

Fermions de Majorana

Eder Ruiz

Universidade de São Paulo
Instituto de Física

20 Junio de 2013



Conteúdo

- 1 Introdução
- 2 Modelo de Kitaev
- 3 Realização experimental
- 4 Bibliografia



Materia e Antimateria



P. Dirac (1902-1984).

Equação de Dirac (1928)

$$(i\gamma^\mu \partial_\mu - m)\psi = 0$$

Conjugação de Carga

$$\psi^c = \eta_\psi C\psi^T$$



Materia e Antimateria



P. Dirac (1902-1984).

Equação de Dirac (1928)

$$(i\gamma^\mu \partial_\mu - m)\psi = 0$$

Conjugação de Carga

$$\psi^c = \eta_\psi C\psi^T$$



Materia e Antimateria



P. Dirac (1902-1984).

Equação de Dirac (1928)

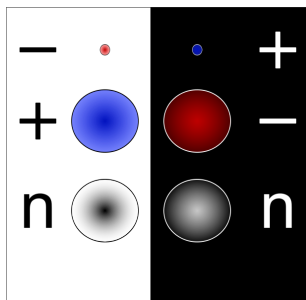
$$(i\gamma^\mu \partial_\mu - m)\psi = 0$$

Conjugação de Carga

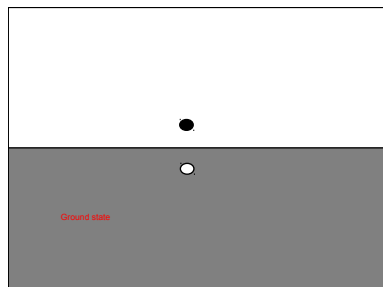
$$\psi^c = \eta_\psi C\psi^T$$



Materia e Antimateria



Partículas e antipartículas.



Mar de eletrons con energia negativa.



Majorana Fermions



E. Majorana (1906-c.a 1938)

Matrizes de Majorana

$$\gamma^0 = \sigma_2 \otimes \sigma_1$$

$$\gamma^1 = i\sigma_1 \otimes 1$$

$$\gamma^2 = i\sigma_3 \otimes 1$$

$$\gamma^3 = \sigma_2 \otimes \sigma_2$$

Particula=Antiparticula

$$\psi^c = \psi$$



Majorana Fermions



E. Majorana (1906-c.a 1938)

Matrizes de Majorana

$$\gamma^0 = \sigma_2 \otimes \sigma_1$$

$$\gamma^1 = i\sigma_1 \otimes 1$$

$$\gamma^2 = i\sigma_3 \otimes 1$$

$$\gamma^3 = \sigma_2 \otimes \sigma_2$$

Particula=Antiparticula

$$\psi^c = \psi$$



Majorana Fermions



E. Majorana (1906-c.a 1938)

Matrizes de Majorana

$$\gamma^0 = \sigma_2 \otimes \sigma_1$$

$$\gamma^1 = i\sigma_1 \otimes 1$$

$$\gamma^2 = i\sigma_3 \otimes 1$$

$$\gamma^3 = \sigma_2 \otimes \sigma_2$$

Particula=Antiparticula

$$\psi^c = \psi$$



Fermions de Majorana em Materia condensada

Majorana returns

Frank Wilczek

There are many categories of scientists: plenty of second and third rank, who do their best, but do not go very far; there are also people of first-class rank, who make great discoveries, fundamental to the development of science. But then there are the geniuses, like Galileo and Newton. Ettore Majorana was one of them.³¹ Enrico Fermi, not known for flightiness or overstatement, is the source of these often-quoted lines.

The bare facts of Majorana's life are well-told. Born in Catania, Italy, on August 1906, into an accomplished family, he rose rapidly through the academic ranks, became a friend and scientific collaborator of Enrico Fermi, Werner Heisenberg and other leading physicists, and produced a stream of high-quality papers. Then, beginning in 1938, things started to go terribly wrong. Complained of gastritis, became increasingly unwell, with no official position, and fished nothing for several years. In 1938, he allowed Fermi to write-up and publish under his (Majorana's) name, his most profound paper — the point of departure of this article — containing results he had derived some years before. On Fermi's urging, Majorana applied for professorships and was awarded the position in Theoretical Physics at Naples,



© KIND CONCESSION OF E. RECAMI ANDE MAJORANA

which he took up in January 1938. Two months later, he embarked on a mysterious trip to Palermo, arrived, then boarded a ship straight back to Naples and disappeared without a trace.

