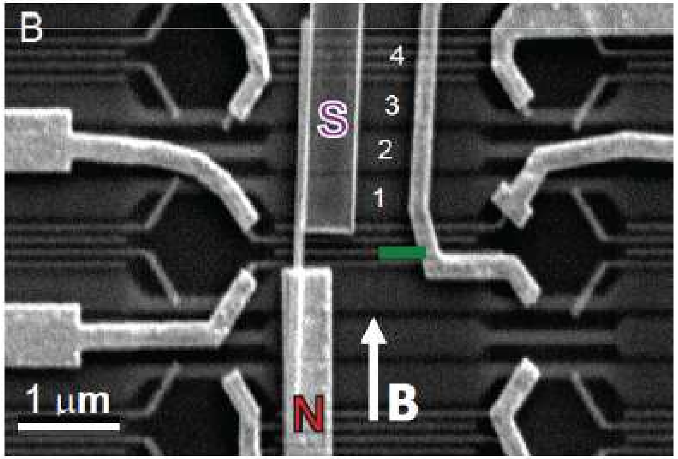
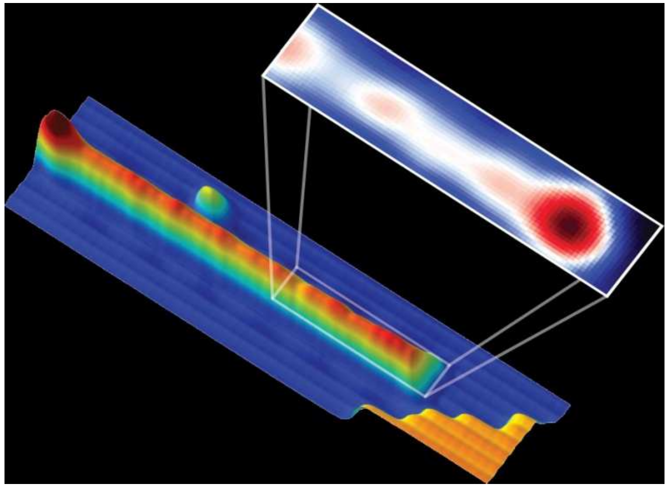
Modelo de Kitaev

Supercondutividade tipo “p-wave” em 1D estados ligados de Majorana





P-wave superconductor in 1D.

s-wave **spinful** (BCS-like)

$$H = \sum_{k,\sigma} (\epsilon_k - \mu)\, c^\dagger_{k,\sigma} c_{k,\sigma} + \left[\Delta_0 c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow} + \Delta_0^* c_{k\uparrow} c_{-k\downarrow}\right]$$

p-wave **spinless**

$$H = \sum_{k} (\epsilon_k - \mu)\, c^\dagger_{k} c_{k} + \left[\Delta(k) c^\dagger_{k} c^\dagger_{-k} + \Delta^*(k) c_{k} c_{-k}\right]$$

"p-wave"

$$\Delta(k) = -\Delta(-k)$$

For small k:

$$\Delta(k) = \Delta k$$

Bogoliubov-deGennes formalism

s-wave **spinful** (BCS-like)

$$H = \sum_{k,\sigma} \bar{\epsilon}_k c^\dagger_{k,\sigma} c_{k,\sigma} + \left[\Delta_0 c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow} + \Delta_0^* c_{k\uparrow} c_{-k\downarrow} \right]$$

$$\Psi_k = \left(c_{k\uparrow} \; c_{k\downarrow} \; c^\dagger_{-k\uparrow} \; c^\dagger_{-k\downarrow} \right)^T$$

"Nambu spinor"

$(\bar{\epsilon}_k = \bar{\epsilon}_{-k})$

$$H = \sum_k \Psi^\dagger_k H_{\text{BdG}} \Psi_k$$

$$H_{\text{BdG}} = \frac{1}{2} \begin{pmatrix} \bar{\epsilon}_k & 0 & 0 & \Delta_0 \\ 0 & \bar{\epsilon}_k & -\Delta_0 & 0 \\ 0 & -\Delta_0^* & -\bar{\epsilon}_{-k} & 0 \\ \Delta_0^* & 0 & 0 & -\bar{\epsilon}_{-k} \end{pmatrix}$$

Diagonalizing, we obtain the familiar result:

$$E_\pm = \sqrt{(\bar{\epsilon}_k)^2 + |\Delta_0|^2}$$

(doubly degenerate)

Note: Diagonalizing is equivalent to a Bogoliubov transformation!

