



Instituto de Física  
Universidade de São Paulo

# EFEITO HALL QUÂNTICO DE SPIN

Carlos Augusto Mera Acosta

*Teoria quântica de Muitos Corpos*

*Prof. Luis Gregorio Dias*

*Departamento de Física dos Materiais*

*Instituto de Física*

*Universidade de São Paulo*

2013

## 1 INTRODUÇÃO

## 2 EFEITO HALL QUÂNTICO

- Fenomenologia
- Estados topológicos na matéria
- Modelo de Hofstadter

## 3 EFEITO HALL QUÂNTICO DE SPIN

- Fenomenologia
- Modelo de Haldane
- Modelo de Kane e Mele

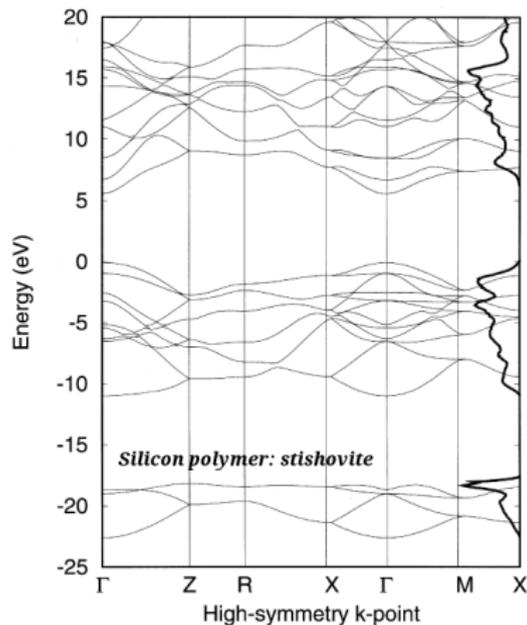
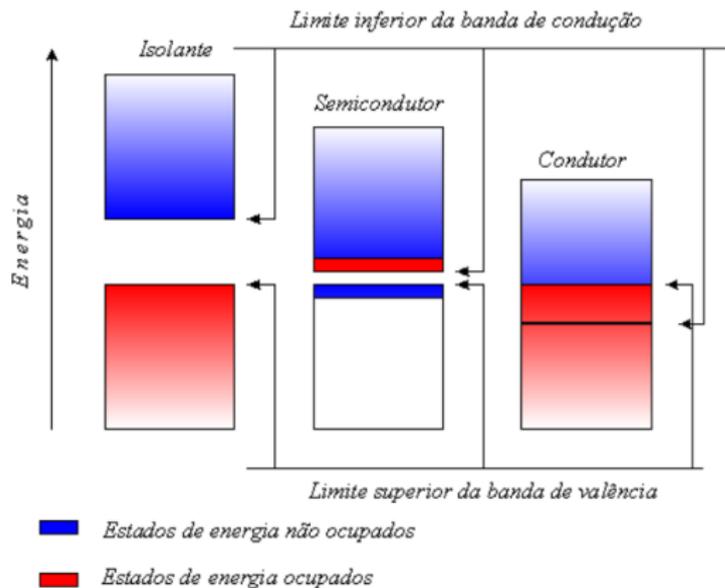
## QUEBRA ESPONTÂNEA DE SIMETRIA - CLASSIFICAÇÃO DA MATÉRIA

## “The broken symmetry principle”

TABELA: Exemplos de quebras de simetria.

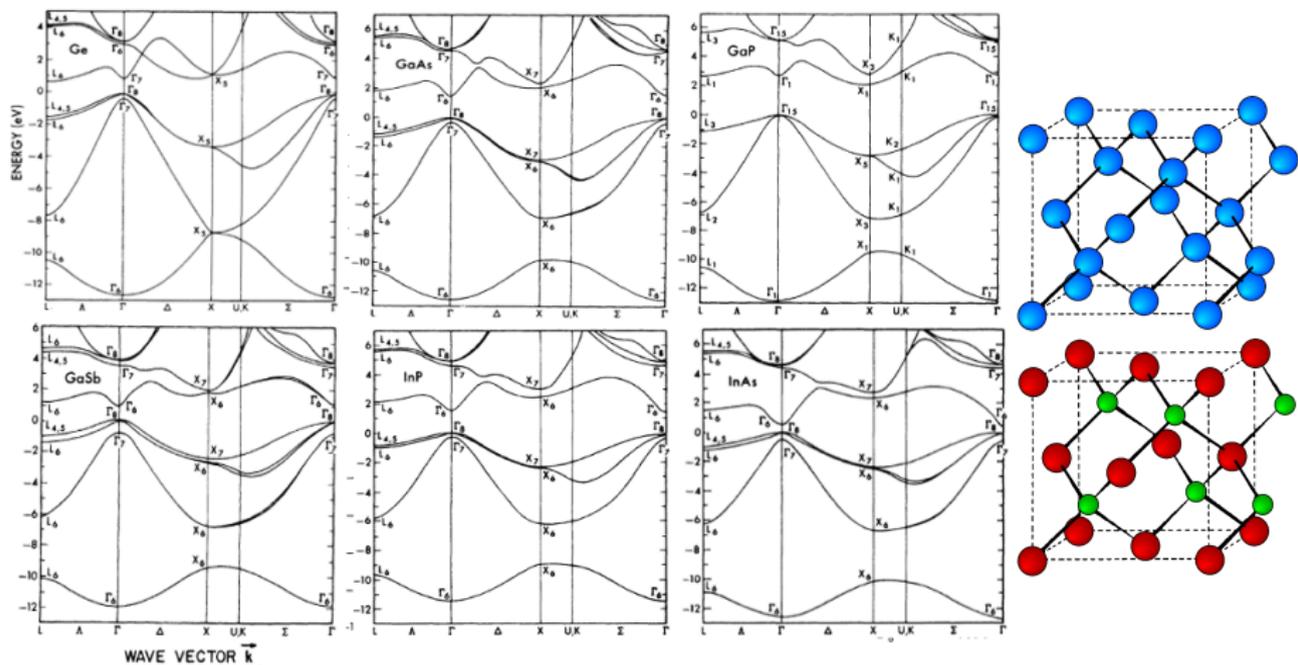
Fenômeno	Parâmetro de ordem	
Cristal	“Onda de densidade”	$\sum_k \langle c_k^\dagger c_{k+G} \rangle$
Ferromagneto	Magnetização	$\sum_k (c_{k\uparrow}^\dagger c_{k\uparrow} - c_{k\downarrow}^\dagger c_{k\downarrow})$
Condensado de Bose-Einstein	População do estado $k = 0$	$\langle a_{k=0}^\dagger \rangle$
Supercondutor	Correlação de pares de Cooper	$\langle c_{k\uparrow} c_{-k\downarrow} \rangle$

## CRISTAL



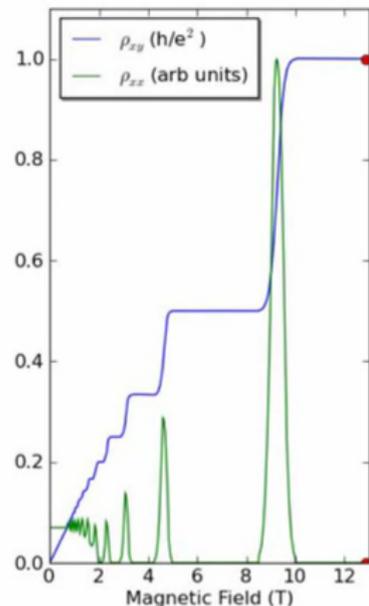
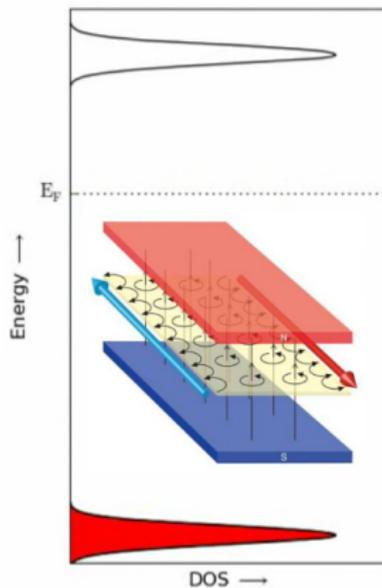
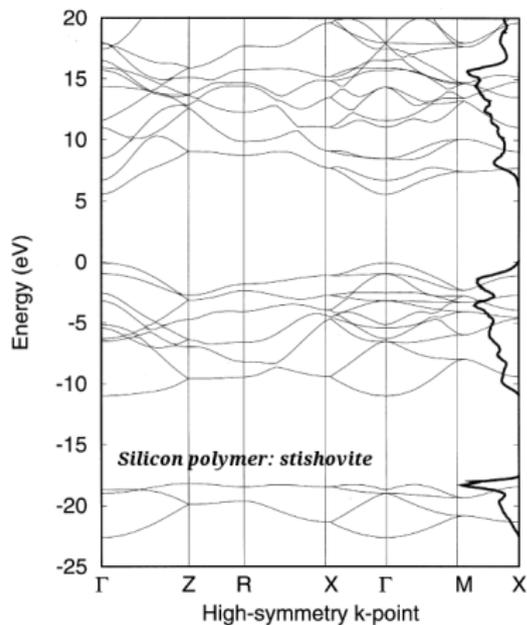
◇ Jose R. Alvarez e Peter Rez. **Solid State Communications**, Vol. 108, No. 1, 37. (1998).