Majorana published only nine papers in his lifetime, none very lengthy. They are collected, with commentaries, all in both Italian and English versions, in a slim volume³⁰. Each is a substantial contribution to quantum physics. At least two are

masterpieces: the last, as mentioned, another on the quantum theory of spin magnetic fields, which anticipates the brilliant development of molecular-beam and magnetic resonance techniques.

In recent years, a small industry has developed, bringing Majorana's unpublished notebooks into print (see for example ref. 31). They are impressive documents, full of original calculations and expositions covering a wide range of physical problems. They leave an overwhelming impression of gathering strength; physics might have advanced more rapidly on several fronts had Majorana pulled this material together and shared it with the world.

How did he vanish? There are two leading theories. According to one, he retired to a monastery, to escape a spin crisis and accept the embrace of his Catholic faith (not unlike another top scientific genius, Blaise Pascal). According to another, he jumped overboard, an act of suicide recalling the alienated superman of fiction, Odd John³². Fermi's appreciation had a wistful conclusion, which is less known: "Majorana had greater gifts than anyone else in the world. Unfortunately he lacked one quality which other men generally have: plain common sense."



Fermions de Majorana em Materia condensada

Operadores fermionicos

$$c \ c^\dagger$$

Partículas e buracos



Fermions de Majorana em Materia condensada

Operadores de Majorana

$$\gamma_1 = \frac{1}{2}(c^\dagger + c)$$

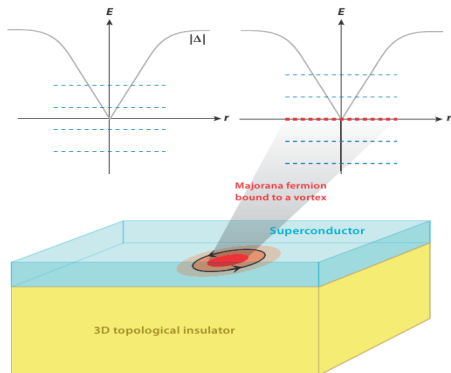
$$\gamma_2 = \frac{i}{2}(c^\dagger - c)$$

