Cadeias de Spins Desordenadas

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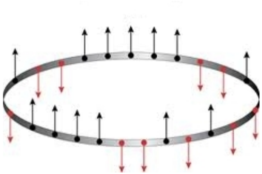
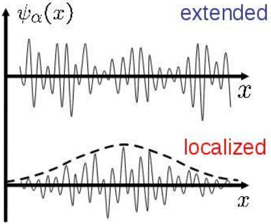
Cadeias de spin



- Possibilidade de resultados exatos;
- Quebra espontânea de simetria, transições de fase, ordem de longo alcance, etc;
- Ansatz de Bethe, grupos de renormalização, Monte Carlo quântico, etc;

Desordem em sistemas de muitos corpos

- Origem: impurezas, vacâncias, defeitos, entre outros;
- Alterações das propriedades do sistema;



- *Toy models*: cadeias de spins desordenadas;

A interação de troca

- Sistema de dois elétrons

$$\psi_1 = \frac{1}{\sqrt{2}}\Big[\phi_1(\mathbf{r}_1)\phi_2(\mathbf{r}_2) + \phi_1(\mathbf{r}_2)\phi_2(\mathbf{r}_1)\Big]\chi_A(\sigma_1, \sigma_2) \quad , \qquad (1)$$

$$\psi_2 = \frac{1}{\sqrt{2}}\Big[\phi_1(\mathbf{r}_1)\phi_2(\mathbf{r}_2) - \phi_1(\mathbf{r}_2)\phi_2(\mathbf{r}_1)\Big]\chi_S(\sigma_1, \sigma_2) \quad , \qquad (2)$$

- Interação coulombiana

$$V(\mathbf{r}_1 - \mathbf{r}_2) = \frac{e^2}{|\mathbf{r}_1 - \mathbf{r}_2|} \quad , \qquad (3)$$

- Diferença de energia $\Delta E = E_1 - E_2$

$$\Delta E = 2\int d^3\mathbf{r}_1 d^3\mathbf{r}_2 \phi_1^*(\mathbf{r}_1)\phi_2^*(\mathbf{r}_2)V(\mathbf{r}_1 - \mathbf{r}_2)\phi_1(\mathbf{r}_2)\phi_2(\mathbf{r}_1) \quad , \qquad (4)$$

A interação de troca

- Hamiltoninano efetivo de spin

$$\mathcal{H}_{spin} = C\mathbf{S}_1 \cdot \mathbf{S}_2 + D \quad , \tag{5}$$

- Diferença de energia

$$\langle \chi_S | \mathcal{H}_{spin} | \chi_S \rangle - \langle \chi_A | \mathcal{H}_{spin} | \chi_A \rangle = C \quad , \tag{6}$$

- Hamiltoniano de Heisenberg

$$\mathcal{H} = J \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j \quad , \tag{7}$$

- J: interação de troca;
- Spins interagentes localizados;

Cadeia de Heisenberg antiferromagnética

- Hamiltoniano da forma

$$\mathcal{H} = J \sum_{j=1}^{N} \mathbf{S}_j \cdot \mathbf{S}_{j+1} \quad , \tag{8}$$

com $J > 0$;

- Operador de spin total

$$\mathbf{S}_{tot} = \sum_{j=1}^{N} \mathbf{S}_j \quad , \tag{9}$$

- Invariância rotacional

$$\left[\mathcal{H}, \mathbf{S}_{tot}\right] = 0 \quad , \tag{10}$$

- Invariância translacional;

Cadeia de Heisenberg antiferromagnética

- Solução exata: ansatz de Bethe;

feature article

The Bethe ansatz after 75 years

Murray T. Batchelor

A 1931 result that lay in obscurity for decades, Bethe's solution to a quantum mechanical model now finds its way into everything from superconductors to string theory.

Murray Batchelor is head of the department of theoretical physics and a member of the Mathematical Sciences Institute at the Australian National University in Canberra.

Hans Bethe introduced his now-famous ansatz to obtain the energy eigenstates of the one-dimensional version of Werner Heisenberg's model of interacting, localized spins in a solid.[1,2] Although it is among Bethe's most cited works and has a wide range of applications, it is rarely included in the graduate physics curriculum except at the advanced level. The 75th anniversary of the Bethe ansatz is appropriately marked by reflecting on the impact of Bethe's result on modern physics, ranging from its profound influence on the field of exactly solved models in statistical mechanics to insights into the subtle nature of quantum many-body effects observed in cold quantum gases.

Bethe (figure 1) completed his PhD in theoretical physics under Arnold Sommerfeld in 1928. During his subsequent time at the University of Munich, Bethe used a traveling fellowship to go to Cambridge in the fall of 1930 and to Rome during the spring terms of 1931 and 1932. His 1931 paper, published in German in *Zeitschrift für Physik*, was submitted from Rome. That period was one of the most exciting times in the history of physics; the theory of quantum mechanics was being refined and applied to reveal the intricate nature of the quantum realm. Those breathtaking developments, along with encounters with pioneers such as Heisenberg, Paul Dirac, and Enrico Fermi, clearly had a strong influence on Bethe's research. (For further biographical details, see the October 2005 Hans Bethe special issue of PHYSICS TODAY; for Bethe's own account of his time spent with Fermi in Rome see PHYSICS TODAY, June 2002, page 28.)

which means that each eigenstate is a linear combination of basis states that all have the same number of down spins. In other words, the Hamiltonian for an array of L spins is a block-diagonal matrix, with one block for each number of down spins. It is useful to think of each down spin as a quasiparticle, and the state of all up spins as the vacuum state.