Bogoliubov-deGennes formalism

p-wave **spinless**

$$H = \sum_k \bar{\epsilon}_k c_k^\dagger c_k + \left[\Delta(k) c_k^\dagger c_{-k}^\dagger + \Delta^*(k) c_k c_{-k} \right]$$

$$\Psi_k = \left(c_k \; c_{-k}^\dagger \right)^T$$

giving a 2x2 matrix

$$H = \sum_k \Psi_k^\dagger H_{\mathrm{BdG}} \Psi_k$$

$(\bar{\epsilon}_k = \bar{\epsilon}_{-k})$

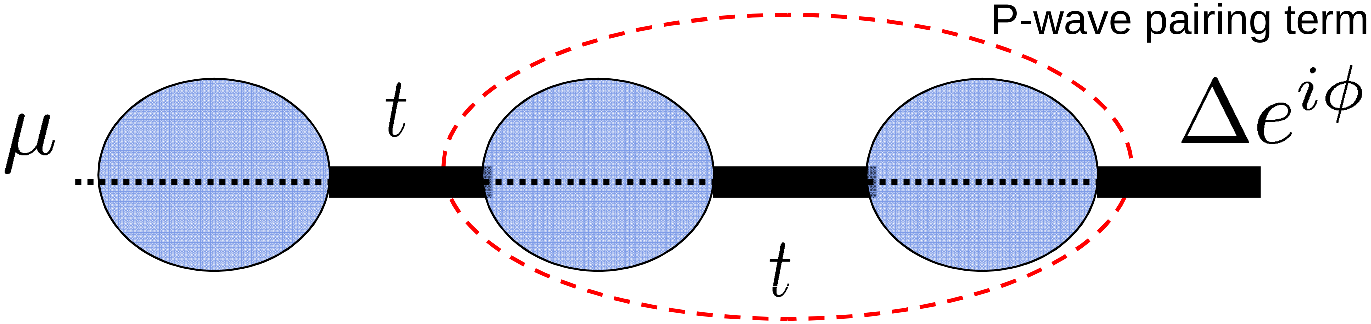
$$H_{\mathrm{BdG}} = \frac{1}{2} \begin{pmatrix} \bar{\epsilon}_k & \Delta(k) \\ \Delta^*(k) & -\bar{\epsilon}_{-k} \end{pmatrix}$$

Diagonalizing, we obtain: $$E_\pm = \pm \sqrt{(\bar{\epsilon}_k)^2 + |\Delta(k)|^2}$$

1D p-wave superconductor (Kitaev model)

$$H = -\mu \sum_x c_x^\dagger c_x - \frac{1}{2} \sum_x (t c_x^\dagger c_{x+1} + \Delta e^{i\phi} c_x c_{x+1} + h.c.)$$

P-wave pairing term (**spinless** fermions)



Using: $c_x = (\sqrt{N})^{-1} \sum_k e^{ikx} c_k$ we write the Hamiltonian as:

$$H = \sum_k \bar{\epsilon}_k c_k^\dagger c_k + \left[\Delta(k) c_k^\dagger c_{-k}^\dagger + \Delta^*(k) c_k c_{-k} \right]$$

with

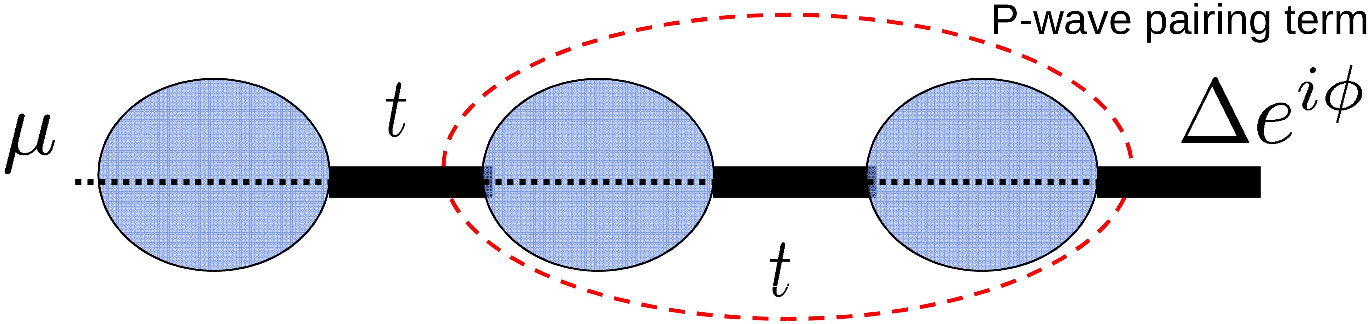
$$\bar{\epsilon}_k = t \cos k + \mu$$

$$\Delta(k) = i \Delta e^{i\phi} \sin k$$

1D p-wave superconductor (Kitaev model)