## SEMICONdutoRES ZINC BLENDE



◇ James R. Chelikowsky and Marvin L. Cohen *PRB* Vol. 14, No. 556, (1976).

# EFEITO HALL QUÂNTICO



- ◇ D. J. Thouless *et al.*, **Phys. Rev. Lett** **49**, 405 (1982).
- ◇ Y. G. Yao *et al.*, **Phys. Rev. Lett** **92**, 037204 (2004).
- ◇ Checkelsky J. G. *et al.*, **Nature Physics** **8**, 729–733. (2012).

VOLUME 45, NUMBER 6

PHYSICAL REVIEW LETTERS

11 AUGUST 1980

---

## **New Method for High-Accuracy Determination of the Fine-Structure Constant Based on Quantized Hall Resistance**

**K. v. Klitzing**

*Physikalisches Institut der Universität Würzburg, D-8700 Würzburg, Federal Republic of Germany, and  
Hochfeld-Magnetlabor des Max-Planck-Instituts für Festkörperforschung, F-38042 Grenoble, France*

and

**G. Dorda**

*Forschungslaboratorien der Siemens AG, D-8000 München, Federal Republic of Germany*

and

**M. Pepper**

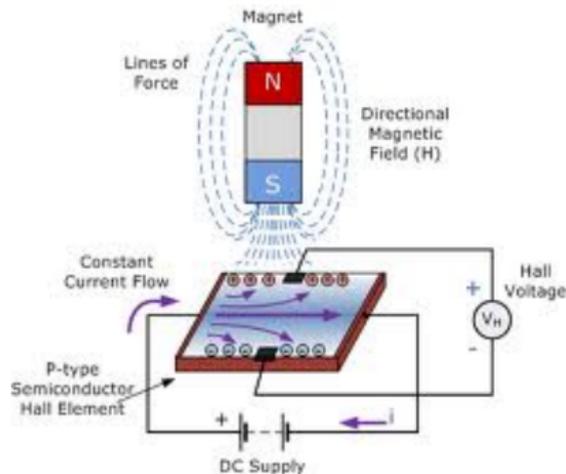
*Cavendish Laboratory, Cambridge CB3 0HE, United Kingdom*

(Received 30 May 1980)

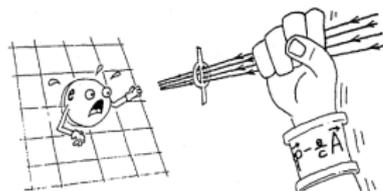
# OUTLINE

- 1 INTRODUÇÃO
- 2 EFEITO HALL QUÂNTICO
  - Fenomenologia
  - Estados topológicos na matéria
  - Modelo de Hofstadter
- 3 EFEITO HALL QUÂNTICO DE SPIN
  - Fenomenologia
  - Modelo de Haldane
  - Modelo de Kane e Mele

## EFETIO HALL QUÂNTICO



$$R_H = \frac{V_H}{I} = \frac{B}{\rho e c}$$



Explicação teórica: Electrodinâmica Clássica

$$\frac{d\vec{p}}{dt} = \vec{F} = q\vec{E} + \frac{qB}{c}(v_y\hat{i} + v_x\hat{j})$$

$$E_y = \frac{B}{c}v_x \Rightarrow E_y = \frac{B}{c}J_x \left( \frac{1}{e\rho} \right)$$

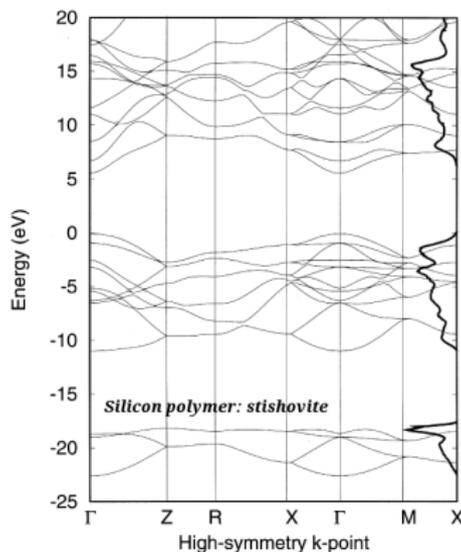
$$\frac{E_y}{J_x} = \frac{V_H}{I} = \frac{B}{\rho e c}$$

Aspectos geométricos:

$$J_i = \sigma_{ij}E_j,$$

sendo  $\sigma_{xx} = \sigma_{yy} = 0$  e  $\sigma_{yx} = -\sigma_{xy} = \frac{ec\rho}{B}$

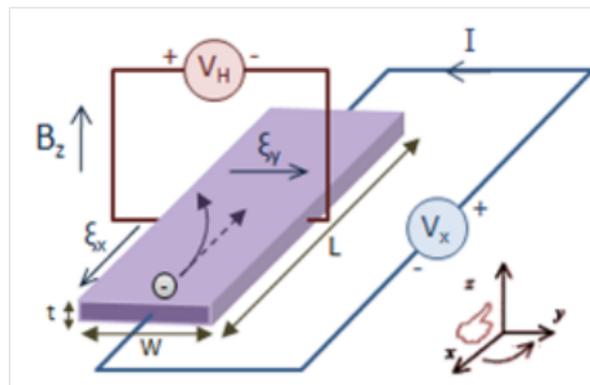
## EFEITO HALL QUÂNTICO



$$\left( \sum_{i=1}^3 \frac{P_i \hat{e}_i}{(2m_i)^{1/2}} \right)^2 \psi = E \psi$$

$$E = \frac{\hbar^2 k_x^2}{2m_1} + \frac{\hbar^2 k_y^2}{2m_2} + \frac{\hbar^2 k_z^2}{2m_3}.$$

- ◇ Jose R. Alvarez e Peter Rez. **Solid State Communications**, Vol. 108, No. 1, 37. (1998).
- ◇ Checkelsky J. G. et al., **Nature Physics** 8, 729–733. (2012).



$$P \rightarrow (P + eA); \quad A = (0, Bx, 0)$$

$$\left( \frac{P_x^2}{2m_1} + \frac{(\hbar k_y + eBx)^2}{2m_2} + \frac{\hbar^2 k_z^2}{2m_3} \right) \psi = E \psi$$

