Reminders

- **RQ#12,13 were due today 07/17 10am**
- **Last Capa assignment HW#9 due today 7/17, 11:59pm**

- **Today: finish content (Ch.8 & 9 - Rotational Kinematics and dynamics).**
- **Tomorrow: Q&A review for the Final (Bring your questions!)**
- **Tomorrow: "Office hours" 5pm-6pm in Clippinger 035**
- **(No Capa sessions tomorrow)**

- **FINAL EXAM: Thursday (07/19).**

  Topics:
  - Intro (Chapter 1)
  - 1D and 2D Kinematics (Chapters 2 and 3).
  - Newton' s Law and Forces (Chapter 4)
  - Torques and equilibrium (Chapters 9, secs 1-3),
  - Uniform Circular motion (chapter 5)
  - Work and Energy (chapter 6)
  - Momentum (Chapter 7)
  - Fluids (Chapter 11) (today and Monday)
  - Rotational Kinematics and Dynamics (Chapter 8 and 9) (Tuesday)

  (including Lab-related material).

Dynamics and kinematics of rotational motion

All motion can be broken into these parts

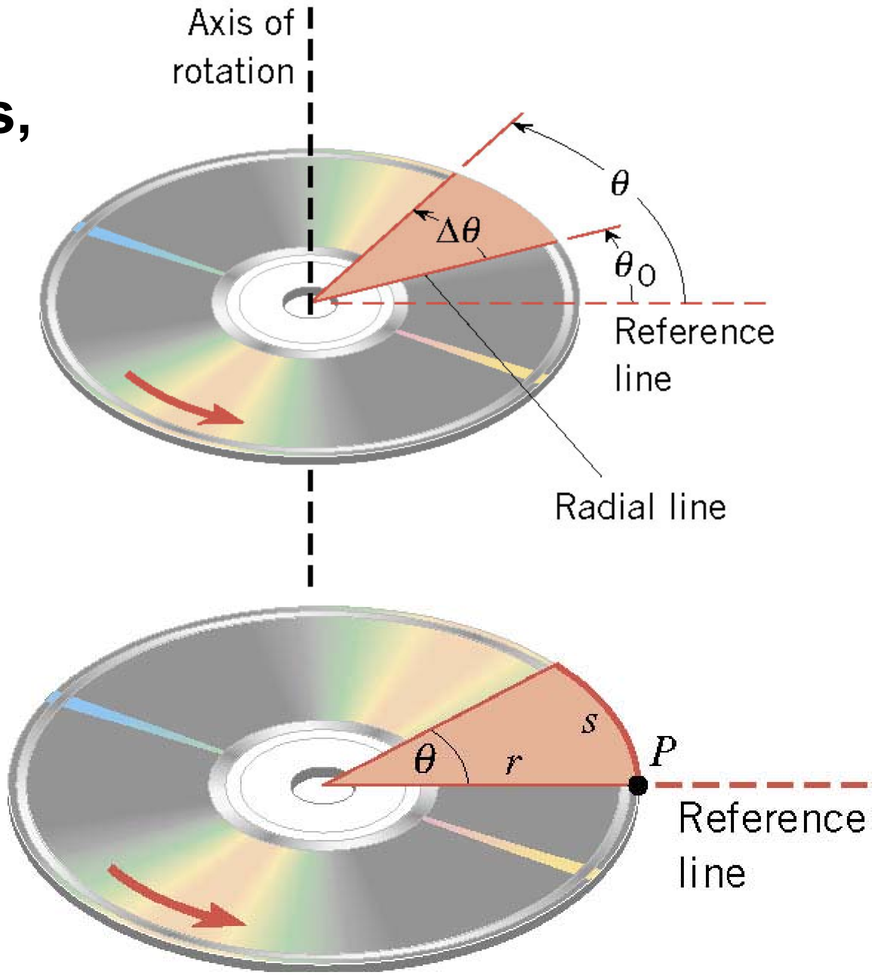
1. Translational – linear (seen in Chapters 2,3)
2. Rotational – object move in circular paths
    - **Motion around an *axis of rotation*.**
    - **There are the rotational equivalents of: displacements, velocities, acceleration and *Forces.***

