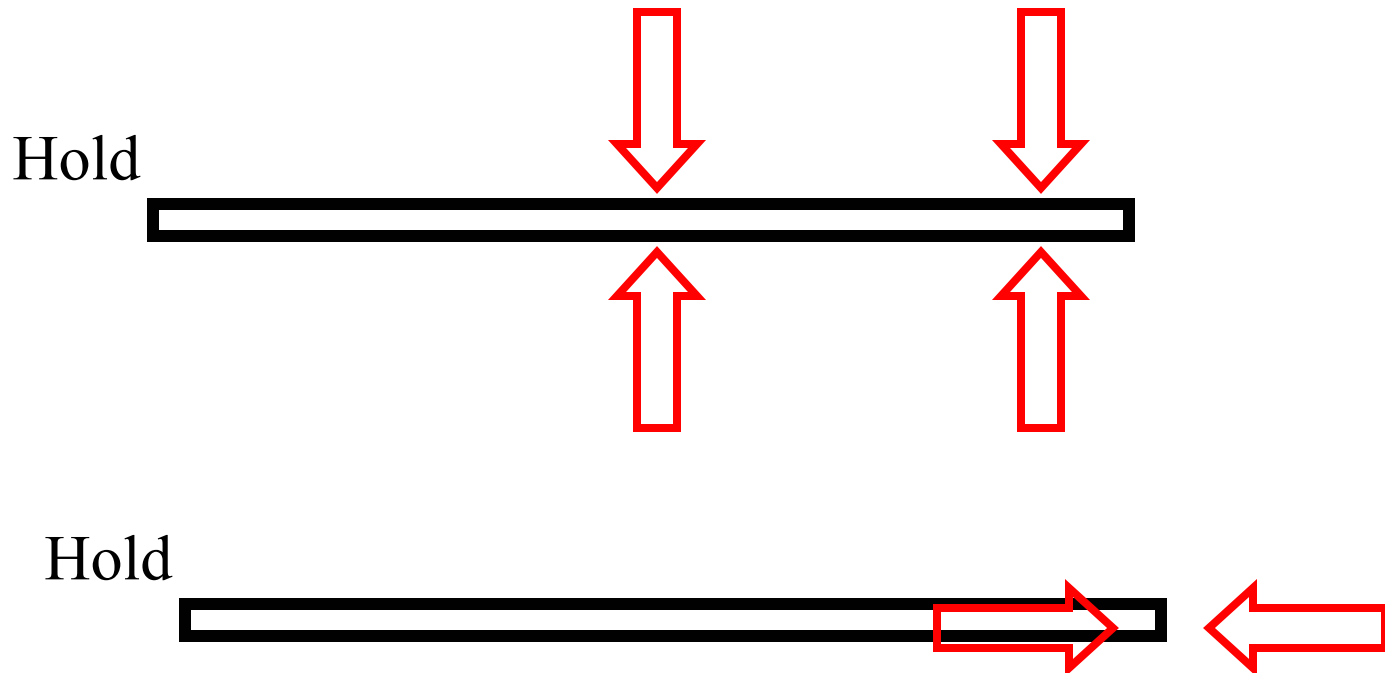


Chapter 9.1-9.3 Torques ('Twists')

- So far, we have considered problems in which it doesn't matter which point we choose to apply the force on an object.
- How about those cases in which the point of application of the force *does* matter?
 - **Example: opening a door**

Chapter 9.1-9.3 Torques ('Twists')

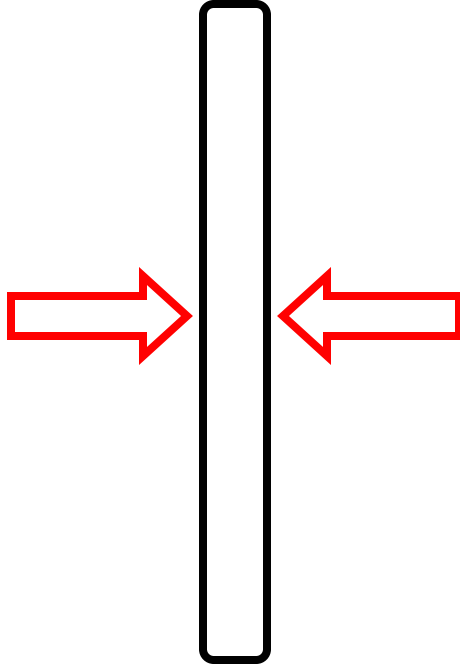
- Hold and push down or up at different places
- What happens if let go of 'hold'?



