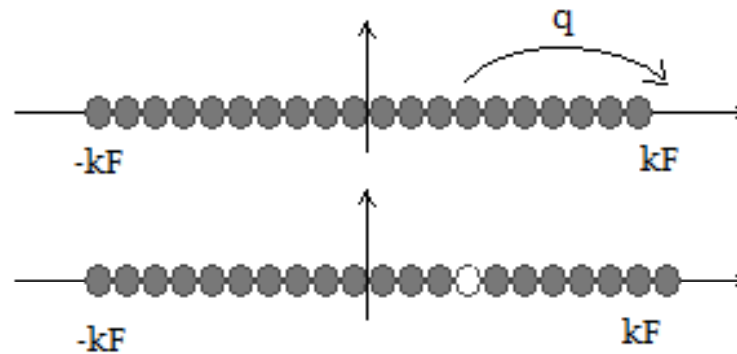
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Electron transport in conductors is usually well described by Fermi-liquid theory, which assumes that the energy states of the electrons near the Fermi level  $E_F$  are not qualitatively altered by Coulomb interactions. In one-dimensional systems, however, even weak Coulomb interactions cause strong perturbations.





# Excitações em 1D



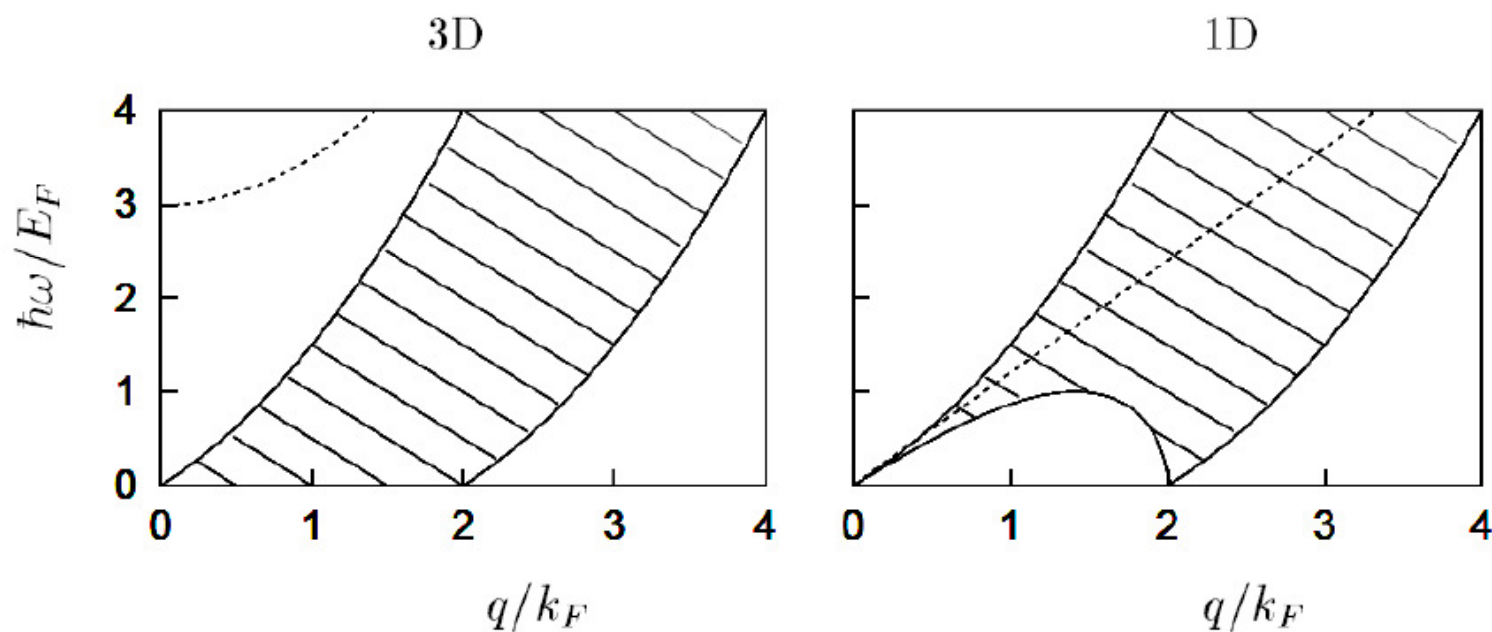
As excitações são sempre coletivas.  
Um operador de partícula única não faz sentido

$$\rho_R(q) = \sum_{k>0} c_{k+q}^\dagger c_k$$
$$\rho_L(q) = \sum_{k<0} c_{k+q}^\dagger c_k$$