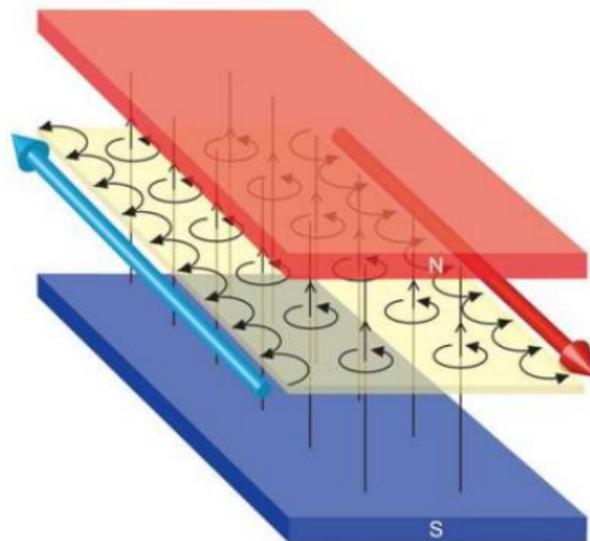
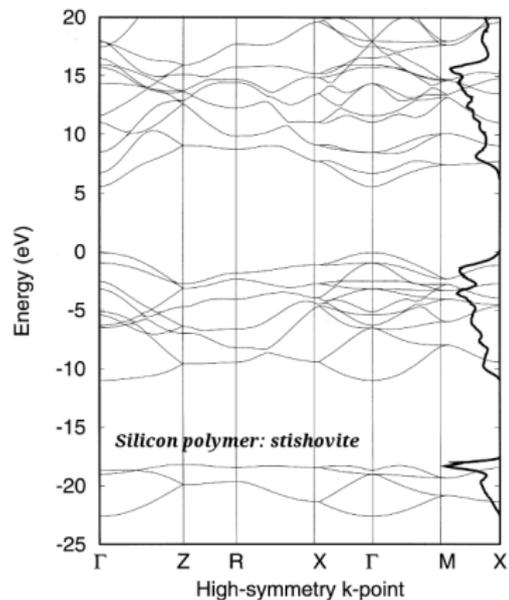
$$\text{Fazendo } E' = E - \left( \frac{\hbar^2 k_z^2}{2m_3} \right) \text{ e } x_0 = - \left( \frac{\hbar k_y}{eB} \right):$$

$$\left( \frac{P_x^2}{2m} + \frac{m\omega_c^2}{2} (x - x_0)^2 \right) \phi = E' \phi$$

$$\text{Oscilador harmônico com } \omega_c = \frac{eB}{(m_1 m_2)^{1/2}}.$$

$$E(n, B, l) = \frac{\hbar^2 k_z^2}{2m_3} + \left( n + \frac{1}{2} \right) \hbar \omega_c$$

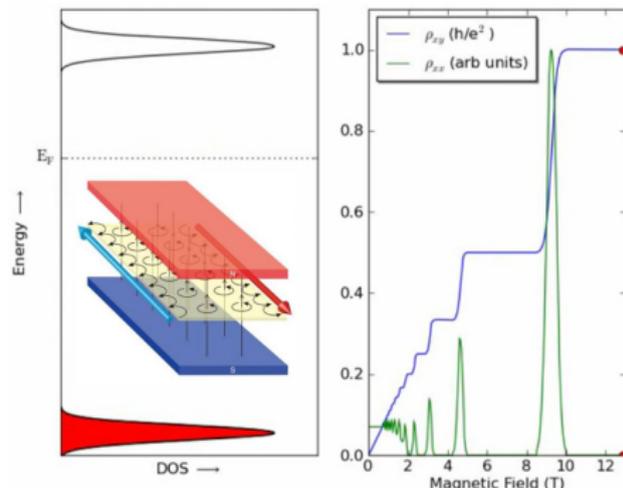
## EFEITO HALL QUÂNTICO



- ◇ Jose R. Alvarez e Peter Rez. **Solid State Communications**, Vol. 108, No. 1, 37. (1998).
- ◇ Checkelsky J. G. et al., **Nature Physics** 8, 729–733. (2012).

## INTERPRETAÇÃO DO EFEITO HALL QUÂNTICO

- ◇ Comprimento magnético  $l_c = \left(\frac{\hbar c}{eB}\right)^{\frac{1}{2}}$ .
- ◇ Se imaginamos os estados de Landau como pequenos discos de área  $2\pi l_c^2$ , em uma área  $A$  da amostra temos  $\frac{A}{2\pi l_c^2}$  estados disponíveis.
- ◇ Degenerescência em cada nível de Landau  $n$ ;  $\rho_L = \frac{1}{2\pi l_c^2} = \frac{eB}{\hbar c}$ .
- ◇ Fator de preenchimento dos níveis de Landau:  $\eta = \frac{\rho}{\rho_L}$ .



Um valor inteiro de  $\eta$  corresponde a um preenchimento completo. Dessa forma:

$$R_H = \frac{B}{\rho e c} = \frac{\hbar \rho_L}{e^2 \rho}$$

INTERPRETAÇÃO!!!

$$R_H = \frac{\hbar}{e^2 \eta} : \quad \sigma_{xy} = \frac{\eta e^2}{\hbar}$$

# OUTLINE

## 1 INTRODUÇÃO

## 2 EFEITO HALL QUÂNTICO

- Fenomenologia
- Estados topológicos na matéria
- Modelo de Hofstadter

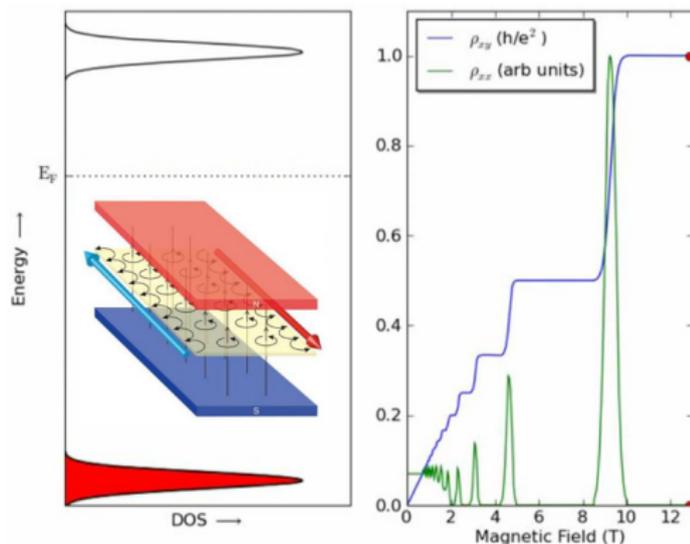
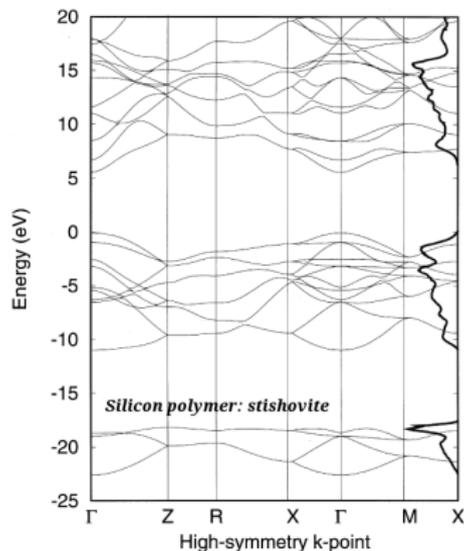
## 3 EFEITO HALL QUÂNTICO DE SPIN

- Fenomenologia
- Modelo de Haldane
- Modelo de Kane e Mele

## ESTADOS TOPOLÓGICOS NA MATÉRIA

## Quantized Hall Conductance in a Two-Dimensional Periodic Potential

D. J. Thouless, M. Kohmoto,<sup>(a)</sup> M. P. Nightingale, and M. den Nijs  
 Department of Physics, University of Washington, Seattle, Washington 98195  
 (Received 30 April 1982)



$$C = \frac{1}{2\pi} \sum_n \int_{BZ} d^2k \Omega_n; \quad \Omega_n(k) = - \sum_{n' \neq n} \frac{2 \text{Im} \langle \psi_{nk} | v_x | \psi_{n'k} \rangle \langle \psi_{n'k} | v_y | \psi_{nk} \rangle}{(\omega_{n'} - \omega_n)^2},$$

- ◇ D. J. Thouless *et al.*, **Phys. Rev. Lett** **49**, 405 (1982).
- ◇ Y. G. Yao *et al.*, **Phys. Rev. Lett** **92**, 037204 (2004).
- ◇ Checkelsky J. G. *et al.*, **Nature Physics** **8**, 729–733. (2012).