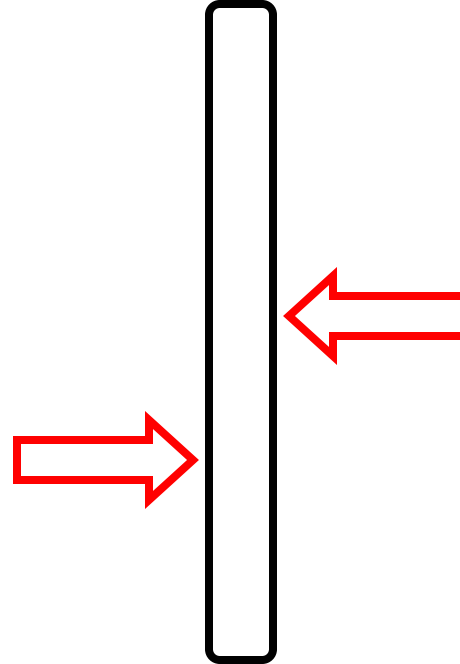
'Twist' depends on force and position force applied

Equilibrium

- Equilibrium – $\text{Accel} = 0$
- Static Equilibrium – $\text{Accel} = 0$ AND $\text{velocity} = 0$
 - **Stationary**



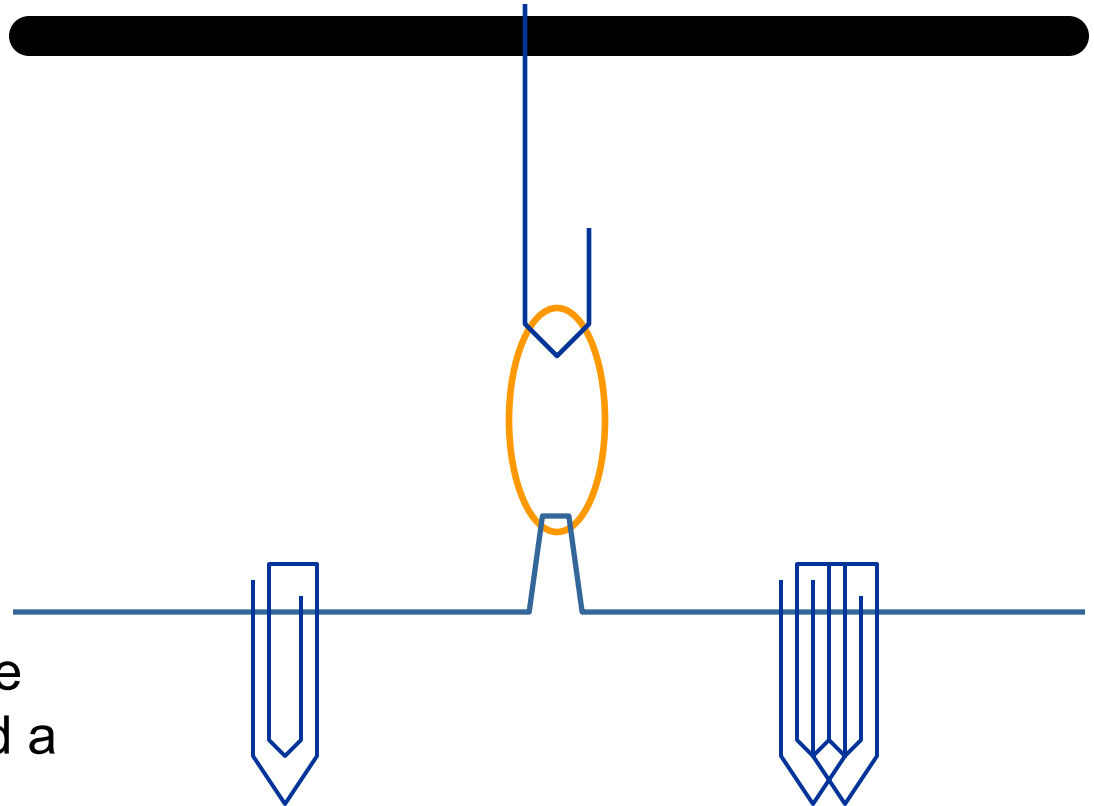
What happens?