Superposição



Dois Fermions de Majorana formam um Fermion normal

Fermions de Majorana em Supercondutores

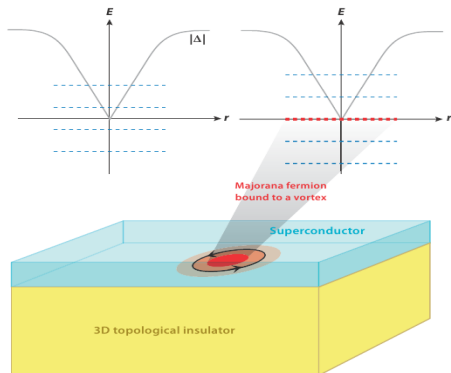


$$\gamma(E) = \gamma^\dagger(-E)$$

$$\gamma = \gamma^\dagger$$



Fermions de Majorana em Supercondutores

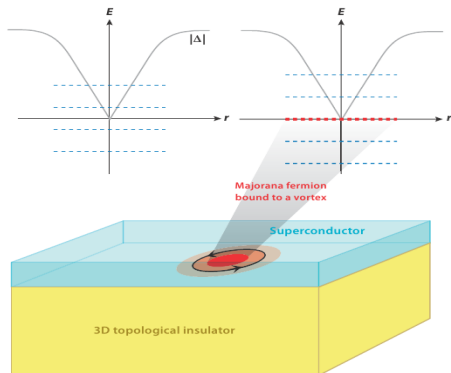


$$\gamma(E) = \gamma^\dagger(-E)$$

$$\gamma = \gamma^\dagger$$



Fermions de Majorana em Supercondutores

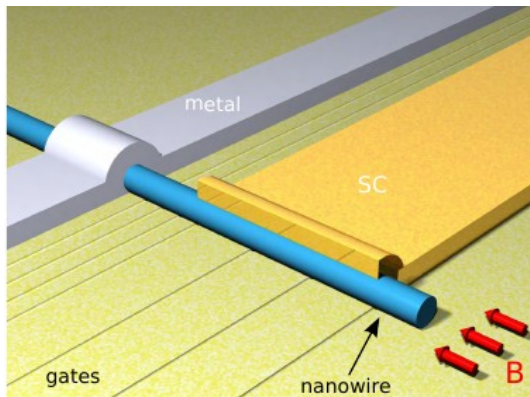


$$\gamma(E) = \gamma^\dagger(-E)$$

$$\gamma = \gamma^\dagger$$



Fermions de Majorana em fios Quânticos



Nanofio ligado a um supercondutor e um metal normal



Fermions de Majorana em Materia condensada

Porque fios?

vantagens

- Fabricação.
- Menos excitações.
- Menor numero de graus de liberdade.



Fermions de Majorana em Materia condensada

Porque fios?

vantagens

- Fabricação.
- Menos excitações.
- Menor numero de graus de liberdade.



Fermions de Majorana em Materia condensada

Porque fios?

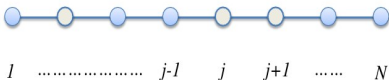
vantagens

- Fabricação.
- Menos excitações.
- Menor numero de graus de liberdade.



Cadeia de tight binding

$$H_1 = \sum_j [-w(c_j^\dagger c_{j+1} + c_{j+1}^\dagger c_j) - \mu(c_j^\dagger c_j - \frac{1}{2}) + \Delta(c_j c_{j+1} + \Delta c_{j+1}^\dagger c_j^\dagger)]$$



Cadeia de tight binding

$$\begin{aligned}\gamma_{2j-1} &= \exp(i\frac{\theta}{2})c_j + \exp(-i\frac{\theta}{2})c_j^\dagger \\ \gamma_{2j} &= -i\exp(i\frac{\theta}{2})c_j + i\exp(-i\frac{\theta}{2})c_j^\dagger\end{aligned}$$

$$H_1 = \frac{i}{2} \sum_j [-\mu\gamma_{2j-1}\gamma_{2j} + (w+|\Delta|)\gamma_{2j}\gamma_{2j+1} + (-w+|\Delta|)\gamma_{2j-1}\gamma_{2j+2}]$$



fase (a)

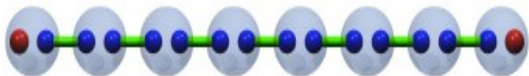
$$|\Delta| = w = 0$$
$$H_1 = \frac{i}{2} \sum_j [-\mu\gamma_{2j-1}\gamma_{2j}]$$



fase (b)

$$|\Delta| = w \Rightarrow 0$$

$$H_1 = iw \sum_j [-\mu\gamma_{2j}\gamma_{2j+1}]$$

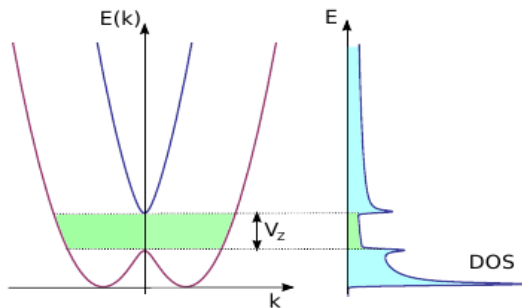


$$H_1 = \frac{i}{2} \sum_j [-\mu \gamma_{2j-1} \gamma_{2j} + (w + |\Delta|) \gamma_{2j} \gamma_{2j+1} + (-w + |\Delta|) \gamma_{2j-1} \gamma_{2j+2}]$$