# Considerações

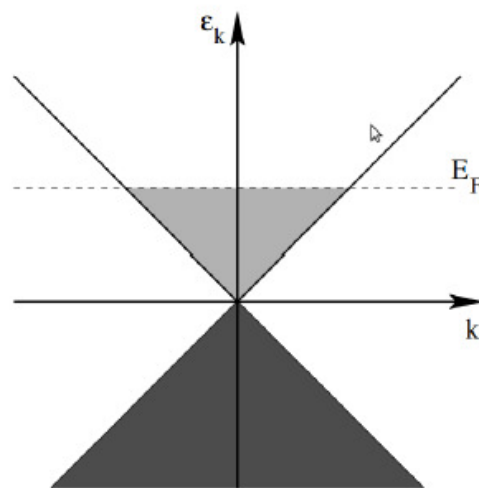
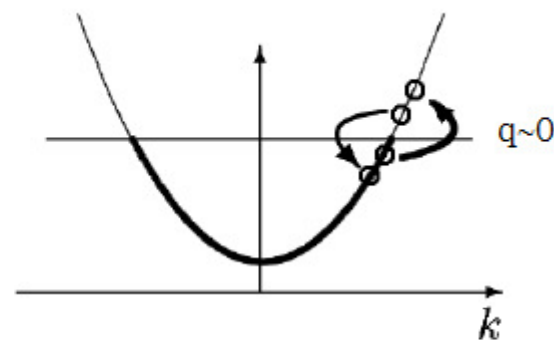
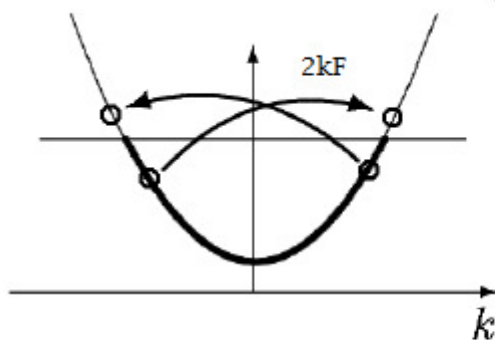
$$\omega_{pl} \propto q.$$



$$\tau^{-1} \propto \xi k$$



# Linearização



$$\rho_R(q) = \sum_{k>0} c_{k+q}^\dagger c_k$$

$$\rho_L(q) = \sum_{k<0} c_{k+q}^\dagger c_k$$



# Construção do hamiltoniano



$$[\rho_r(q), \rho_{r'}(-q')] = 2 \sum_k (n_r(k) - n_r(k+q)) \approx \delta_{r,r'} \delta_{q,q'} \frac{rLq}{2\pi}$$

$$[H_0, \rho_r(q)] = -rqv_F \rho(q)$$

$$H_0 = \frac{2\pi v_F}{L} \sum_{q>0} [\rho_R(-q)\rho_R(q) + \rho_L(q)\rho_L(-q)]$$

$$H_{I,1} = \frac{1}{2L} \sum_q V_q (\rho_R(q) + \rho_L(q)) (\rho_R(-q) + \rho_L(-q))$$

$$H_{I,2} = -\frac{1}{2L} \sum_q V_{2k_F} (\rho_R(-q)\rho_L(q) + \rho_R(q)\rho_L(-q))$$



# “Bosonização”

$$b_{R,q} = \rho_R(q) \sqrt{\frac{2\pi}{qL}}, \quad b_{R,q}^\dagger = \rho_R(-q) \sqrt{\frac{2\pi}{qL}}$$
$$b_{L,q} = \rho_L(-q) \sqrt{\frac{2\pi}{qL}}, \quad b_{L,q}^\dagger = \rho_L(q) \sqrt{\frac{2\pi}{qL}}$$

Fazendo uma transformação de Bogoliunov

$$E_q = q\tilde{v}, \quad \tilde{v} = v_F \sqrt{1 + \frac{V_0 - V_{2k_F}}{\pi v_F}}$$



# Formulação no espaço real

$$[\hat{A}(x), \rho_{x'}] = i\delta(x - x') \quad e^{i\hat{A}(x_0)}|0\rangle = |x_0\rangle$$

$$\rho_r(x) = \frac{1}{L} \sum_q e^{iqx} \rho_r(q)$$

$$[\rho_r(x), \rho'_r(x')] = \frac{r}{2\pi i} \delta_{rr'} \frac{\partial}{\partial x} \delta(x - x')$$

$$A(x) = \int_{-\infty}^x \rho_r(x') dx'$$

$$\psi_r(x) \propto U_r e^{\int_{-\infty}^x \rho(x') dx'}$$



# Separação Carga-espín

$$H_0 = \frac{2\pi v_F}{L} \sum_{q>0,s} [\rho_{R,s}(-q)\rho_{R,s}(q) + \rho_{L,s}(q)\rho_{L,s}(-q)]$$

$$H_{int} = \frac{V}{L} \sum_q \sum_{s,s'} [(\rho_{R,s}(q) + \rho_{L,s}(q))(\rho_{R,s'}(-q)\rho_{L,s'}(-q))]$$

Com a transformação

$$\rho = \rho_{\downarrow} + \rho_{\uparrow}, \quad \sigma = \rho_{\downarrow} - \rho_{\uparrow},$$

$$H_0 = \frac{2\pi v_F}{L} \sum_q [\rho_R(-q)\rho_R(q) + \rho_L(q)\rho_L(-q) + \sigma_R(-q)\sigma_R(q) + \sigma_L(q)\sigma_L(-q)]$$

$$H_{int} = \frac{V}{L} \sum_q [(\rho_R(q) + \rho_L(q))(\rho_R(-q)\rho_L(-q))]$$

$$v_F^2 = v_{\sigma} v_{\rho}$$

$\pi$



OBRIGADO