## 2-d Kinematics

- We'll consider motion on a flat plane rather than on a straight line. (2 dimensions instead of 1)
- Key Concept: will be able to simplify motion to two linked one dimensional problems:
- Break into components
- link with time!




## Definitions

- Remember: velocity and displacement are

VECTORS! (they have components).

$$
\overrightarrow{\vec{v}}=\frac{\vec{r}-\vec{r}_{0}}{t-t_{0}}=\frac{\Delta \vec{r}}{\Delta t}
$$

$$
\vec{r}=\vec{r}_{0}+\overline{\vec{v}} \Delta t+\begin{aligned}
& x=x_{0}+v_{x} \Delta t \\
& y=y_{0}+v_{y} \Delta t
\end{aligned}
$$

$$
\overrightarrow{\vec{a}}=\frac{\Delta \vec{v}}{\Delta t}
$$

$$
\vec{v}=\vec{v}_{0}+\overline{\vec{a}} \Delta t, \begin{aligned}
& v_{x}=v_{x 0}+a_{x} \Delta t \\
& v_{y}=v_{y 0}+a_{y} \Delta t
\end{aligned}
$$

- Note: Two sides equal if and only if COMPONENTS ARE EQUAL


## Constant Acceleration

- If acceleration constant:

$$
\vec{v}=\vec{v}_{0}+\vec{a} t
$$

$$
\vec{r}=\frac{1}{2}\left(\vec{v}_{0}+\vec{v}\right) t
$$

$$
\vec{r}=\vec{v}_{0} t+\frac{1}{2} \vec{a} t^{2}
$$

$$
v^{2}=v_{0}^{2}+2 a r
$$

$$
v_{y}=v_{y 0}+a_{y} t
$$

$$
y=\frac{1}{2}\left(v_{y 0}+v_{y}\right) t
$$

$$
x=v_{x 0} t+\frac{1}{2} a_{x} t^{2}
$$

$$
y=v_{y 0} t+\frac{1}{2} a_{y} t^{2}
$$

$$
v_{x}^{2}=v_{x 0}^{2}+2 a_{x} x
$$

$$
v_{y}^{2}=v_{y 0}^{2}+2 a_{y} y
$$

A plane is landing at a speed of $50 \mathrm{~m} / \mathrm{s}$. The plane is 1000 m from the runway. The plane is descending at an angle of 20 deg. As part of a stunt for a movie, a jeep wants to travel directly underneath the plane all the way to the runway. What speed will the jeep need to travel?
(1) $50 \mathrm{~m} / \mathrm{s}$
(2) $(50 \mathrm{~m} / \mathrm{s}) \sin 20^{\circ}$
(3) $(50 \mathrm{~m} / \mathrm{s}) \cos 20^{\circ}$
(4) $(50 \mathrm{~m} / \mathrm{s}) / \mathrm{sin} 20^{\circ}$
(5) $(50 \mathrm{~m} / \mathrm{s}) / \cos 20^{\circ}$


940 m

Find horizontal component of velocity of plane and match that.

A plane is landing at a speed of $50 \mathrm{~m} / \mathrm{s}$. The plane is 1000 m from the runway. The plane is descending at an angle of 20 deg . As part of a stunt for a movie, a jeep wants to travel directly underneath the plane all the way to the runway. From it's current position, how long will it take for the plane to touch down?
(1) 10.0 s
(3) 21.3 s (5) 77 s
 time $=$ distance/velocity

Can also find horiz component of velocity and use horiz displacement
Or use vert component of velocity and vert displacement

## Projectile Motion

- If free-fall, acceleration $9.8 \mathrm{~m} / \mathrm{s}^{2}$ downward AND
- IF choose y-axis vertical (+y up typically)
- $a_{x}=0 \mathrm{~m} / \mathrm{s}^{2}, \quad a_{y}=9.8 \mathrm{~m} / \mathrm{s}^{2}$ downward
http://capa2.phy.ohiou.edu/res/ohiou/physlets/2dkinematics/proj2.html

$$
\vec{v}=\vec{v}_{0}+\vec{a} t
$$

$$
\vec{r}=\frac{1}{2}\left(\vec{v}_{0}+\vec{v}\right) t
$$

$$
\vec{r}=\vec{v}_{0} t+\frac{1}{2} \vec{a} t^{2}
$$

$$
v^{2}=v_{0}^{2}+2 a r
$$

$$
\frac{v_{x}^{2}-v_{x 0}^{2}+2 a_{x} x}{}
$$

$$
v_{y}^{2}=v_{y 0}^{2}+2 a_{y} y
$$

$$
y=\frac{1}{2}\left(v_{y 0}+v_{y}\right) t
$$

$$
y=v_{y 0} t+\frac{1}{2} a_{y} t^{2}
$$

## A soccer ball is kicked on a level playing field. It's initial velocity is $20 \mathrm{~m} / \mathrm{s}$ at an angle of $30^{\circ}$ above the horizontal. Which of the following is the most accurate statement about the speed of the ball?