What happens?

'Paper clip Twists'

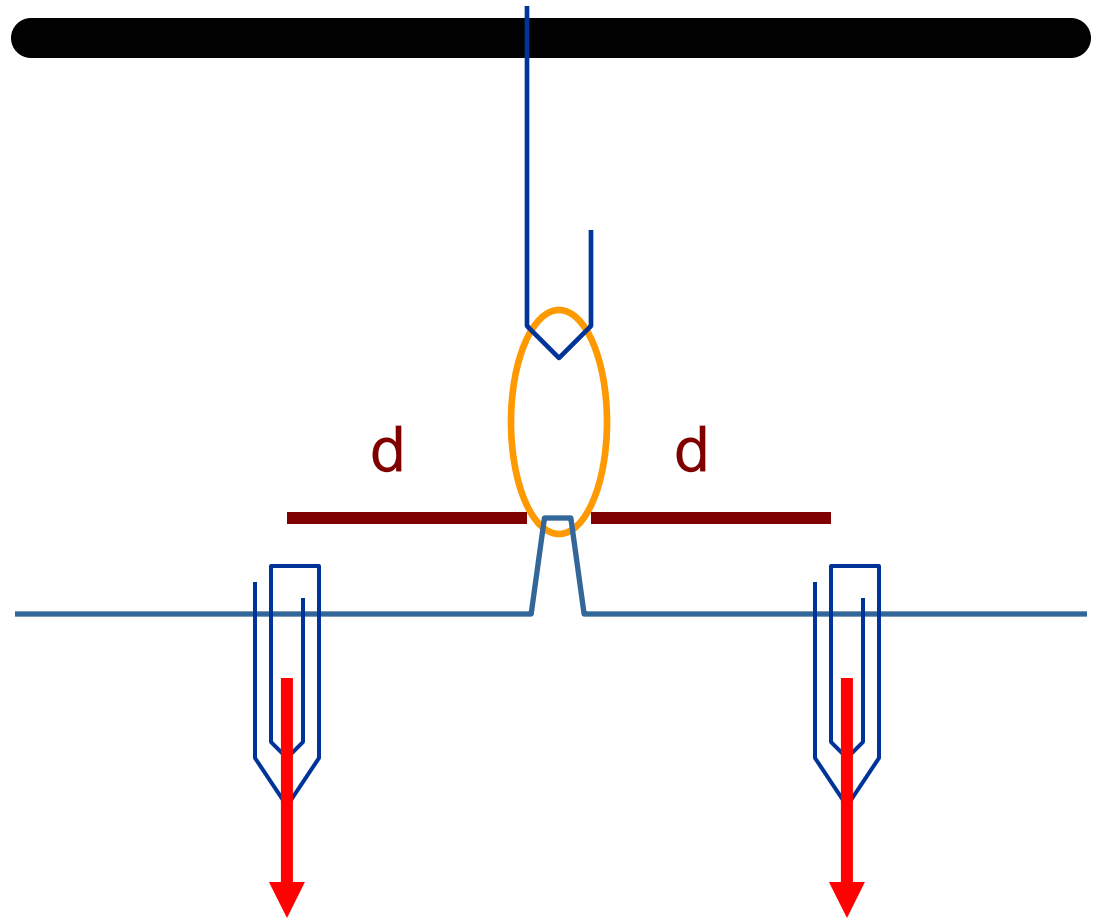
- Work in pairs.
- Get 5 paper clips, a rubber band and a pen/pencil.
- **Paper-clip scale:**
 1. Unbend paper clip, fold it in the mid-section, as shown
 2. Unbend another one and “wrap it” around a pen/pencil;
 3. Join them by a rubber band
 4. Attach paper clips on lever arm.