# RELAÇÃO DE UM OBSERVÁVEL COM UM INVARIANTE TOPOLÓGICO

## QUANTIZAÇÃO DA CONDUCTIVIDADE HALL

$$\sigma_{xy} = \frac{e^2}{\hbar} \times \text{integer}$$

$$\mathcal{A}_\alpha = i \langle u_{k\alpha} | \nabla_{\mathbf{k}} | u_{k\alpha} \rangle \quad \mathcal{F}_\alpha(\mathbf{k}) = \nabla \times \mathcal{A}_\alpha$$

Berry connection and Berry curvature

$$\text{Ch}N_\alpha = \int_{\mathbb{T}^2} d\mathbf{k} \mathcal{F}_\alpha(\mathbf{k})$$

$$\text{integer} = \sum_{\alpha \text{ occup}} \text{Ch}N_\alpha$$



$$\int_S K_g dS = 4\pi$$

$$\int_S K_g dS = 2\pi\chi_E = 2\pi(2 - 2g)$$

Euler characteristic  
 $g$  genus (# holes)



$$\int_S K_g dS = 0$$

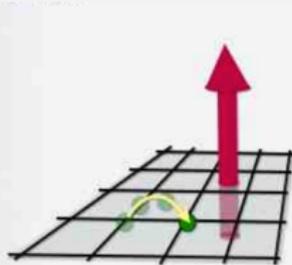


# OUTLINE

- 1 INTRODUÇÃO
- 2 EFEITO HALL QUÂNTICO
  - Fenomenologia
  - Estados topológicos na matéria
  - Modelo de Hofstadter
- 3 EFEITO HALL QUÂNTICO DE SPIN
  - Fenomenologia
  - Modelo de Haldane
  - Modelo de Kane e Mele

# MODELO DE HOFSTADTER

FÉRMIONS EM UMA REDE QUADRADA BIDIMENSIONAL SOB AÇÃO DE UM CAMPO MAGNÉTICO  $B(\mathbf{r})$ .



$$H = -t \sum_{\langle r, r' \rangle} c_{r'}^\dagger c_r + \text{h.c.}$$

↓ R. Peierls '33

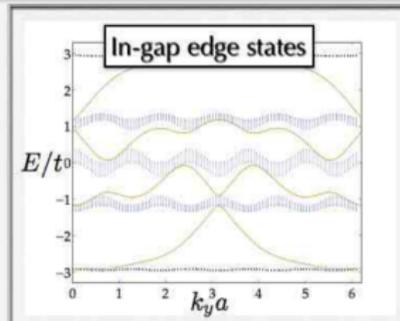
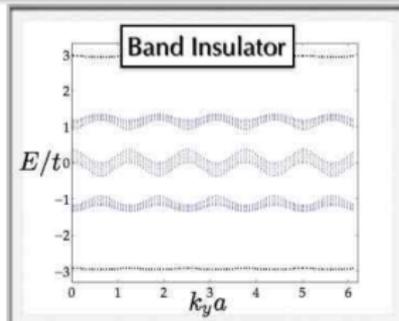
$$H = -t \sum_{\langle r, r' \rangle} c_{r'}^\dagger e^{-i \int A dl} c_r + \text{h.c.}$$

Landau gauge

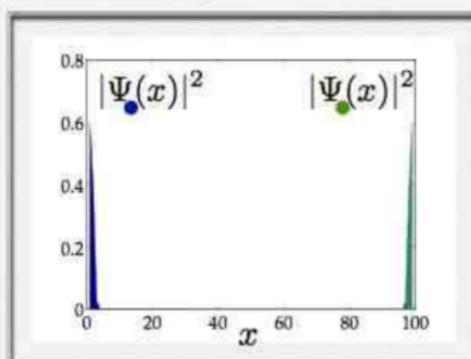
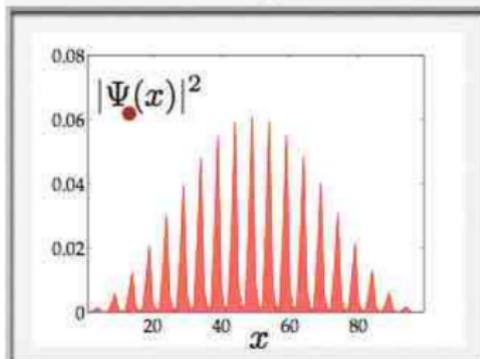
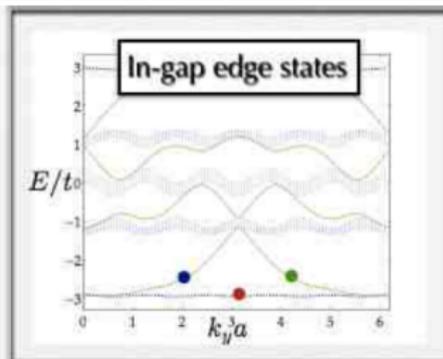


↓  $\Phi \sim \#$  flux quanta per unit cell

$$H = -t \sum_r c_{r+\hat{x}}^\dagger c_r + e^{i\Phi x} c_{r+\hat{y}}^\dagger c_r + \text{h.c.}$$



## ESTADOS DE BORDA



$$\sigma_{xy} = \frac{e^2}{h} \times \text{integer} \xrightarrow[\text{"holographic"}]{\text{bulk-edge}} \text{integer} = \sum_{\text{right edge}} \text{sgn} \left( \frac{d\epsilon_{\text{edge}}(k)}{dk} \right)$$

correspondance

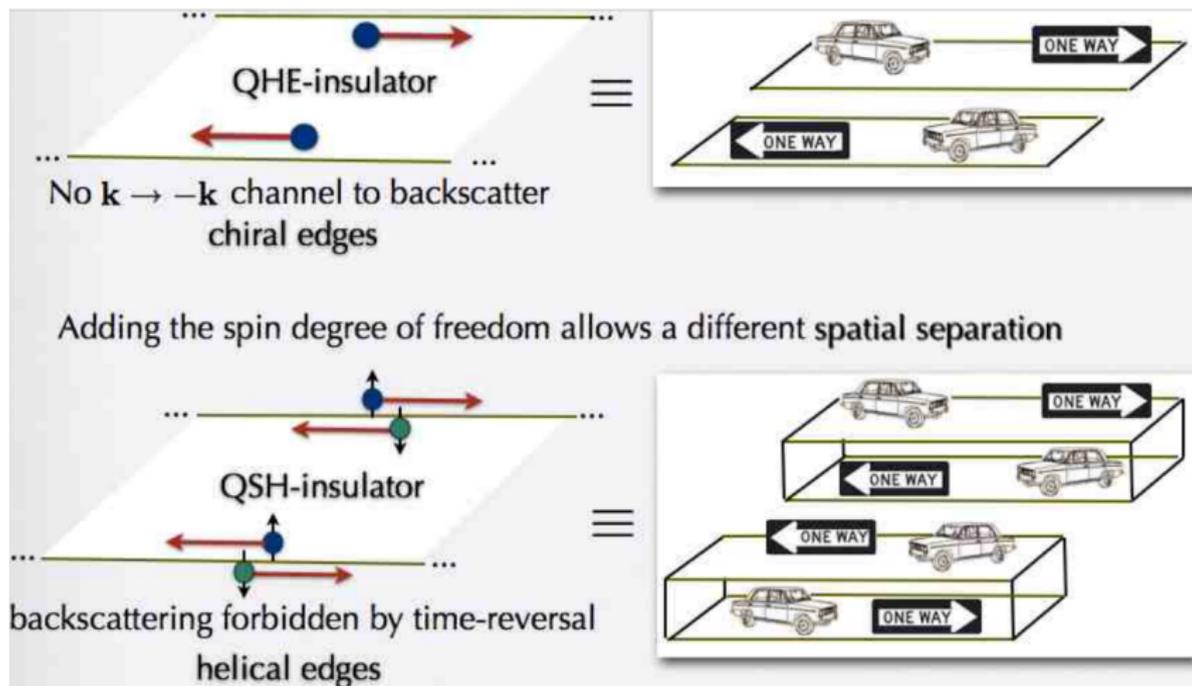
R. Rammal et al. '83 Y. Hatsugai '93



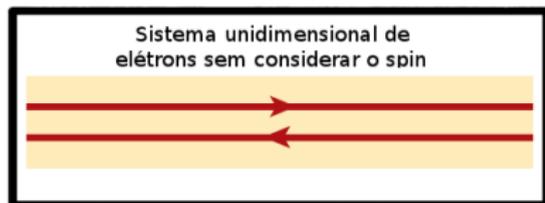
# OUTLINE

- 1 INTRODUÇÃO
- 2 EFEITO HALL QUÂNTICO
  - Fenomenologia
  - Estados topológicos na matéria
  - Modelo de Hofstadter
- 3 EFEITO HALL QUÂNTICO DE SPIN
  - Fenomenologia
  - Modelo de Haldane
  - Modelo de Kane e Mele