1. it is a minimum at maximum height
2. it is zero at maximum height
3. it is a minimum just before it lands
4. it is a minimum just after it is kicked
5. it has the same speed throughout the trajectory

$\mathrm{v}_{\mathrm{x}}$ constant. Speed depends on both $\mathrm{v}_{\mathrm{x}}$ and $\mathrm{v}_{\mathrm{y}}$. Smallest speed when $\mathrm{v}_{\mathrm{y}}=0$.

## Example: Soccer Ball

## http://capa2.phy.ohiou.edu/res/ohiou/physlets/2dkinematics/proj2.html

A soccer ball is kicked on a level playing field. It's initial velocity is $20 . \mathrm{m} / \mathrm{s}$ at an angle of $30^{\circ}$ above the horizontal.

- What is the maximum height the ball reaches? 5.1 m
- How long is the ball in the air?
2.04 s
- What is the final speed of the ball just before it hits the ground? $20 . \mathrm{m} / \mathrm{s}$
- How far away does it hit the ground? 35.3 m

$$
\begin{aligned}
& v_{x}=v_{x 0} \\
& x=v_{x 0} t
\end{aligned}
$$

$$
v_{y}=v_{y 0}+a_{y} t
$$

$$
y=\frac{1}{2}\left(v_{y 0}+v_{y}\right) t
$$

$$
y=v_{y 0} t+\frac{1}{2} a_{y} t^{2}
$$

$$
v_{y}^{2}=v_{y 0}^{2}+2 a_{y} y
$$

## Reminders

- Review on Chaps 1\&2 next Wed.
- New deadline on "Active" questions (Avg velocity+ follow moving ball): today 4pm.
- RQ\#4 due tomorrow 10am
- HW\#3 due Tuesday 6/26, 11:59pm.
- HW\#4 due Sunday 7/01, 11:59pm
- Lab sessions:
- PRINT OUT Lab Report before going to labs
- Download lab report from LONCAPA website
- Fill out the Pre-lab questions BEFORE going to lab!
- CAPA sessions: Clippinger 036, M-Th, 1-5pm
- EXAM 1: Next Thursday (06/29).
- Topics: Chapters 1 and 2 (including related Lab material)

Three projectiles are launched as described. Which one reaches the highest altitude?

1. A 5 kg ball launched straight up at $8 \mathrm{~m} / \mathrm{s}$
2. A 4 kg ball launched at $9 \mathrm{~m} / \mathrm{s}$ at an angle $70^{\circ}$ above the horizontal
3. A 2 kg ball launched at $10 \mathrm{~m} / \mathrm{s}$ at an angle $50^{\circ}$ above the horizontal
4. All three reach the same maximum altitude

Compare vertical components of velocity. Largest $\mathrm{v}_{\mathrm{y}}$ will go the highest.

A cart travels at constant speed along a level track. At a given point, a ball pops straight up from the cart. Which of the following is the best option for the behavior of the ball.

## Demo 1

1. The ball will land in front of the cart
2. The ball will land in the cart
3. The ball will land behind the cart
4. The ball at the point on the track where the ball was launched

Horizontal motion independent of vertical motion.
In this case, both objects have constant horizontal velocity

A firefighter is spraying water on a building. Water leaves the hose at $35 \mathrm{~m} / \mathrm{s}$ and an angle of $30^{\circ}$ above the horizontal. The nozzle of the hose is 1.0 m above the ground, and the building is 22 m away. The building is 40 m tall.

## How long does it take for the water to reach the building?

(1) 0.63 s (2) 0.72 s (3) 1.00 s (4) 1.32 s (5) 1.78 s

You have enough information to determine this from the horizontal part of the problem:

You know $\mathrm{v}_{\mathrm{x}}=35^{*} \cos 30$ and that the water travels 22 m

A firefighter is spraying water on a building. Water leaves the hose at $35 \mathrm{~m} / \mathrm{s}$ and an angle of $30^{\circ}$ above the horizontal. The nozzle of the hose is 1.0 m above the ground, and the building is 22 m away. The building is 40 m tall.


## How is the water traveling when it hits the building?

## (1) up (2) level (3) down

Find $v_{y}$ at $t=0.72 \mathrm{~s}$. If it is positive, water traveling up. If negative, water traveling down.
$v_{y}=v_{y 0}+$ at $=35^{*} \sin (30)+(-9.8)^{*} 0.72 \mathrm{~s}=+10.4 \mathrm{~m} / \mathrm{s}$
Upward

A firefighter is spraying water on a building. Water leaves the hose at $35 \mathrm{~m} / \mathrm{s}$ and an angle of $30^{\circ}$ above the horizontal. The nozzle of the hose is 1.0 m above the ground, and the building is 22 m away. The building is 40 m tall.