$$H_{\text{canonical}} = \frac{i}{2} \sum_m \varepsilon_m b_m^1 b_m^2$$

$$\varepsilon(q) = \pm \sqrt{(2w \cos q + \mu)^2 + 4|\Delta|^2 \sin^2 q}$$



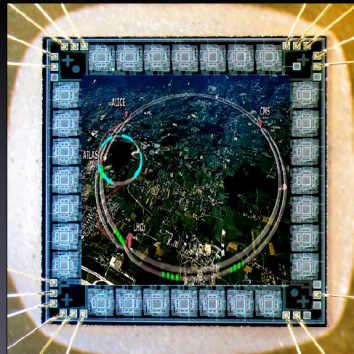


Espectro de excitação de um nanofio semiconductor



Particle Physics on a Chip

“The Search for Majorana Fermions”



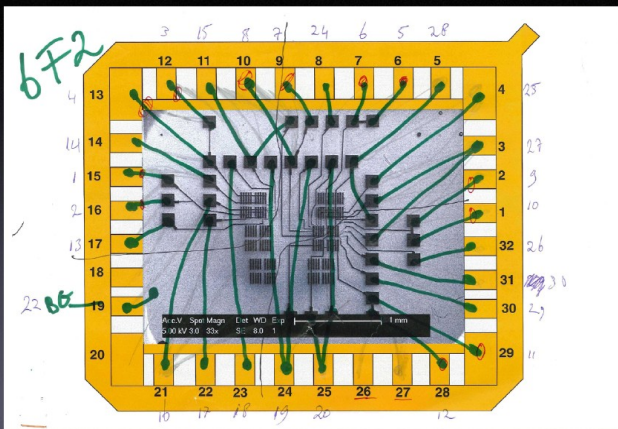
Leo Kouwenhoven, *Kavli Institute of NanoScience, Delft, The Netherlands*



Underground Majorana Lab



Majorana lab on a nano-chip



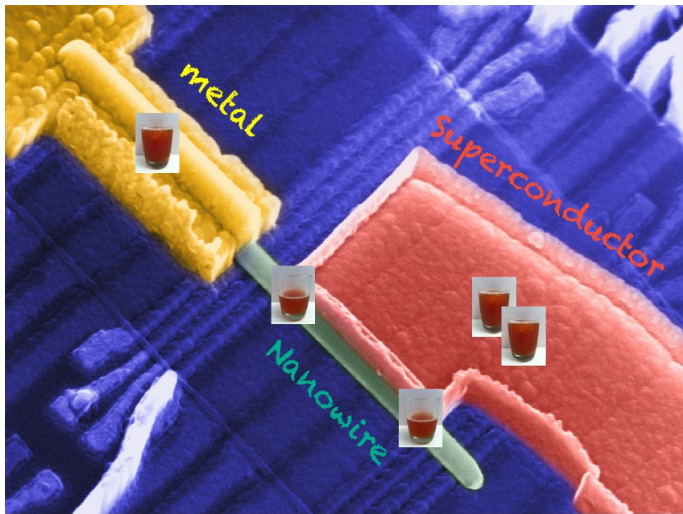
Signatures of Majorana Fermions in Hybrid Superconductor-Semiconductor Nanowire Devices

V. Mourik,^{1*} K. Zuo,^{1*} S. M. Frolov,¹ S. R. Plissard,² E. P. A. M. Bakkers,^{1,2} L. P. Kouwenhoven^{1†}

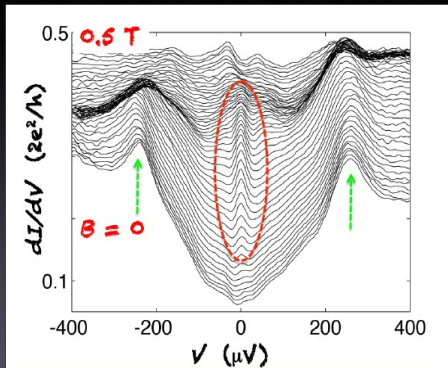
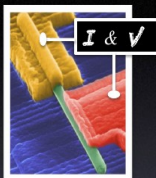
Majorana fermions are particles identical to their own antiparticles. They have been theoretically predicted to exist in topological superconductors. Here, we report electrical measurements on indium antimonide nanowires contacted with one normal (gold) and one superconducting (niobium titanium nitride) electrode. Gate voltages vary electron density and define a tunnel barrier between normal and superconducting contacts. In the presence of magnetic fields on the order of 100 millitesla, we observe bound, midgap states at zero bias voltage. These bound states remain fixed to zero bias, even when magnetic fields and gate voltages are changed over considerable ranges. Our observations support the hypothesis of Majorana fermions in nanowires coupled to superconductors.