Chapter 8 - Rotational Kinematics
Describing Rotation

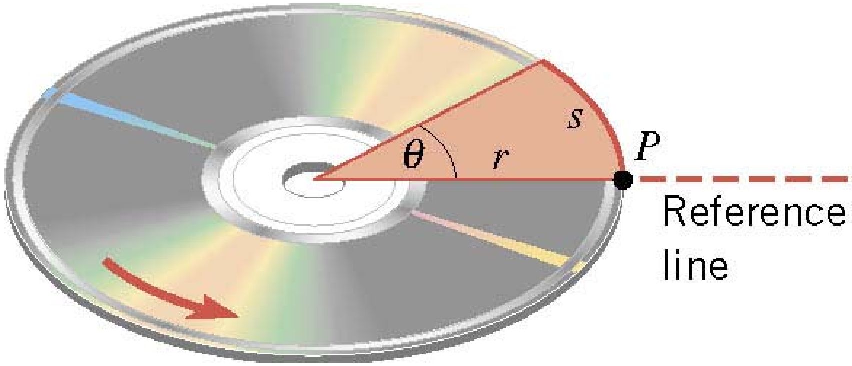
- Angular ***Displacement*** $\Delta\theta$
  - **measure in radians, degrees, or revolutions**
- Angular ***velocity***

  $\omega = \Delta\theta / \Delta t$
  - **rad/s, rpm, deg/sec**
- Angular ***acceleration***

  $\alpha = \Delta\omega / \Delta t$
  - **rad/s^2**
- θ(rad) = arc length/radius = s/r
  - **ratio: dimensionless**
  - **rad simple placeholder**



Angles in Radians



- θ(rad) = arc length/radius = s/r
  - **ratio: dimensionless**
  - **rad simple placeholder**
- Full circle: θ(rad) = circumference/radius = $2\pi r/r = 2\pi$
- 360° = 2π rad   180° = π rad
- 135° = ?

  $(3/4)\pi$ rad or 2.36 rad

A wheel undergoes an angular displacement of π/3. What is this in degrees?

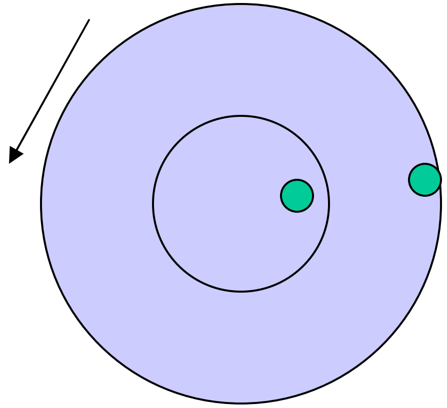
(1) 15 (2) 30 (3) 45 (4) 60 (5) 75

(6) 90 (7) 105 (8) 120 (9) 135 (0) 150

$\pi \text{ rad} = 180^\circ$ $(\pi/3)(180^\circ/\pi \text{ rad}) = 60^\circ$

Two objects are sitting on a rotating turntable. One is much further out from the axis of rotation. Which one has the larger angular velocity?

1) the one nearer the disk center
2) the one nearer the disk edge
3) they both have the same angular velocity



All points on rigid object have same angular displacement ($\Delta\theta$),
same angular velocity (ω), and same angular acceleration (α)

This is why angular quantities are so useful!

Linear ("along the rim") quantities

- Arc length or linear distance travelled

$$s = r\,\Delta\theta$$

- Tangential velocity or speed

$$(s/\Delta t) = r\,(\Delta\theta/\Delta t)$$

$$v_t \quad = r\,\omega$$

- Tangential acceleration

$$(v_t\,/\,\Delta t) = r\,(\omega/\Delta t)$$

$$a_t \quad = r\,\alpha$$

Centripetal Acceleration

- Circular motion – net force towards center required
- $a_c = v^2 / r \quad = (r\omega^2)/r = r\,\omega^2$

- $a_c = v^2 / r$ or $a_c = r\,\omega^2$

Use whichever is most convenient!