## EFEITO HALL QUÂNTICO DE SPIN

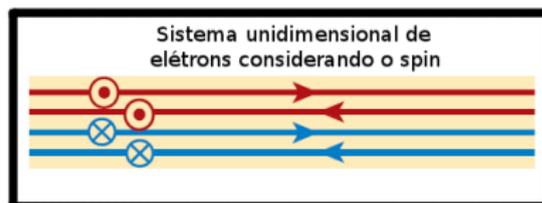
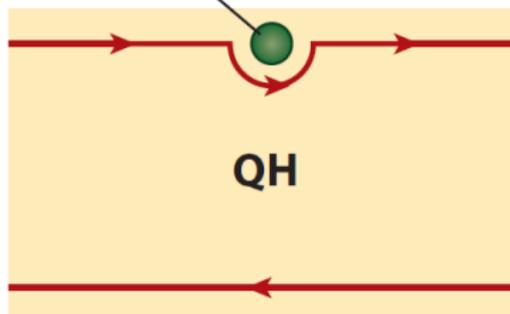


## EFEITO HALL QUÂNTICO DE SPIN

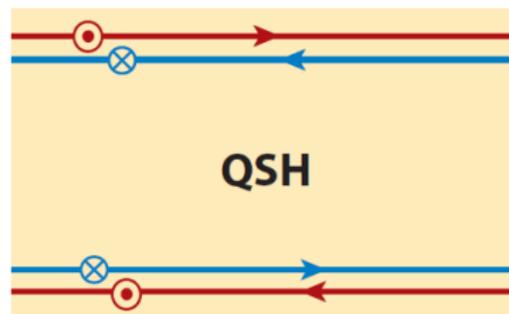


$$2 = 1 + 1$$

Impureza

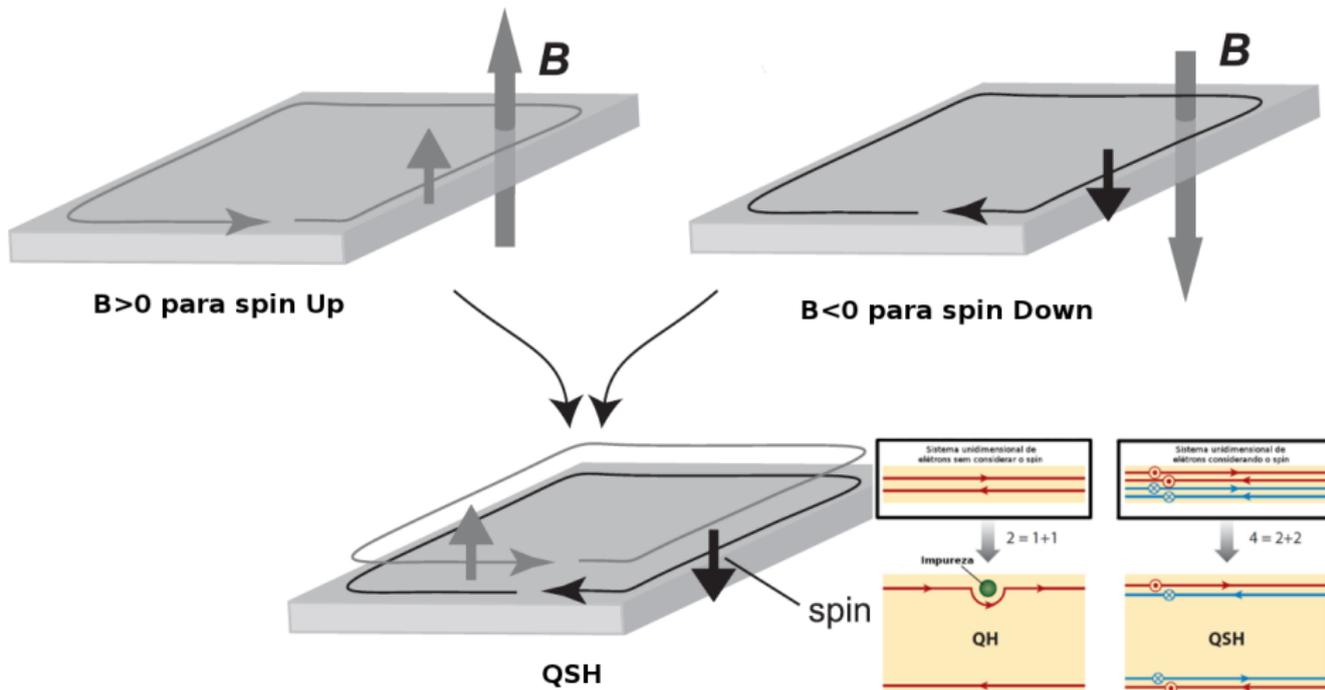


$$4 = 2 + 2$$

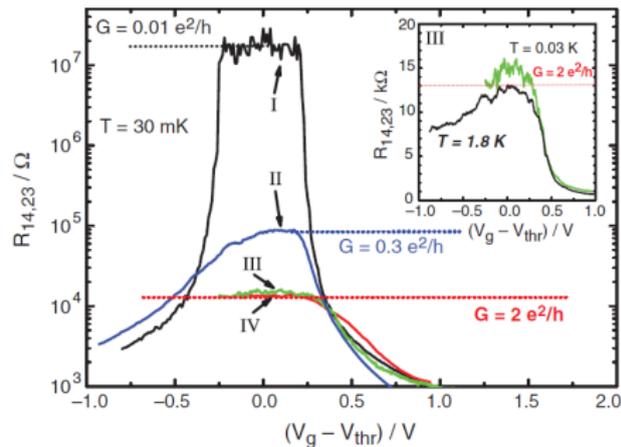
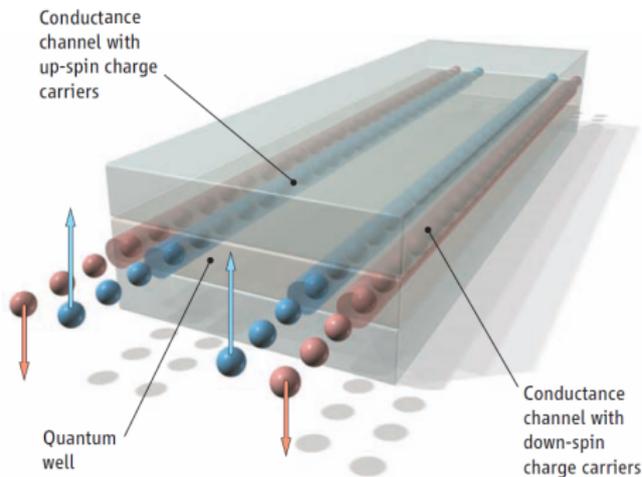


◇ J. Maciejko, T. L. Hughes e S. C. Zhang, *Annu. Rev. Condens. Matter Phys.* **2**, 31-53 (2011).

## EFEITO HALL QUÂNTICO DE SPIN



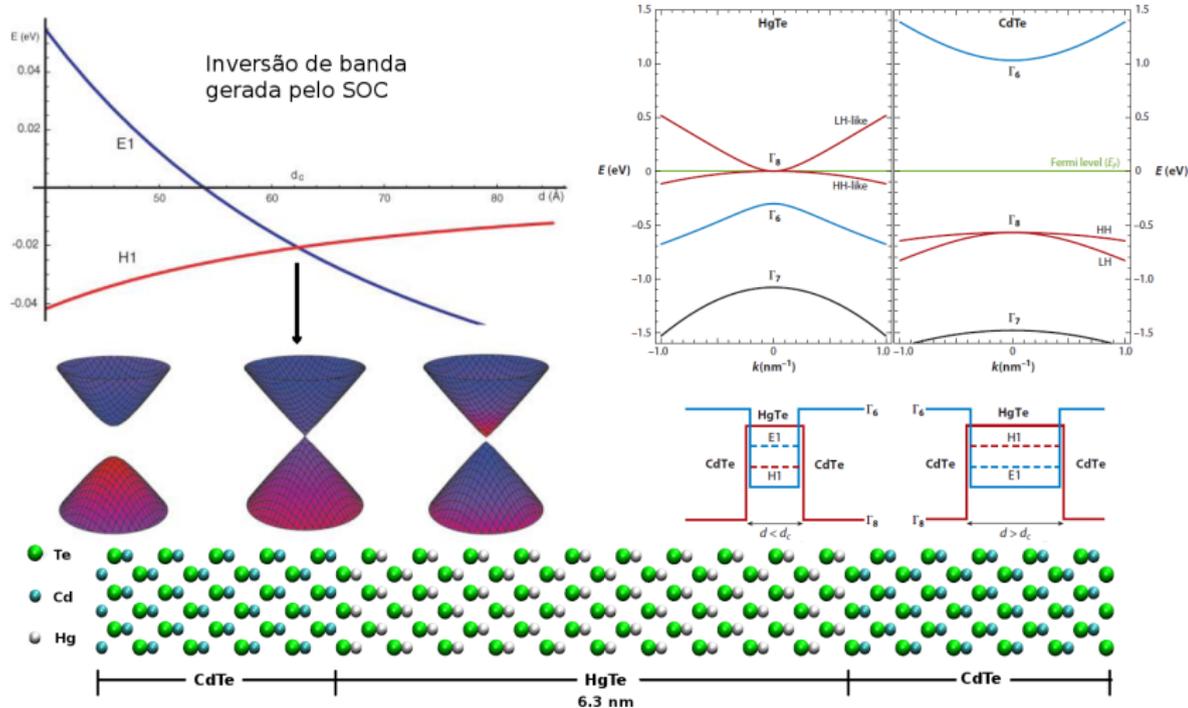
◇ J. Maciejko, T. L. Hughes e S. C. Zhang, *Annu. Rev. Condens. Matter Phys.* **2**, 31-53 (2011).