of the nanowire (i.e., a Zeeman field), a gap is opened at the crossing between the two spin-orbit bands. If the Fermi energy μ is inside this gap, the degeneracy is twofold, whereas outside the gap it is fourfold. The next ingredient is to connect the semiconducting nanowire to an ordinary s-wave superconductor (Fig. 1A). The proximity of the superconductor induces pairing in the nanowire between electron states of opposite momentum and opposite spins and induces a gap, Δ . Combining this twofold degeneracy with an induced gap creates a topological superconductor (4–11). The condition for a topological phase is $E_Z > (\Delta^2 + \mu^2)^{1/2}$, with the Zeeman energy $E_Z = g\mu_B B/2$ (g is the Landé g factor, μ_B is the Bohr magneton). Near the ends of the wire, the electron density is reduced to zero, and subsequently, μ will drop below the subband energies such that μ^2 becomes large. At the points in space where $E_Z = (\Delta^2 + \mu^2)^{1/2}$, Majoranas arise as zero-energy (i.e., midgap) bound states—one





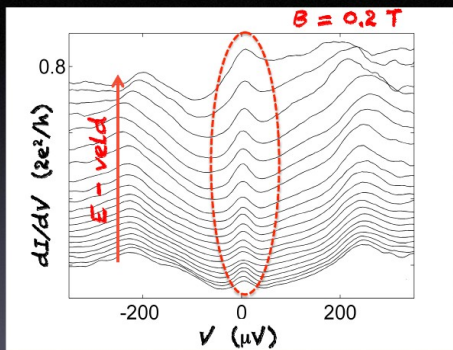
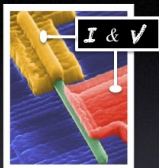
current & voltage measurement



→ Particle is **not** magnetic

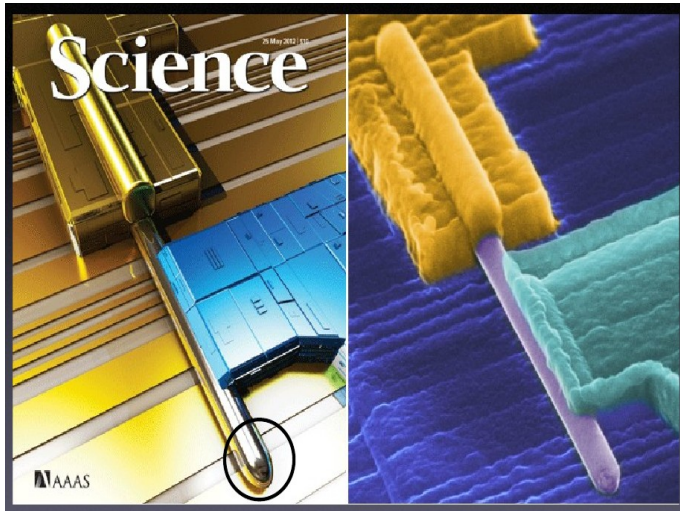


current & voltage measurement



→ Particle is **not** electrically charged









Ettore Majorana



-  F.Wilczek,Nature Physics 5, 614 (2009).
-  M.Franz, Physics 3, 24 (2010).
-  V.Mourik, K. Zou, S.M. Frolov, S.R Plissard, E.P.A. M. Bakkers, and L.P. Kouwenhoven, Science 336, 1003 (2013).
-  C.W.J. Beenakker Annu. Rev. Condens. Matter Phys. 2013.
-  M. Leijnse and K. Flensberg Semicond. Sci. Technol. 27 (2012) 124003 (11pp).
-  Leo Kouwenhoven presentation.



¡Muito Obrigado!