An object starts at rest and undergoes an average angular acceleration of 0.5 rad/s^2 for 10 seconds. What is the angular speed after 10 seconds?

(1) 0.05 rad/s (2) 0.5 rad/s (3) 5 rad/s

(4) 10 rad/s (5) 20 rad/s (6) 50 rad/s

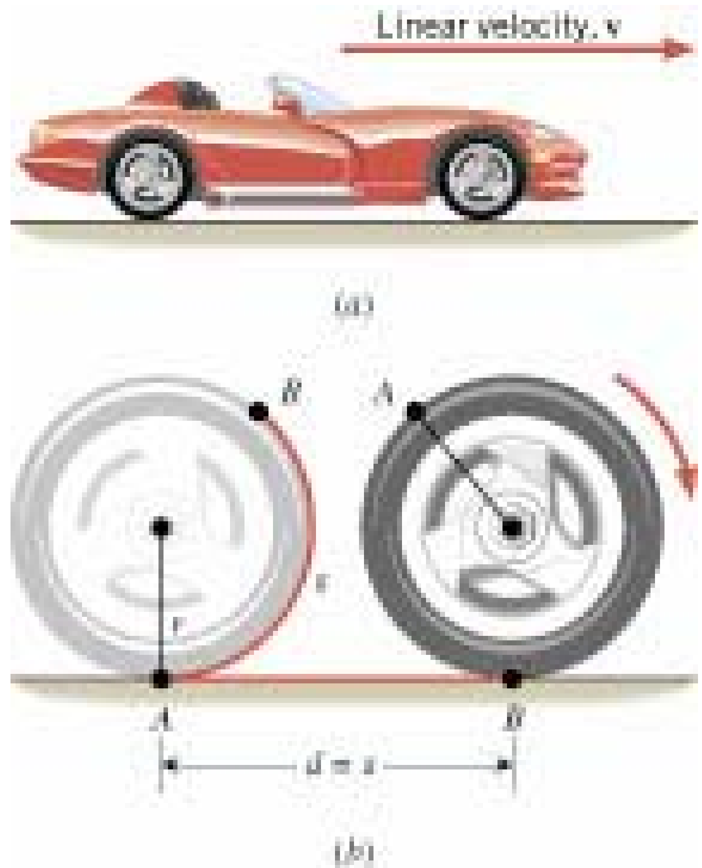
$\alpha = \Delta\omega / \Delta t$

$\Delta\omega = \alpha \; \Delta t = (0.5 \text{ rad/s}^2) \; 10 \text{ s} = 5 \text{ rad/s}$

Since $\omega_0 = 0$, $\omega_f = 5$ rad/s

- If object rolls without slipping, linear distance traveled is equal to arc length of rotation, so:

$$x = r\,\Delta\theta$$



- Divide by Δt:

$$v_{LINEAR} = r\,\omega$$

- Again:

$$a_{LINEAR} = r\,\alpha$$

A pulley of radius 0.10m has a string wrapped around the rim. If the pulley is rotating on a fixed axis at an angular speed of 0.5rad/s, what is the length of the string that comes off the reel in 10 seconds?