$$H = -\mu \sum_x c_x^\dagger c_x - \frac{1}{2}\sum_x (t c_x^\dagger c_{x+1} + \Delta e^{i\phi} c_x c_{x+1} + h.c.)$$

P-wave pairing term (**spinless** fermions)



In Bogoliubov-deGennes form: we write the Hamiltonian as:

$$H = \sum_k \Psi_k^\dagger H_{\mathrm{BdG}} \Psi_k$$

$$H_{\mathrm{BdG}} = \frac{1}{2}\begin{pmatrix} t\cos k + \mu & i\Delta e^{i\phi}\sin k \\ -i\Delta e^{-i\phi}\sin k & -t\cos k - \mu \end{pmatrix}$$

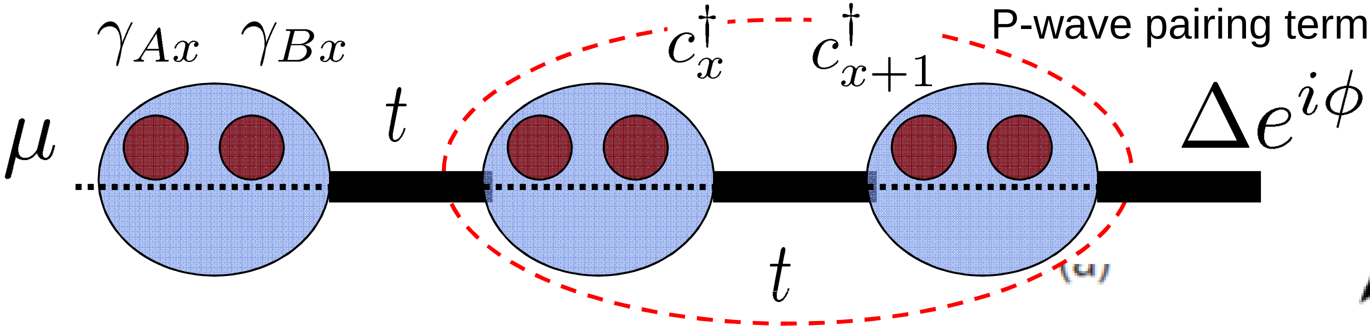
and the spectrum is (two bands):

$$E_\pm = \pm\sqrt{(t\cos k + \mu)^2 + |\Delta|^2 \sin k^2}$$

1D p-wave superconductor (Kitaev model)

$$H = -\mu \sum_x c_x^\dagger c_x - \frac{1}{2} \sum_x (t c_x^\dagger c_{x+1} + \Delta e^{i\phi} c_x c_{x+1} + h.c.)$$

P-wave pairing term (**spinless** fermions)



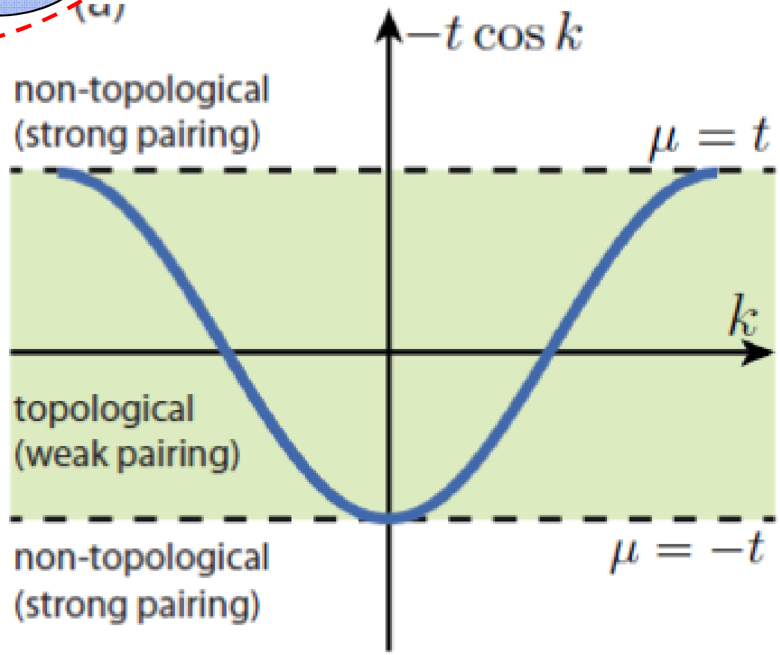
Energy spectrum:

$$E(k) = \pm\sqrt{(t\cos k + \mu)^2 + (\Delta \sin k)^2}$$

$|\mu| > t$ Gapped (E_+-E_->0): **trivial**

$\mu = \pm t$ Gappless modes (E=0) : $k = \pm\pi$ or $k = 0$

$|\mu| < t$ Gapped: **topological** (Δ≠0)



Jason Alicea **arXiv:1202.1293v1**

Majorana states in the Kitaev model.

Map into a "chain of Majorana modes" using:

$$\begin{cases} c_x = \frac{e^{-i\phi/2}}{2}\left(\gamma_{B,x} + i\gamma_{A,x}\right) \\ c_x^\dagger = \frac{e^{+i\phi/2}}{2}\left(\gamma_{B,x} - i\gamma_{A,x}\right) \end{cases}$$

$$H = -\mu \sum_x c_x^\dagger c_x - \frac{1}{2}\sum_x \left(t c_x^\dagger c_{x+1} + \Delta e^{i\phi} c_x c_{x+1} + h.c.\right)$$

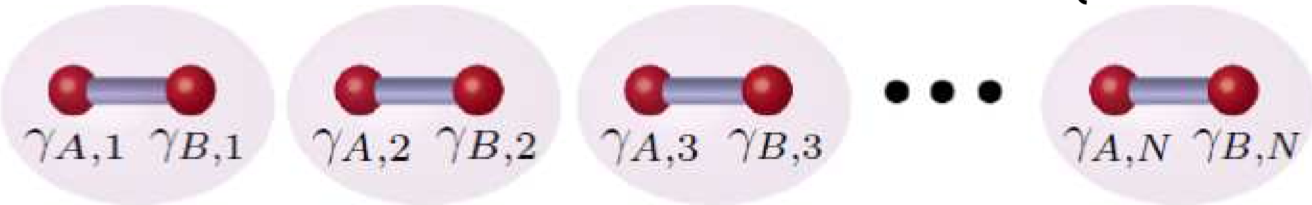
↓

$$H = -\frac{\mu}{2}\sum_x^N \left(1 + i\gamma_{B,x}\gamma_{A,x}\right) - \frac{i}{4}\sum_x^{N-1} \left(\Delta + t\right)\gamma_{B,x}\gamma_{A,x+1} + \left(\Delta - t\right)\gamma_{A,x}\gamma_{B,x+1}$$

Majorana states in the Kitaev model.

$$H = -\frac{\mu}{2}\sum_{x}^{N}(1 + i\gamma_{B,x}\gamma_{A,x}) - \frac{i}{4}\sum_{x}^{N-1}(\Delta + t)\,\gamma_{B,x}\gamma_{A,x+1} + (\Delta - t)\,\gamma_{A,x}\gamma_{B,x+1}$$

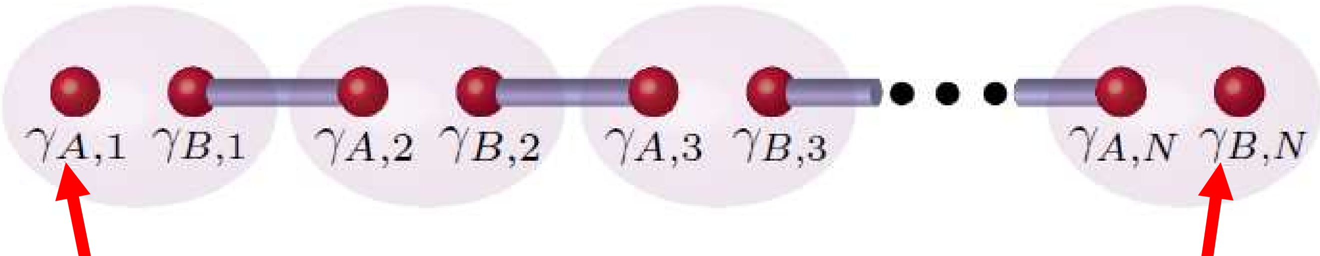
$|\mu| > t$ Gapped: **trivial.** Special case: $\begin{cases} \mu \neq 0 \\ t = \Delta = 0 \end{cases}$



$$E_+ - E_- = 2\mu$$

$|\mu| < t$ Gapped: **topological.** Special case: $\begin{cases} \mu = 0 \\ t = \Delta \neq 0 \end{cases}$

$$E_+ - E_- = 2\Delta$$



Topological regime: Majorana fermions (e=μ=0!!!) at the *edges* of the chain!