The wavefunction for a single quasiparticle looks very much like the wavefunction for a free particle in a ring: a plane wave of the form $\exp(ikx)$, with an energy that depends on the wavenumber k, which itself must be an integer multiple of $2\pi/L$. Eigenstates with two or more particles are more complicated because the particles interact—but the interaction is short-range, since only pairs of nearest-neighbor spins contribute to the Hamiltonian.

Bethe developed his ansatz by looking at those regions, in the configuration space of quasiparticles, where all the quasiparticles are separated from one another. For n quasiparticles, there are $n!$ such regions in the n-dimensional configuration space, one for each ordering of the quasiparticles. Bethe's big idea was to hope that the wavefunction in each region is a superposition of plane waves, as if the quasiparticles were not interacting at all. By matching the wavefunctions on the interfaces between those regions, he worked out a system of equations, shown in box 1, for the coefficients and wavenumbers of the plane waves. The wavenumbers were then no longer integer multiples of $2\pi/L$. Taking the place of that constraint is a complicated set of equations known as the Bethe-ansatz equations, or simply the Bethe equations.

Figura: M. Batchelor. *Physics Today* 60, 36 (2007).

Cadeia de Heisenberg antiferromagnética

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NATURE PHYSICS | ARTICLE

Fractional spinon excitations in the quantum Heisenberg antiferromagnetic chain

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Abstract

Abstract • References • Author information • Supplementary information

One of the simplest quantum many-body systems is the spin-1/2 Heisenberg antiferromagnetic chain, a linear array of interacting magnetic moments. Its exact ground state is a macroscopic singlet entangling all spins in the chain. Its elementary excitations, called spinons, are fractional spin-1/2 quasiparticles created and detected in pairs by neutron scattering. Theoretical predictions show that two-spinon states exhaust only 71% of the spectral weight and higher-order spinon states, yet to be experimentally located, are predicted to participate in the remaining. Here, by accurate absolute normalization of our inelastic neutron scattering data on a spin-1/2 Heisenberg antiferromagnetic chain compound, we account for the full spectral weight to within 99(8)%. Our data thus establish and quantify the existence of higher-order spinon states. The observation that, within error bars, the experimental line shape resembles a rescaled two-spinon one with similar boundaries allows us to develop a simple picture for understanding multi-spinon excitations.

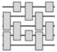
Selected feature

Nature Physics Insight: Quantum simulation

... with ultracold quantum gases

Immanuel Bloch *et al.*

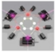
Detection possibilities and extreme parameter regimes make these 'artificial solids' an exciting complement to condensed-matter systems.



... with trapped ions

R. Blatt and C. F. Roos

A variety of interactions can be engineered with high precision, and the first proof-of-principle experiments have been performed.



... with photonic systems

Alán Aspuru-Guzik and Philip Walther

Mobile over free space and in waveguide structures, these enable the natural investigation of quantum transport phenomena.

See complete feature ▸

Cadeia de Heisenberg antiferromagnética

- Hamiltoniano local

$$\mathcal{H}_{j,j+1} = \frac{1}{2} J \left[\left(\mathbf{S}_j + \mathbf{S}_{j+1} \right)^2 - \mathbf{S}_j^2 - \mathbf{S}_{j+1}^2 \right] \quad , \qquad (11)$$

- Estado fundamental local: singleto

$$|s\rangle = \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) \quad , \qquad (12)$$

- Estados excitados locais: tripleto

$$|t\rangle = \begin{cases} |\uparrow\uparrow\rangle \\ \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle \right) \\ |\downarrow\downarrow\rangle \end{cases} \quad ,$$

Cadeia de Heisenberg antiferromagnética

- Estado fundamental do sistema: singleto macroscópico (todos os spins estão emaranhados);
- Excitações elementares

$$\omega_k = \frac{\pi}{2} |\sin(k)| \quad , \tag{13}$$

  onde k é o momento da cadeia,
- Excitações com spin total não nulo;
- *Gapless* : Teorema de Lieb-Schultz-Mattis;

Cadeia de Heisenberg antiferromagnética: susceptibilidade magnética

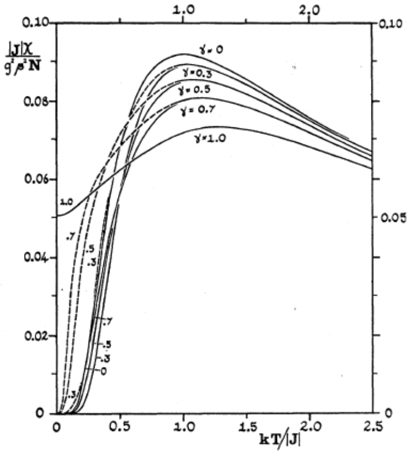


Figura: J. C. Bonner e M. E. Fisher. Phys. Rev. 135, A640 (1964).