(1) 0.005 m (2) 0.05 m (3) 0.5 m

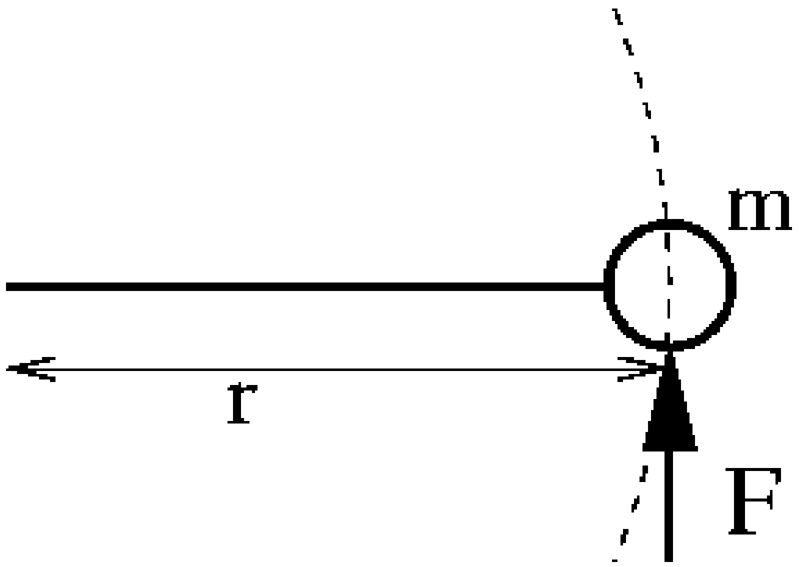
(4) 5.0 m (5) 50 m (6) 500 m

$\Delta\theta = \omega\,(\Delta t) = 5$ rad

arc length $= r\,(\Delta\theta(\text{rad})) = (0.10\text{m})\,(5\text{ rad}) = 0.5$ m

Chapter 9- Rotational Dynamics
Newton's Second Law Analog for Rotational Motion

A puck on a frictionless tabletop attached to center post by massless rod of length r.

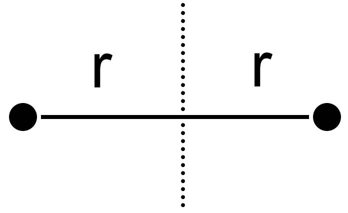


$$F_t = m\, a_t$$
$$F_t = m\,(r\alpha)$$
$$(F_t\, r) = m\, r\, r\, \alpha$$
$$\tau = (m\, r^2)\, \alpha$$
$$\tau = I\, \alpha$$

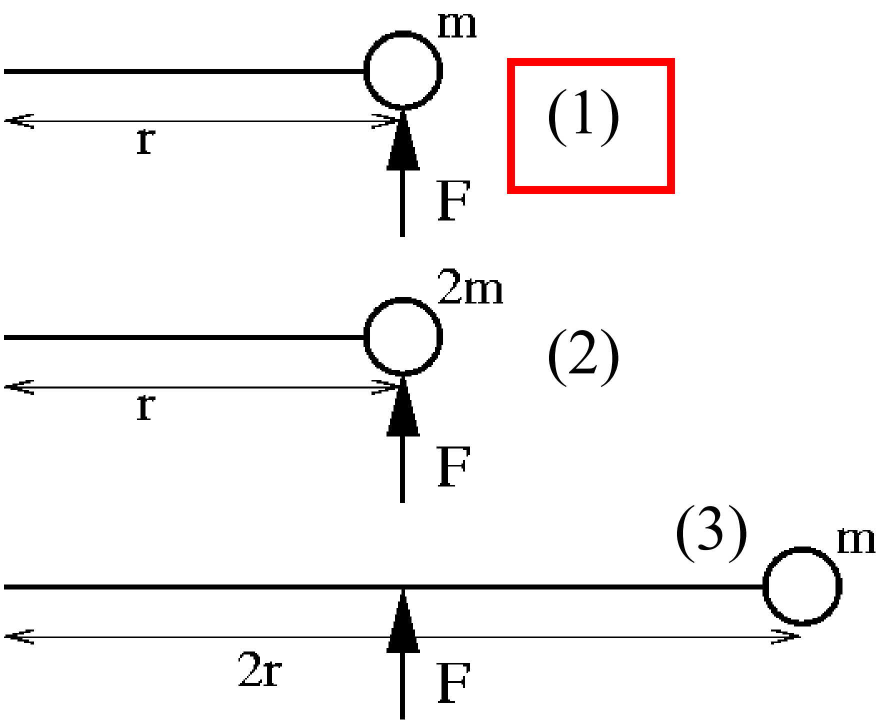
I = moment of Inertia (depends on object geometry)

$I=\Sigma m_i\, (r_i)^2$ ($r_i \rightarrow$ dist of particle "i" to axis)

$I_{POINT} = m\, r^2$; $I_{dumbbell} = 2\, m\, r^2$



Which of the three objects will undergo the greatest angular acceleration?