OBSERVAÇÃO EXPERIMENTAL DO QSH NO POÇO QUÂNTICO DE *HgTe/CdTe*

O acoplamento spin-órbita é fundamental para a descrição destes sistemas!

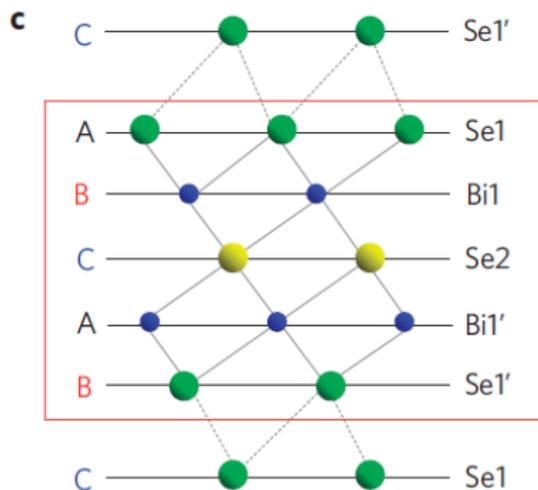
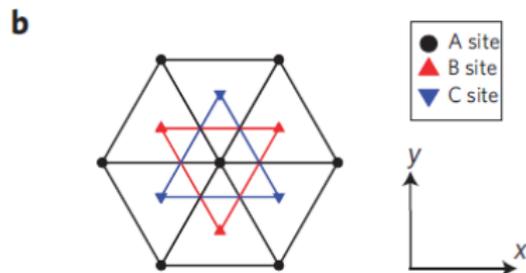
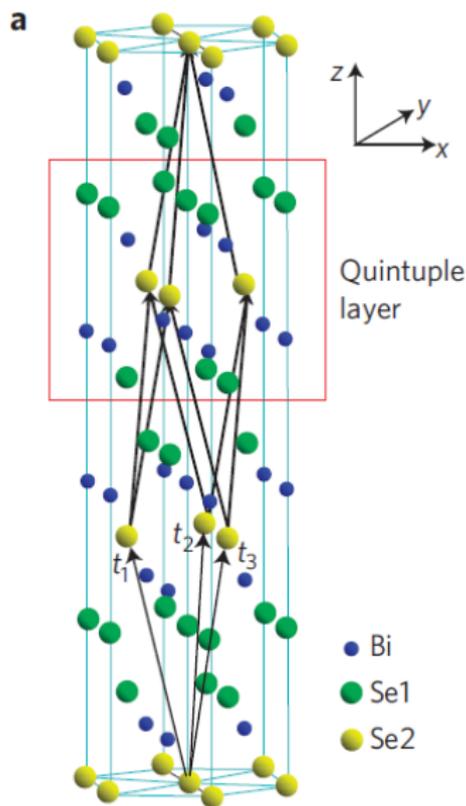
◇ M. König *et al.*, *Science* **318**, 766 (2007).

# PREDIÇÃO TEÓRICA DO QSH NO POÇO QUÂNTICO DE *HgTe/CdTe*

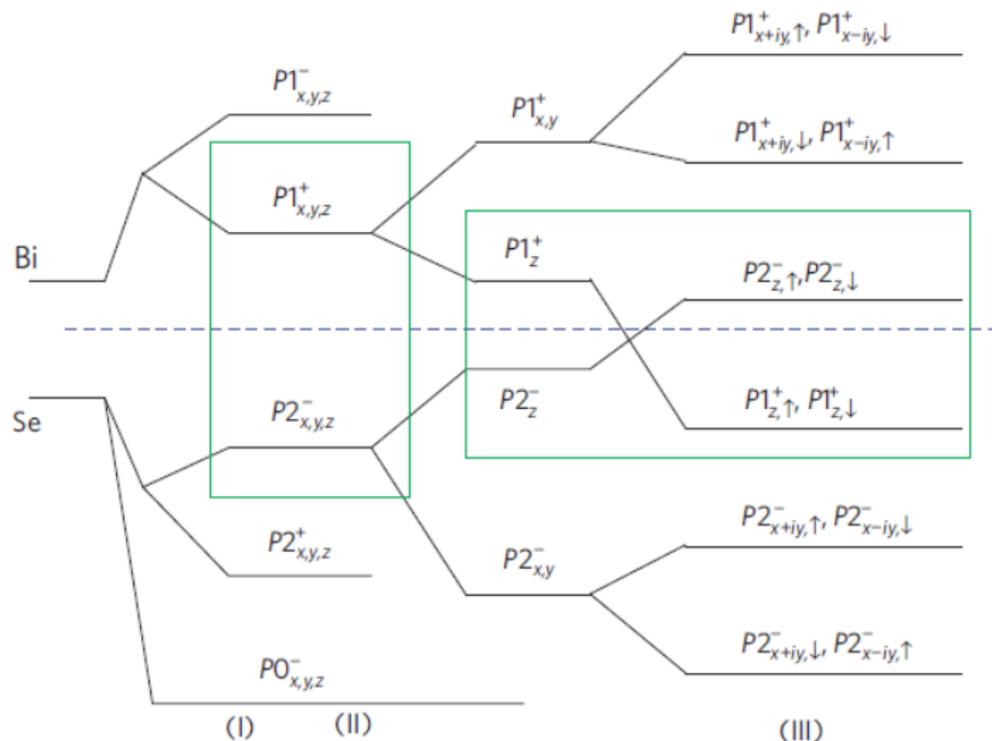


◇ B. A. Bernevig, T. L. Hughes, S. C. Zhang. *Science* **314**, 1757 (2006).