What are Majorana fermions anyway?

Dirac Equation

- Dirac's equation: $\left[c\mathbf{p}\cdot\vec{\alpha}+mc^2\beta\right]\Psi = E\Psi$

Dirac matrices: Clifford algebra

$$\alpha_i^2 = \beta^2 = 1$$
$$\alpha_i\alpha_j(\beta) = -\alpha_j(\beta)\alpha_i$$

Ex:. 1 and 2-D: Pauli matrices (complex)

- Compact form (c=1): $\left[i\gamma^\mu\partial_\mu - m\right]\Psi = 0$

http://en.wikipedia.org/wiki/Paul_Dirac /

$$H = \gamma^0\left[\gamma^i p^i + m\right]$$

4x4 matrices satisfying

$$\gamma_0^\dagger = \gamma_0$$
$$\gamma_i^\dagger = -\gamma_i$$

Solutions: "particle" and "anti-particle"

	Mass	Energy
$\Psi(\mathbf{r})$	m	E
$\Psi^\dagger(\mathbf{r})$	m	$-E$

Majorana Fermions

Majorana solution: Representarions of Dirac matrices with *only imaginary non-zero elements* while still satisfying

$$\begin{array}{l}\tilde{\gamma}_0^\dagger = \tilde{\gamma}_0 \\ \tilde{\gamma}_i^\dagger = -\tilde{\gamma}_i\end{array} \Rightarrow [i\tilde{\gamma}^\mu \partial_\mu - m]\,\Psi_{\mathrm{Maj}} = 0$$

http://www.giornalettismo.com/archives/255332/il-ritorno-di-ettore-majorana/

Real solutions: $\psi_{\mathrm{Maj}} = \psi_{\mathrm{Maj}}^\dagger$ *m*=0,*E*=0 $\psi_{\mathrm{Maj}} \equiv \gamma$

- A Dirac fermion can be "written" in terms of two Majoranas fermions

$$\begin{cases}\Psi = \frac{1}{2}(\gamma_1 + i\gamma_2) \\ \Psi^\dagger = \frac{1}{2}(\gamma_1 - i\gamma_2)\end{cases}$$

or

$$\gamma_1 = \frac{1}{2}\left(\Psi^\dagger + \Psi\right)$$

Can there be non-elementary Majorana fermions?

Majorana "quasiparticles" in condensed matter systems

- Fractional Quantum Hall liquids (ν=5/2): "non-Abelian anyons".

  Moore and Read, *Nucl. Phys. B* **360** 362 (1991).

- Quantum spin systems.

  Kitaev, *Ann. Phys.* **303** 2 (2003).

- Interface of topological insulators with BCS superconductors

  Fu and Kane, *Phys. Rev. Lett.* **100** 096407 (2008).

- Spin-polarized ("spinless") p-wave superconductors.

  Read and Green, *Phys. Rev. B* **61** 10267 (2000).

  Kitaev, *Phys. Usp.* **44** 131 (2001).

Motivation:

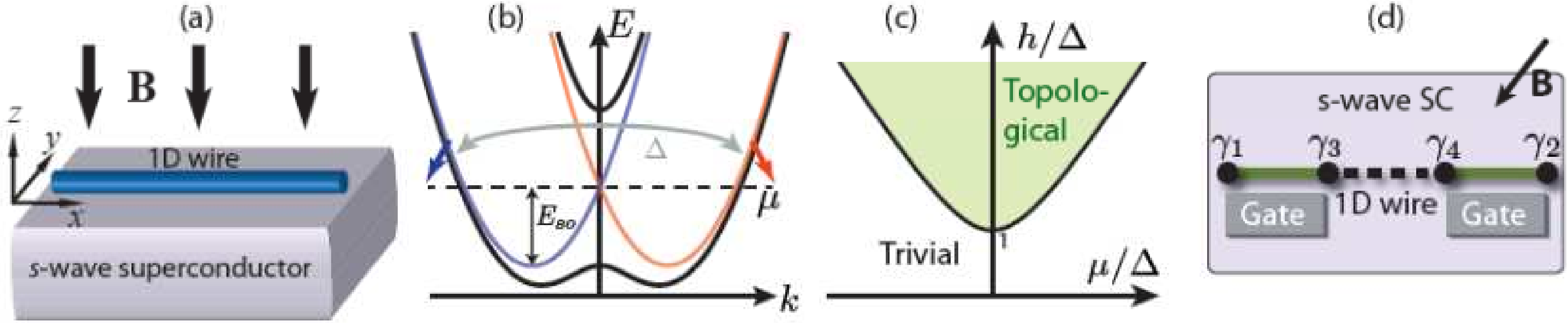
entanglement of particles with non-abelian statistics=