(4) (1) and (3) same

(5) (1) and (2) same

(6) all the same

(1) has smallest moment of inertia

all have same torque

(1) will have largest acceleration

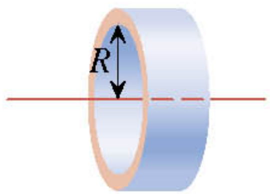
Moment of Inertia – Multiple or Compound Objects

Add! (or integrate)

TABLE 9.1 Moments of Inertia for Various Rigid Objects of Mass *M*

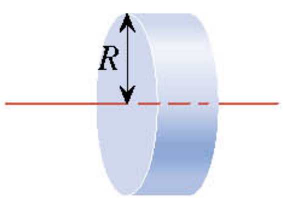
Thin-walled hollow cylinder or hoop

$I = MR^2$



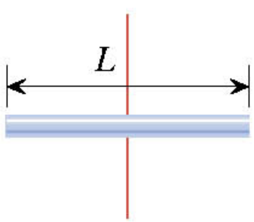
Solid cylinder or disk

$I = \frac{1}{2}MR^2$



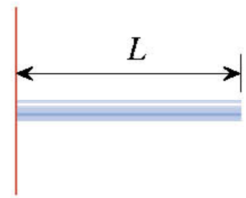
Thin rod, axis perpendicular to rod and passing through center

$I = \frac{1}{12}ML^2$



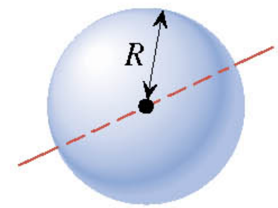
Thin rod, axis perpendicular to rod and passing through one end

$I = \frac{1}{3}ML^2$



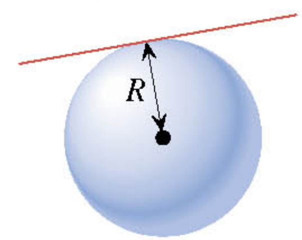
Solid sphere, axis through center

$I = \frac{2}{5}MR^2$



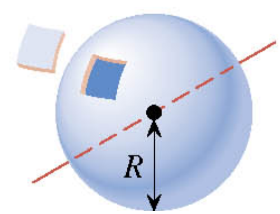
Solid sphere, axis tangent to surface

$I = \frac{7}{5}MR^2$



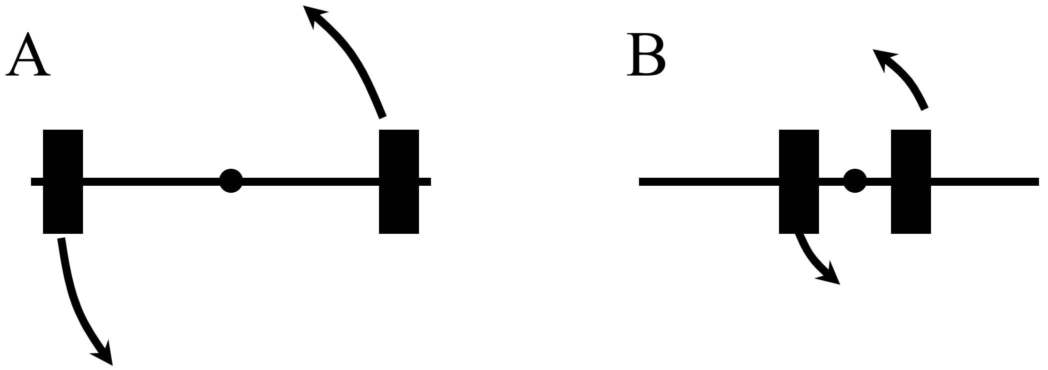
Thin-walled spherical shell, axis through center

$I = \frac{2}{3}MR^2$



Consider two masses on a rod rotating as shown. In case A, the masses are at the end of the rod. In case B the same 2 masses are closer to the center. If the same torque is applied to each situation, which arrangement of masses has the lowest angular acceleration?