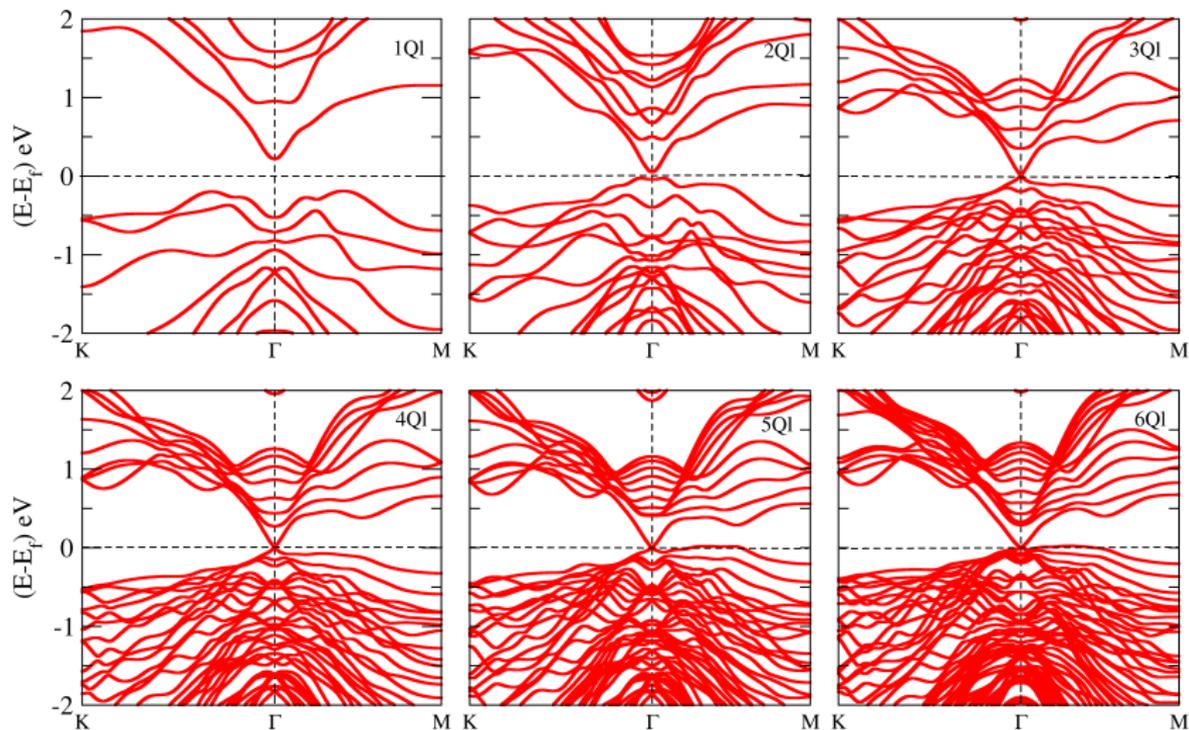
## INVERSÃO DE BANDA



## INVERSÃO DE BANDA



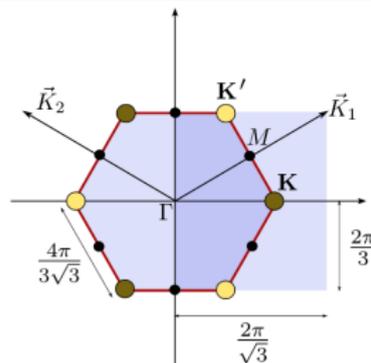
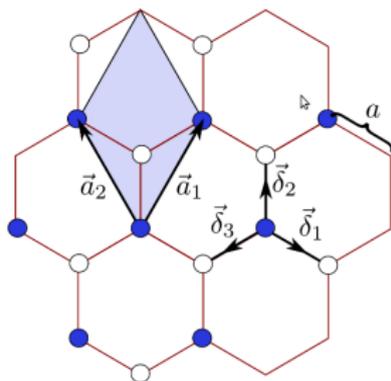
## INVERSÃO DE BANDA



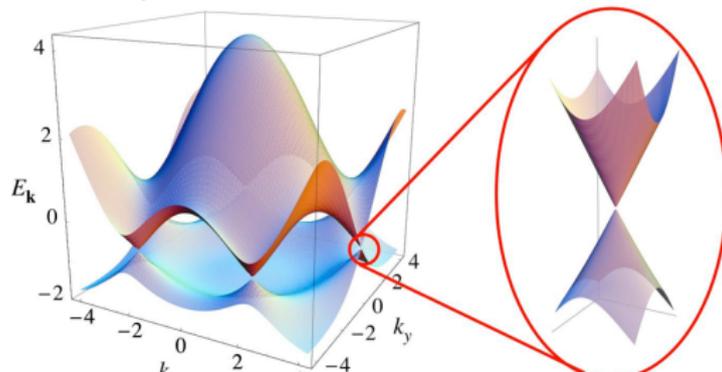
# OUTLINE

- 1 INTRODUÇÃO
- 2 EFEITO HALL QUÂNTICO
  - Fenomenologia
  - Estados topológicos na matéria
  - Modelo de Hofstadter
- 3 EFEITO HALL QUÂNTICO DE SPIN
  - Fenomenologia
  - **Modelo de Haldane**
  - Modelo de Kane e Mele

## GRAFENO



$$H = -t \sum_{\langle i,j \rangle} c_i^\dagger c_j + h.c. \quad \xrightarrow{\mathbf{K}} \quad E_q^\pm = \hbar v_F (\boldsymbol{\sigma} \cdot \mathbf{p})$$



# MODELO DE HALDANE

VOLUME 61, NUMBER 18

PHYSICAL REVIEW LETTERS

31 OCTOBER 1988

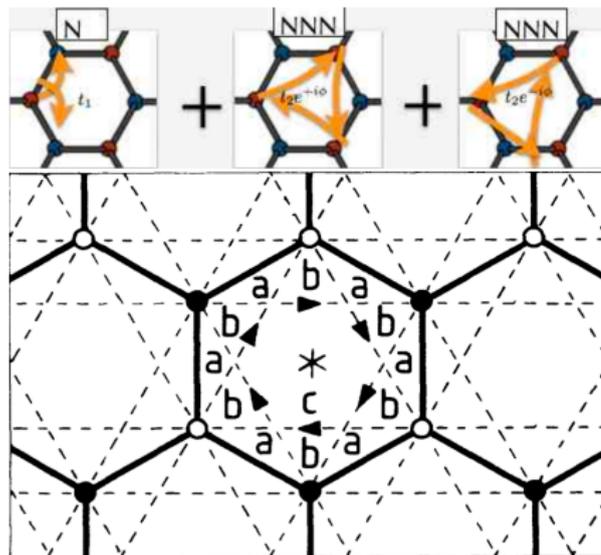
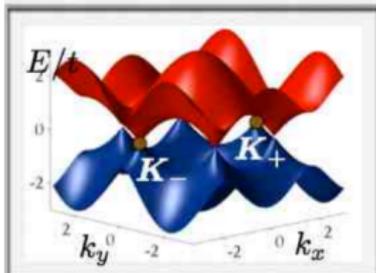
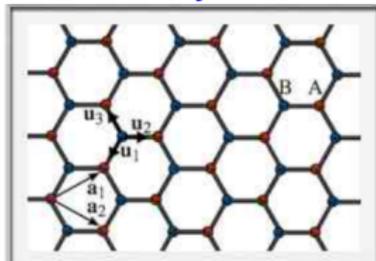
## Model for a Quantum Hall Effect without Landau Levels: Condensed-Matter Realization of the "Parity Anomaly"

F. D. M. Haldane

Department of Physics, University of California, San Diego, La Jolla, California 92093

(Received 16 September 1987)

*"The external B-field is not necessary to obtain quantized  $\sigma_{xy}$ .  
One just needs a time-reversal breaking perturbation."*



## MODELOS DE HALDANE

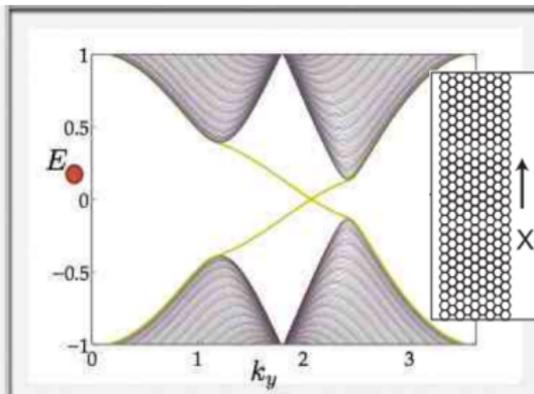
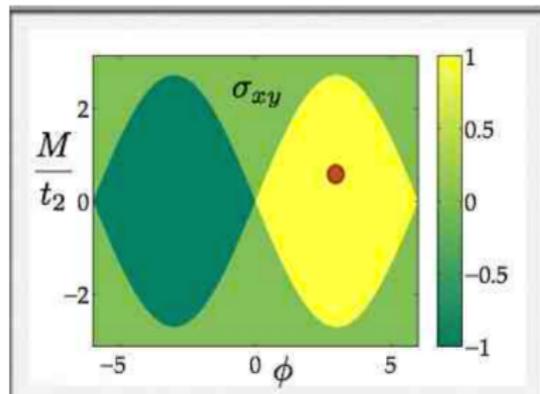
$$H = t_1 \sum_{\langle i,j \rangle} c_i^\dagger c_j + t_2 \sum_{\langle\langle i,j \rangle\rangle} e^{-iv_{ij}\phi} c_i^\dagger c_j + M \sum_i \xi_i c_i^\dagger c_i \rightarrow_{\mathbf{K}} H^{\pm K} = \hbar v_F (\boldsymbol{\sigma} \cdot \mathbf{p}) + M_{K_{\pm}} \sigma_z$$

Sendo  $M_{K_{\pm}} = M \mp 3\sqrt{3}t_2 \sin \phi$ .