Torque – Tendency to rotate object

- 1 clip on each side
- Same distance from center.
- What happens? **Why?**

1. No movement.
Net torque is zero.
2. Positive net torque: rotates counterclockwise
3. Negative net torque: rotates clockwise

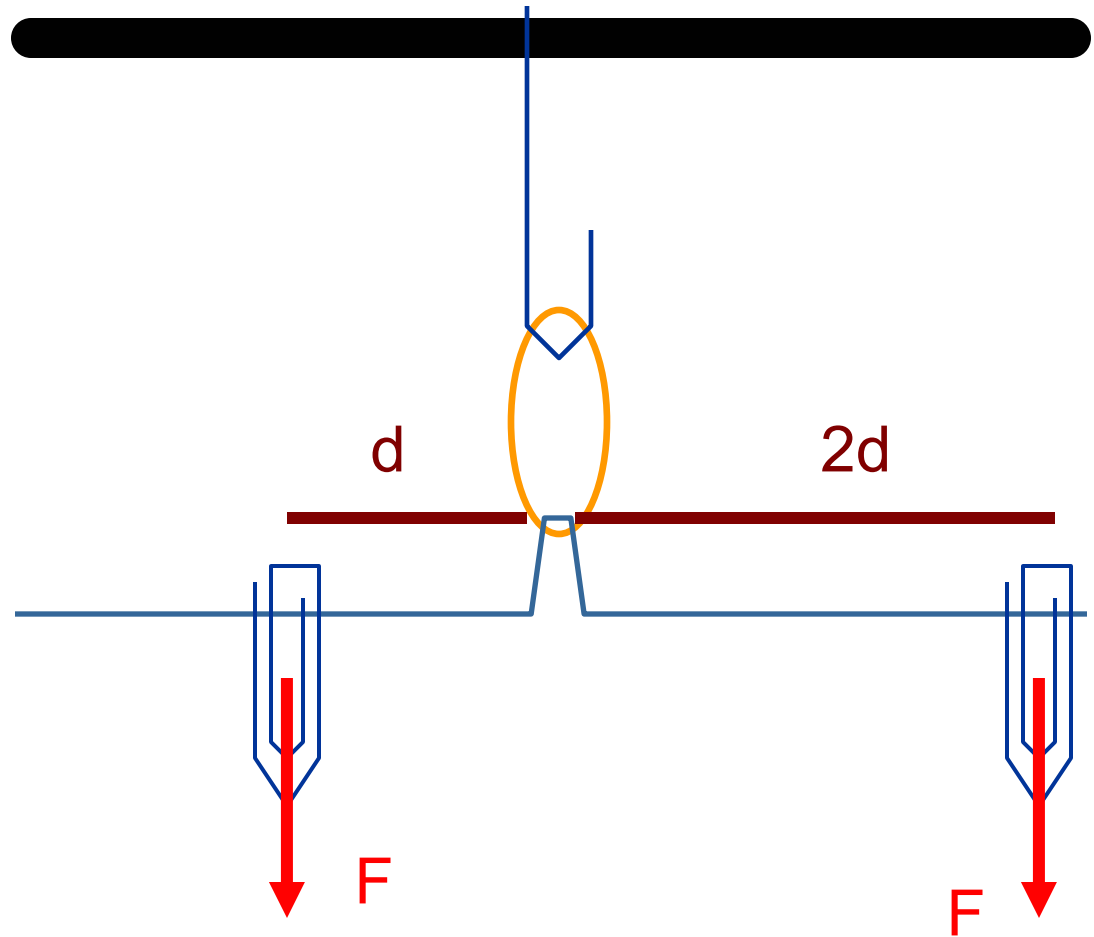


No movement. “Torques and forces are balanced.”