1. A
2. B
3. Both the same



$I=\Sigma m_i (r_i)^2$ ($r_i \rightarrow$ dist of particle "i" to axis)

Largest moment of Inertia, lowest angular acceleration (same torque)

Rotational Variables:
Similar formulas as in linear dynamics!

Linear	Angular
x	θ
v	ω
a	α

$x \leftrightarrow \theta$

$v \leftrightarrow \omega$

$a \leftrightarrow \alpha$

Newton's 2nd Law

Force Torque

$$F \leftrightarrow \tau$$

$$\Sigma F = ma \leftrightarrow \Sigma\tau = I\alpha$$

mass mom. inertia

$$m \leftrightarrow I$$

$$I_{POINT} = MR^2$$
$$I_{HOOP} = MR^2$$
$$I_{CYL} = ½MR^2$$

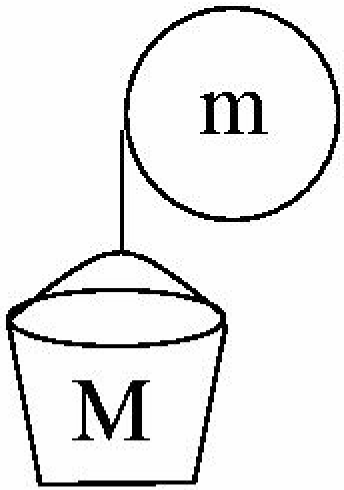
Kinetic energy and momentum

$$½mv^2 \leftrightarrow ½I\omega^2$$

$$p \leftrightarrow L$$

$$mv \leftrightarrow I\omega$$

A M=0.50kg mass is hung from a massive, frictionless pulley of mass m=1.5kg and radius R=0.10m. Starting from rest, how long will it take for the mass to fall 1.0 m?



Torque

$$\tau = M.g.R$$

Moment of Inertia

$$I_{\text{disk}} = \frac{1}{2} m R^2$$

Linear displacement = d =1m

Angular displacement= d/R = 10 rad

Angular variables

$$\tau = I.\alpha \qquad \theta_f = \frac{1}{2}.\alpha.t^2$$

Solve for α, t: α=65.3 rad/s^2 ; t=0.55 s

(if there was no pulley: t=0.45s)

Conservation of Energy

Total Mechanical Energy = KE + PE

$= (KE_{TRANS} + KE_{ROT}) + PE$

$KE_{ROT} = \frac{1}{2} I\omega^2$

If rotating, it has KE_{ROT}

If $W_{NC} = 0$,

$(KE_{TRANS} + KE_{ROT} + PE)_{INIT} = (KE_{TRANS} + KE_{ROT} + PE)_{FINAL}$

A block starting at rest slides down a frictionless ramp. A hoop rolls without slipping down an identically shaped ramped. Both have the same total mass. Which one has the greatest linear speed when it reaches the bottom?

1. Block
2. Hoop
3. Both the same

Part of loss of PE has to go into rotational energy of hoop.