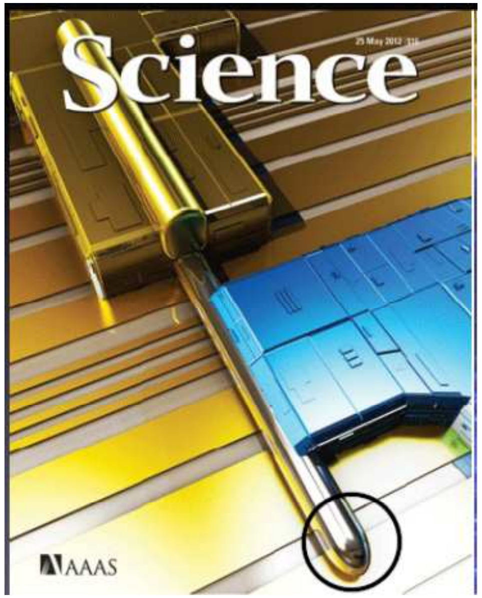
"topologically protected" quantum computation.

Can the Kitaev model be realized experimentally?

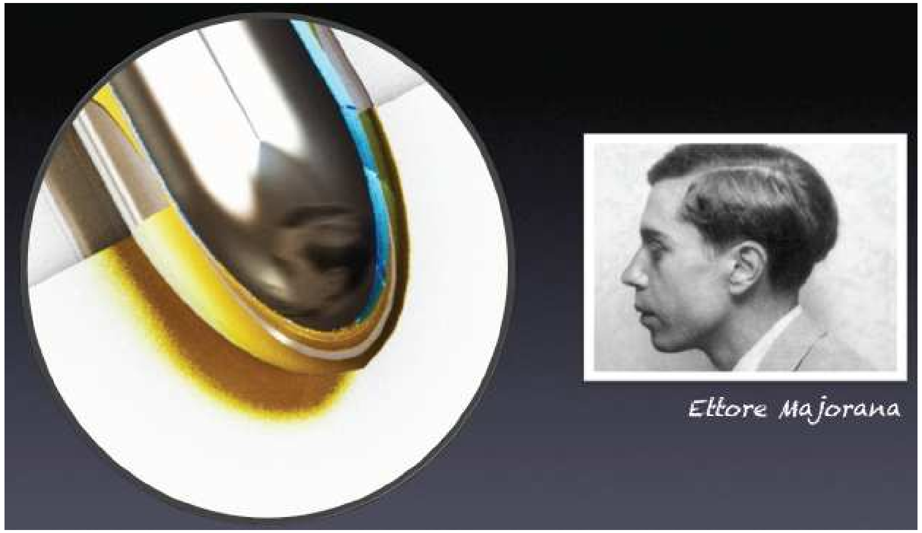
How to realize a p-wave superconductor: Quantum wires.

Theory: Lutchyn et al. PRL, **105**, 077001 (2010); Oreg et al. PRL,**105**, 077002 (2010);