Torque – Tendency to rotate object

- 1 clip on each side
- Different (d and $2d$) distances from center.
- What happens?

1. No movement.
The weight is equal on both sides.
2. No movement.
Net torque is zero.
3. Positive net torque: rotates counterclockwise
4. Negative net torque: rotates clockwise



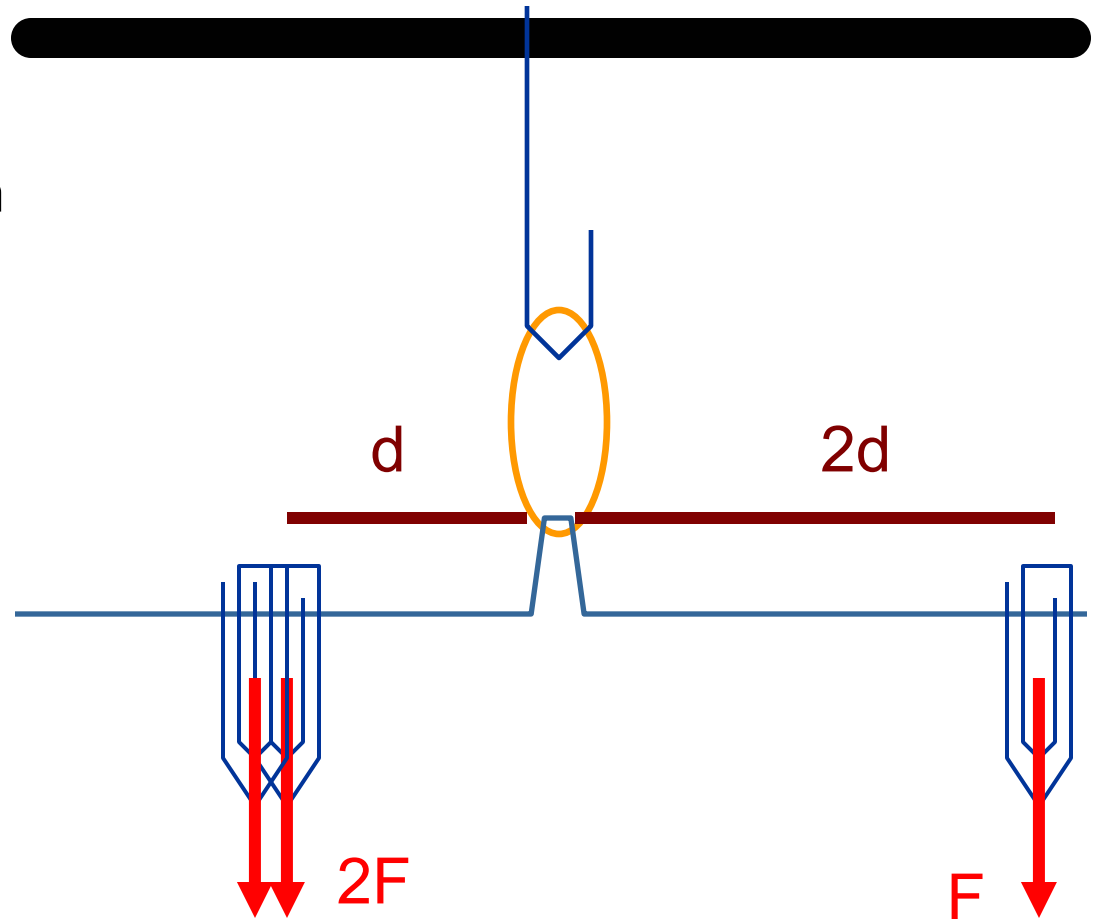
Rotates! Unbalanced torque.

Sign Convention: (+) for CCW rotation; (-) for CW

Torque – Tendency to rotate object

- 1 clip on one side, two clips on the other
- Different distances from center.
- What happens?