Conservation of Angular Momentum

$$L = I\omega$$

If ***no net external torque***,

$$I_i\,\omega_i = I_f\,\omega_f$$

Directions and forces are unusual

Gyroscope

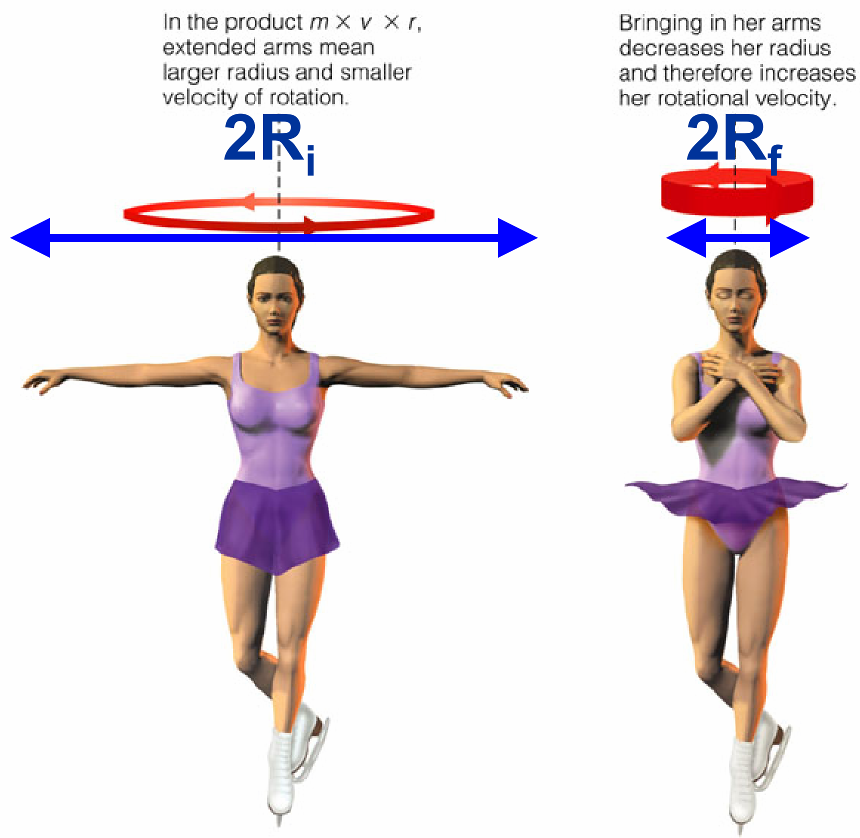
Bicycle tire

Example: Spinning Skater

- Moment of Inertia: ΣMR^2
- R: distance from spinning axis.
- Angular momentum is conserved:

$$I_i \omega_i = I_f \omega_f$$

  - Arms fold in: R decreases, the moment of inertia decreases.
  - Angular velocity *increases.*



$I_f < I_i$ **therefore** $\omega_f > \omega_i$

A student stands on a turntable holding two 2-kg masses in their hands. His/her arms are stretched out, away from his/her body and he/she is rotating. When the student folds his/her arms in (with the masses in hand), the angular velocity of the student will:

1. Increase
2. Decrease
3. Stay the Same

Moment of Inertia decreases, angular speed increases

Chapter 8/9b

$$\boldsymbol{\omega = \Delta\theta / \Delta t}$$

$$\boldsymbol{\alpha = \Delta\omega / \Delta t}$$

$$\theta(\text{rad}) = s/r$$

$$a_c = v^2 / r$$

$$a_c = r\,\omega^2$$

$$\Sigma\tau = I\alpha$$

$$KE_{TRANS} = \tfrac{1}{2}mv^2$$

$$KE_{ROT} = \tfrac{1}{2}I\omega^2$$

$$\text{Ang Mom} = I\omega$$

$$I_{POINT} = MR^2$$

$$I_{HOOP} = MR^2$$

$$I_{CYL} = \tfrac{1}{2}MR^2$$

$$E_{TOTAL} = (KE_{TRANS} + KE_{ROT}) + PE$$

$$\tau = F\,d$$

$$180^\circ = \pi \text{ rad}$$

$$v = r\,\omega$$

$$a = r\,\alpha$$

Some kids roll two tires down the hill. One is empty. The other has one of the kids curled up inside the tire. Which tire is moving the fastest at the bottom of the hill?

1. The empty tire
2. The tire with the kid
3. Both have the same speed

More mass towards center. Easier to get rolling.

Remember that mass still cancels, so total mass doesn't matter