Para férmions “spinless”:

Para  $\phi = 0, \pi, \dots$ ,  $K_+ \xleftrightarrow{\tau} K_-$ ;  $M_{K_+} = M_{K_-}$

No caso contrario, quebra de  $\tau$ ;  $M_{K_+} \neq M_{K_-}$



# OUTLINE

- 1 INTRODUÇÃO
- 2 EFEITO HALL QUÂNTICO
  - Fenomenologia
  - Estados topológicos na matéria
  - Modelo de Hofstadter
- 3 EFEITO HALL QUÂNTICO DE SPIN
  - Fenomenologia
  - Modelo de Haldane
  - Modelo de Kane e Mele

# MODELO DE KANE E MELE

PRL **95**, 226801 (2005)

PHYSICAL REVIEW LETTERS

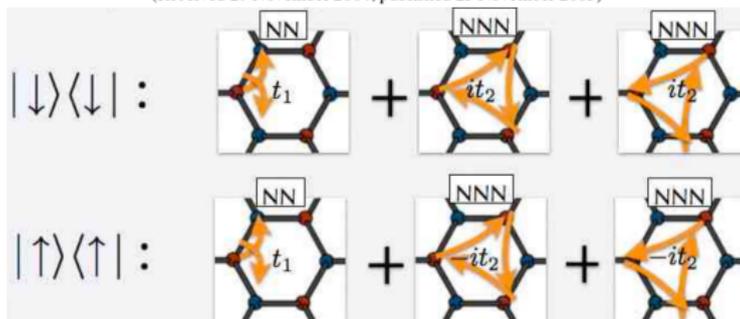
week ending  
25 NOVEMBER 2005

## Quantum Spin Hall Effect in Graphene

C. L. Kane and E. J. Mele

Dept. of Physics and Astronomy, University of Pennsylvania, Philadelphia, Pennsylvania 19104, USA

(Received 29 November 2004; published 23 November 2005)



MODELO DE KANE E MELE → DUAS COPIAS  $H_{\text{Haldane}}$

$$\mathcal{H}_{KM} = H_{\text{spin}} \otimes H_{\text{Haldane}}$$

$$H = t_1 \sum_{\langle i,j \rangle \alpha} c_{i\alpha}^\dagger c_{j\alpha} + \sum_{\langle\langle i,j \rangle\rangle \alpha\beta} it_2 v_{ij} s_{\alpha\beta}^z c_{i\alpha}^\dagger c_{j\beta} + M \sum_{i\alpha} \xi_i c_{i\alpha}^\dagger c_{i\alpha}$$

## MODELO DE KANE E MELE

$$\mathcal{H}_{KM}^{eff} = \mathcal{L} \otimes \hbar v_F (\tau_x P_x \mp \tau_y p_y) + M_{K\pm} \sigma_z \otimes \tau_z + M \mathcal{L} \otimes \tau_z$$

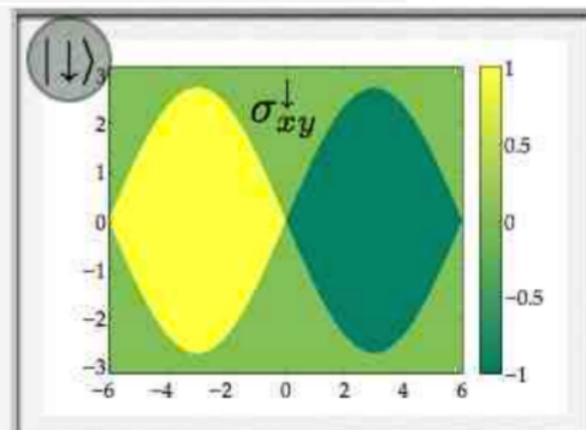
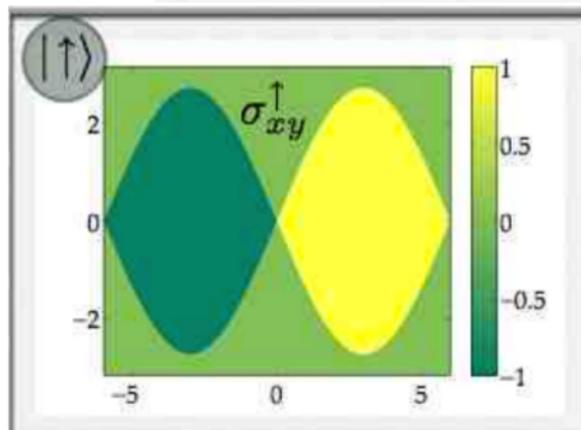
$$M_{K\pm\uparrow} = M \pm 3\sqrt{3}t_2$$

$$M_{K\pm\downarrow} = M \mp 3\sqrt{3}t_2$$

$$K_{\pm\uparrow} \xleftrightarrow{T} K_{\mp\downarrow}$$

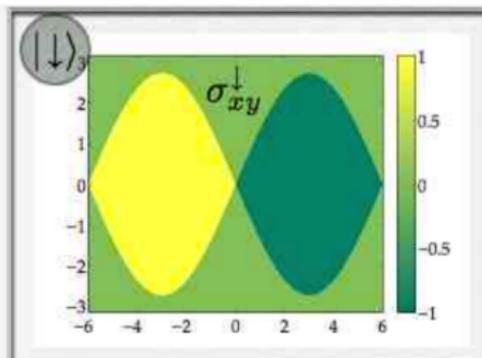
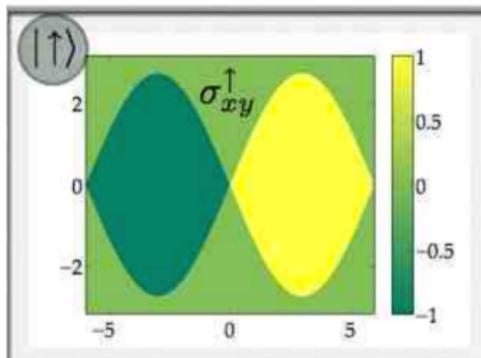
$$M_{K\pm\uparrow} = M_{K\mp\downarrow}$$

$T$ -preserved



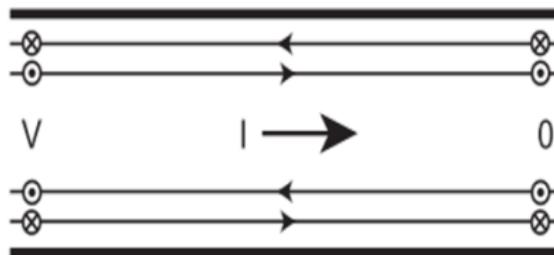
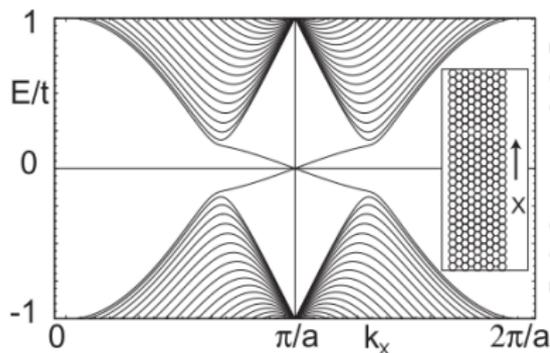
$$j_x = (\sigma_{xy}^{\uparrow} + \sigma_{xy}^{\downarrow}) E_y = 0; \quad j_x^s = \frac{\hbar}{2e} (\sigma_{xy}^{\uparrow} - \sigma_{xy}^{\downarrow}) E_y = \pm \frac{e}{2\pi} E_y$$

## MODELO DE KANE E MELE



$$j_x = (\sigma_{xy}^{\uparrow} + \sigma_{xy}^{\downarrow})E_y = 0;$$

$$j_x^s = \frac{\hbar}{2e}(\sigma_{xy}^{\uparrow} - \sigma_{xy}^{\downarrow})E_y = \pm \frac{e}{2\pi}E_y$$



## MODELO DE KANE E MELE



$$V_{SO} = \frac{\hbar}{4m^2c^2} \mathbf{S} \cdot (\nabla V \times \mathbf{p})$$

OBRIGADO ...