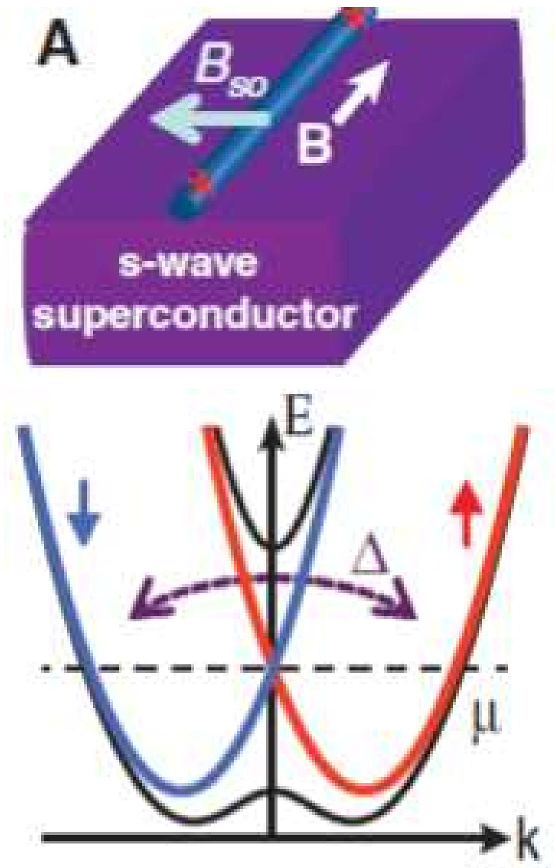
Experiment: V. Mourik et al. Science **336** 1003 (2012)

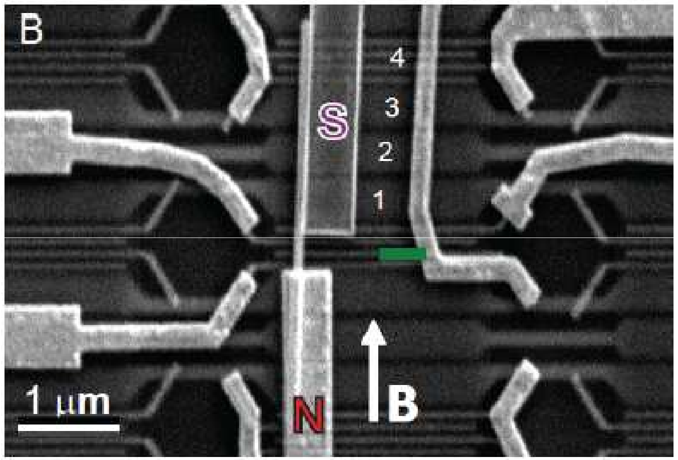


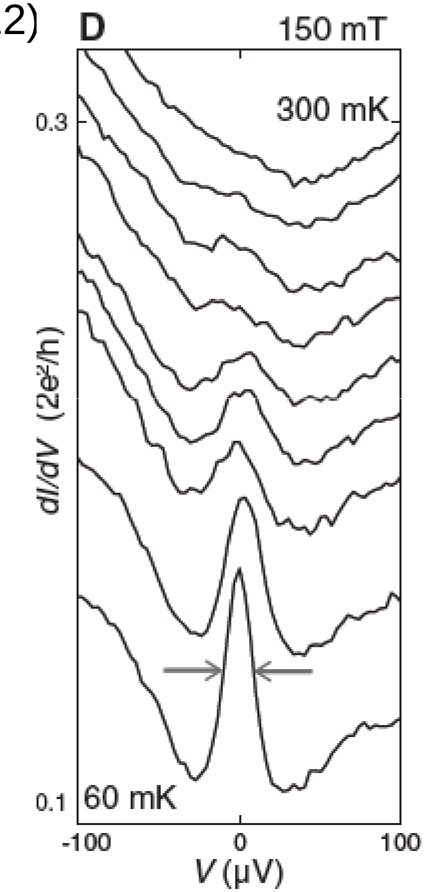
- Experiment: “Majorana is found at the ends of a quantum wire”

How to realize a p-wave superconductor: Quantum wire?