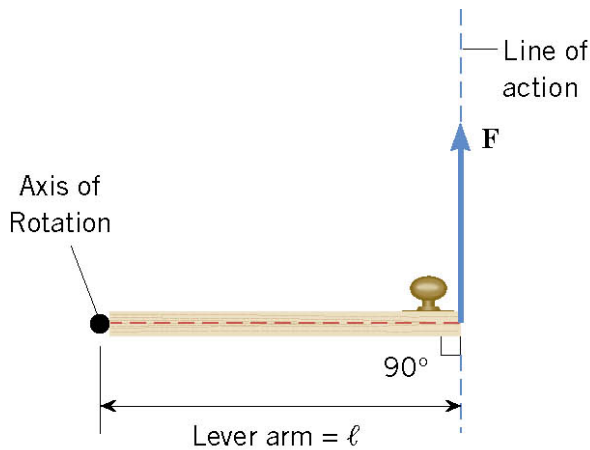
1. No movement.
The weight is equal on both sides.
2. No movement.
Net torque is zero.
3. Positive net torque: rotates counterclockwise
4. Negative net torque: rotates clockwise



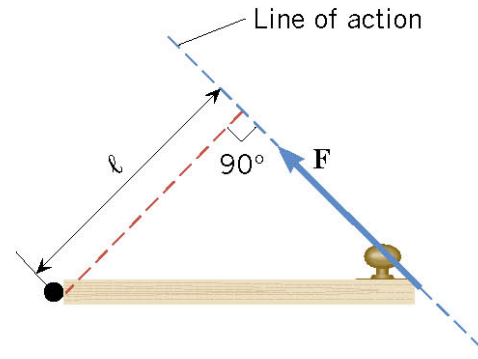
No movement. “Torques are *balanced*.”

Torque – More Formal

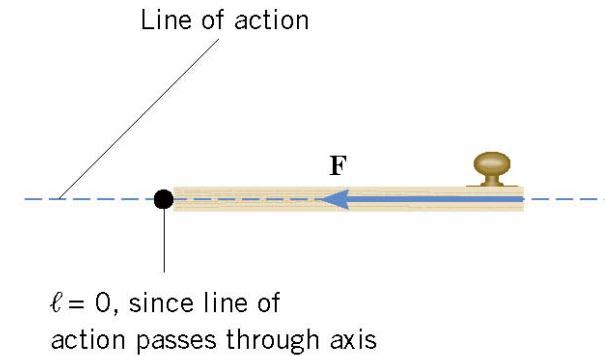
- Pick axis. Can calculate torque due to any force.
- **Torque (τ) = (mag of Force) x (lever arm)**



(a)



(b)



(c)

Lever arm – line drawn through hinge and *perpendicular* to line of action.

And remember!!

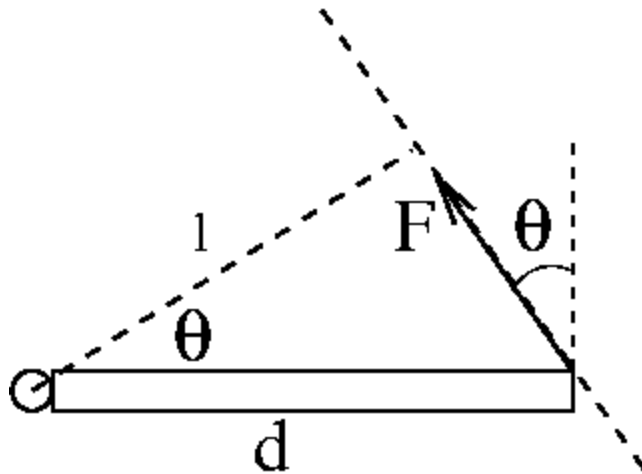
Sign Convention: (+) for CCW rotation; (-) for CW