What is the speed of the water just before it hits the building?
$\begin{array}{ll}\text { (1) } 17.5 \mathrm{~m} / \mathrm{s} & \text { (2) } 30.3 \mathrm{~m} / \mathrm{s} \\ \text { 3) } 32.0 \mathrm{~m} / \mathrm{s} & \text { (4) } 35.0 \mathrm{~m} / \mathrm{s}\end{array}$
(5) $47.8 \mathrm{~m} / \mathrm{s}$

Need $x$ and $y$ components of speed. $v_{x}=35^{*} \cos (30)$
$\mathrm{v}_{\mathrm{y}}$ just found as 10.4
Use Pythagorean's theorem to find speed: $32.0 \mathrm{~m} / \mathrm{s}$

Shoot the monkey: A monkey is hanging from a tree. A hunter takes aim directly at the monkey with a laser pointer. When the shot is fired, the monkey lets go of the branch at the same time the bullet is released from the gun. The bullet will:

1. pass above the monkey
2. Hit the monkey
3. pass below the monkey

Both monkey and bullet fall at same rate.

## Relative Velocity

You are standing on the side of a railroad and a train passes by at a velocity $\mathrm{v}_{\mathrm{TG}}=+9.0 \mathrm{~m} / \mathrm{s}$ (relative to the ground). A passenger walks towards the front of the moving train. To the people sitting in the train, his velocity is $\mathrm{v}_{\mathrm{PT}}=+2.0 \mathrm{~m} / \mathrm{s}$ (relative to the train).


The velocity of the passenger relative the ground $\left(v_{\mathrm{PG}}\right)$ is:

$$
\begin{aligned}
& \mathrm{v}_{\mathrm{PG}}=\mathrm{v}_{\mathrm{PT}}+\mathrm{v}_{\mathrm{TG}} \\
& \mathrm{v}_{\mathrm{PG}}=(+2 \mathrm{~m} / \mathrm{s})+(+9 \mathrm{~m} / \mathrm{s})=+11 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

## 1-d Relative Velocity

You are driving at $25 \mathrm{~m} / \mathrm{s}$ in the $+x$ direction. Your friend is driving along the same road in the same direction at $28 \mathrm{~m} / \mathrm{s}$.
What is $v_{Y F}$ : the velocity of You relative to your Friend?
$\begin{array}{llll}\text { (1) }-53 \mathrm{~m} / \mathrm{s} & (2)-3 \mathrm{~m} / \mathrm{s} & \text { (3) } 3 \mathrm{~m} / \mathrm{s} & \text { (4) } 53 \mathrm{~m} / \mathrm{s}\end{array}$

Assume a stationary observer on side of road.

$$
\begin{aligned}
& \mathrm{v}_{\mathrm{YO}}=\mathrm{v}_{\mathrm{YF}}+\mathrm{v}_{\mathrm{FO}} \\
& 25 \mathrm{~m} / \mathrm{s}=\mathrm{v}_{\mathrm{YF}}+28 \mathrm{~m} / \mathrm{s} \\
& \mathrm{v}_{\mathrm{VF}}=-3 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

A moving sidewalk at an airport is traveling with a speed of $2.0 \mathrm{~m} / \mathrm{s}$. It is 100 m long.
You can cover this distance in 50s on foot without the sidewalk.
How long does it take you to travel this distance if you are walking on the moving sidewalk?
(1) $10 \mathrm{~s} \quad$ (2) $20 \mathrm{~s} \quad$ (3) 25 s (4) $40 \mathrm{~s} \quad$ (5) 50 s

Your walking speed is $2 \mathrm{~m} / \mathrm{s}$. If walking $2 \mathrm{~m} / \mathrm{s}$ on sidewalk moving $2 \mathrm{~m} / \mathrm{s}$, you are moving $4 \mathrm{~m} / \mathrm{s}$ relative to ground.
$100 \mathrm{~m} / 4 \mathrm{~m} / \mathrm{s}=25 \mathrm{~s}$

## $\mathrm{g}=9.8 \mathrm{~m} / \mathrm{s}^{2}$

## Chapter 3

## downward

$\overrightarrow{\vec{v}}=\frac{\vec{r}-\vec{r}_{0}}{t-t_{0}}=\frac{\Delta \vec{r}}{\Delta t}$

$$
\overrightarrow{\vec{a}}=\frac{\Delta \vec{v}}{\Delta t}
$$

$$
c^{2}=a^{2}+b^{2}
$$

$\sin \theta=\frac{\text { opposite }}{\text { hypotenuse }}$

$$
\begin{array}{|l|}
\hline \cos \theta=\frac{\text { adjacent }}{\text { hypotenuse }} \\
\hline \tan \theta=\frac{\text { opposite }}{\text { adjacent }} \\
\hline
\end{array}
$$

$$
v_{x}=v_{x 0}+a_{x} t
$$

$$
x=\frac{1}{2}\left(v_{x 0}+v_{x}\right) t
$$

$$
x=v_{x 0} t+\frac{1}{2} a_{x} t^{2}
$$

$$
v_{x}^{2}=v_{x 0}^{2}+2 a_{x} x
$$

$$
v_{y}=v_{y 0}+a_{y} t
$$

$$
y=\frac{1}{2}\left(v_{y 0}+v_{y}\right) t
$$

$$
y=v_{y 0} t+\frac{1}{2} a_{y} t^{2}
$$

$$
v_{y}^{2}=v_{y 0}^{2}+2 a_{y} y
$$