Cadeia de Heisenberg antiferromagnética: função de correlação

- Função de correlação spin-spin

$$\langle S_j^z S_{j+r}^z \rangle \sim \frac{(-1)^{|r|}}{|r|} \quad , \tag{14}$$

- Decaimento algébrico das correlações: *ordem de quasi-longo alcance*;
- $T = 0$ é a temperatura crítica do sistema;

Cadeia de Heisenberg com interações aleatórias

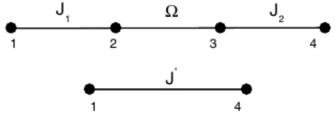
- Hamiltoniano da forma

$$\mathcal{H}_D = \sum_{j=1}^{N} J_j \mathbf{S}_j \cdot \mathbf{S}_{j+1} \quad , \tag{15}$$

onde $J_j > 0$ e fixos no tempo;

- Distribuição de acoplamentos $P(J; \Omega_0)$, com $0 < J < \Omega_0$;
- Distribuição larga: desordem forte (*strong disorder*);

Renormalização no espaço real (Ma e Dasgupta)



- $J_1, J_2 < \Omega$;
- Teoria de perturbação de segunda ordem

$$\mathcal{H}_{eff}^{local} = const. + J'\mathbf{S}_1 \cdot \mathbf{S}_4 \quad , \tag{16}$$

- Acoplamento efetivo

$$J' = \frac{J_1 J_2}{2\Omega} \ll J_1, J_2, \Omega \quad , \tag{17}$$

Renormalização no espaço real (Ma e Dasgupta)

- Redução da escala de energia;
- Alteração da distribuição de probabilidades $P(J;\Omega)$;

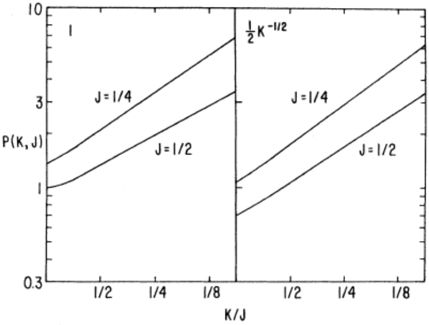


Figura: S. K. Ma, C. Dasgupta e C.-K. Hu. Phys. Rev. Lett. 43, 1434 (1979).

Renormalização no espaço real (Ma e Dasgupta)

- Equação de fluxo

$$
-\frac{\partial}{\partial\Omega}P(J;\Omega) = P(\Omega;\Omega)\int_0^{\Omega} dJ_1 dJ_2 P(J_1;\Omega) \times P(J_2;\Omega)\delta\left(J - \frac{J_1 J_2}{2\Omega}\right) \quad , \quad (18)
$$

- Solução de ponto fixo (Daniel Fisher)

$$
P^*(J;\Omega) = \frac{\alpha(\Omega)}{\Omega}\left[\frac{\Omega}{J}\right]^{1-\alpha(\Omega)} \theta(\Omega - J) \quad , \quad (19)
$$

onde

$$
\alpha(\Omega) = \frac{1}{\ln(\Omega_0/\Omega)} \quad , \quad (20)
$$

- Renormalização assintoticamente exata em baixíssimas temperaturas;

Fase de singleto aleatório

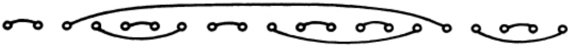


Figura: D. S. Fisher. Phys. Rev. B 50, 3799 (1994).

- Spins arbitrariamente distantes formam pares singleto;
- Caráter localizado;

Previsões sobre o comportamento do sistema

- Fração de spins ativos na escala de energia $\Omega = \Omega_0 e^{-\Gamma}$

$$n(\Gamma) = \frac{1}{\Gamma^2} \quad , \tag{21}$$

- Distância típica entre os spins

$$L(\Gamma) \sim \frac{1}{n(\Gamma)} \sim \left[\ln \left(\frac{\Omega_0}{\Omega} \right) \right]^2 \quad , \tag{22}$$

Previsões sobre o comportamento do sistema

- Susceptibilidade numa escala de energia $\Omega = T$

$$\chi(T) \sim \frac{n(\Gamma)}{T} \sim \frac{1}{T\Big[\ln(\Omega_0/T)\Big]^2} \quad , \qquad (23)$$

- $\chi(T)$ diverge quando a temperatura dimunui;

Previsões sobre o comportamento do sistema

- Correlações médias

$$\overline{\langle \mathbf{S}_j \cdot \mathbf{S}_{j+r} \rangle} \sim \frac{1}{r^2} \quad , \tag{24}$$

- Raros pares singleto fortemente acoplados;
- Correlações típicas muito fracas

$$C_{tic}(p) \sim e^{-a\sqrt{r}} \quad . \tag{25}$$

Muito obrigado!