V. Mourik et al. Science **336** 1003 (2012)

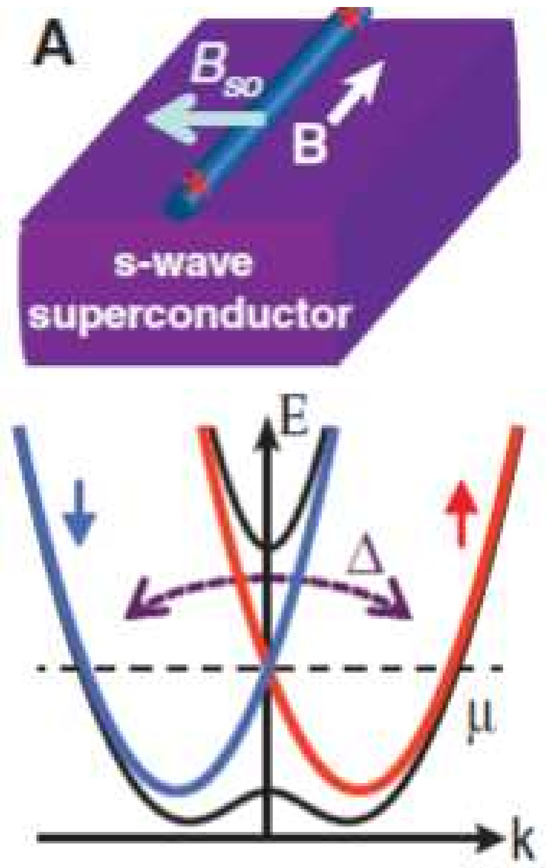




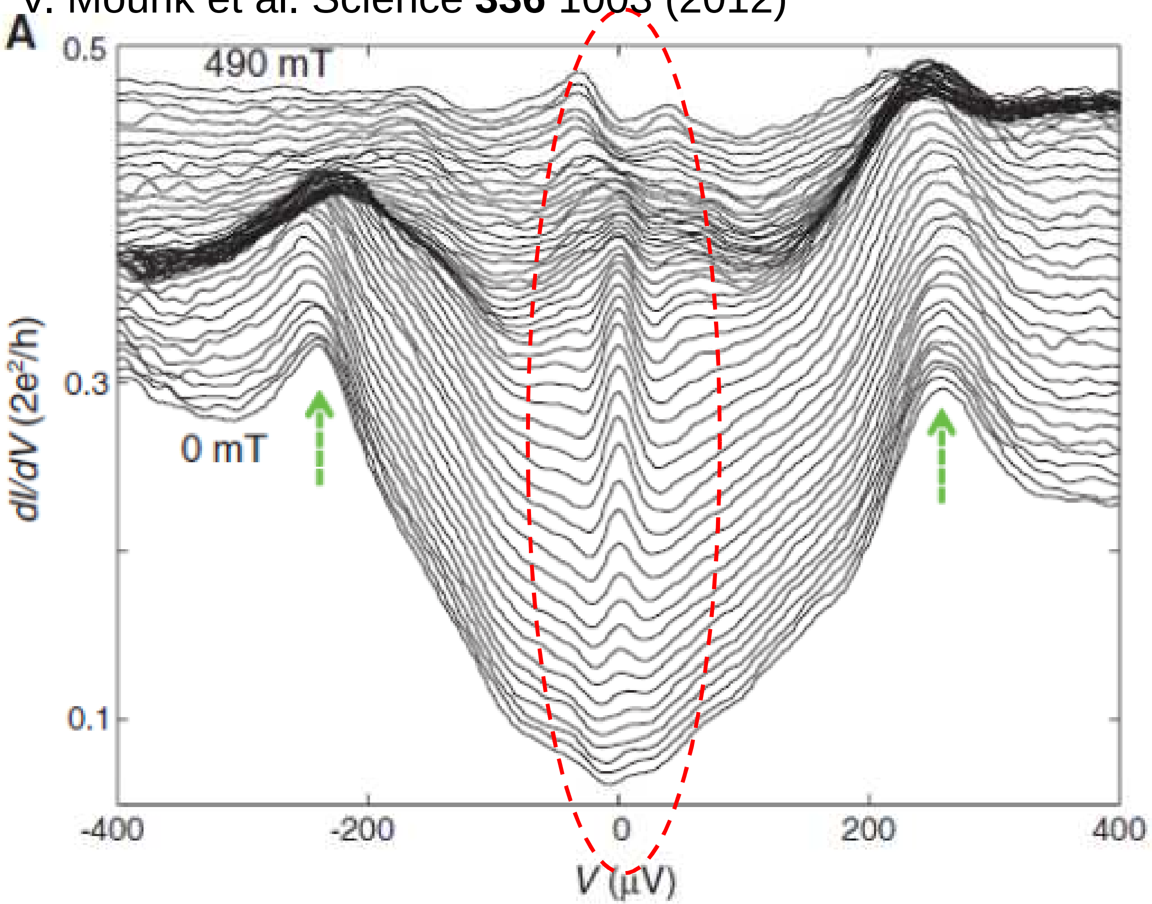


- Nice, but... Are these zero-energy peaks **really** Majorana fermions???
- Other possibilities: Andreev bound states? **Kondo?** Disorder??

How to realize a p-wave superconductor: Quantum wire?



V. Mourik et al. Science **336** 1003 (2012)



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