Two Different Approaches

- Find lever arm through trig
- Look at Components of force

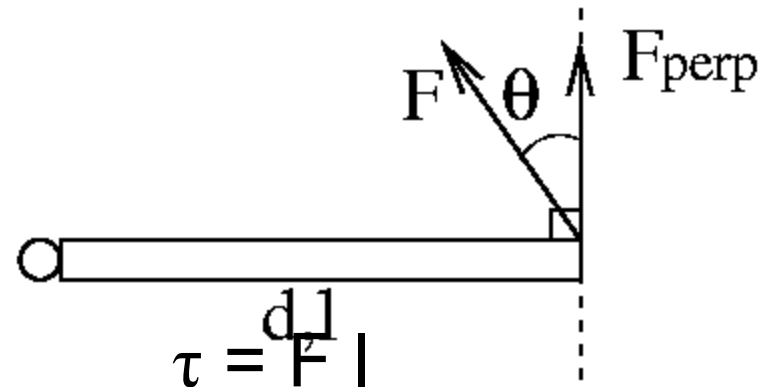
Equilibrium

$$\Sigma F = 0 \quad \text{and} \quad \Sigma \tau = 0$$



$$\tau = F l$$

$$\tau = F (d \cos \theta)$$



$$\tau = F_{\text{perp}} d$$

$$\tau = (F \cos \theta) d$$

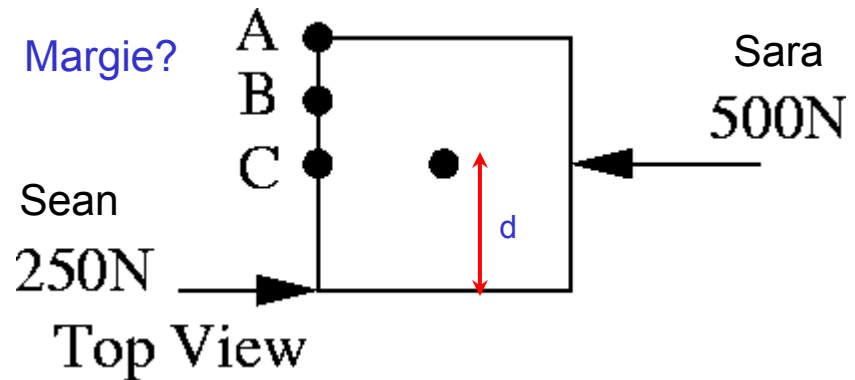
$$\tau = (F \cos \theta) d$$

ALSO: Torque (τ) = (component of Force perp) x (d)

Use the component of the force that is perpendicular to the line connecting hinge and point of application

You are looking at the top view of a crate. Sara is pushing to the left towards the center of the box. Sean is pushing to the right at the bottom left corner of the box. With what force and at what point should Margie push to keep the box stationary?

- (1) 125 at A (2) 125 at B
 (3) 125 at C (4) 250 at A
 (5) 250 at B (6) 250 at C
 (7) 500 at A (8) 500 at B
 (9) 500 at C



Need to balance *forces* AND *torques*: Forces: $F_{\text{Margie}} = F_{\text{Sara}} - F_{\text{Sean}} = 250 \text{ N}$

Torques: (pick axis of rotation, I choose middle point:)

$$\tau_{\text{Sean}} = + 250 \times d, \tau_{\text{Sara}} = 0 \text{ (lever arm} = 0)$$

$$\tau_{\text{Sean}} + \tau_{\text{Sara}} + \tau_{\text{Margie}} = 0 \rightarrow \tau_{\text{Margie}} = - 250 \times d \text{ (CW rotation, point A)}$$

NOTE: If static equilibrium, can pick any point as axis of rotation

Applying the conditions of equilibrium

1. Draw a FBD, choosing a convenient set of x,y axes to resolve forces into components.

2. Balance *forces*:

$$\sum F_x = 0 \quad \sum F_y = 0$$

3. Choose a convenient axis of rotation and identify the point where each external force acts on the object

4. Calculate the torque due to each force

5. Assign the correct sign (+ for CCW and – for CW).

6. Balance *torques*: $\sum \tau = 0$

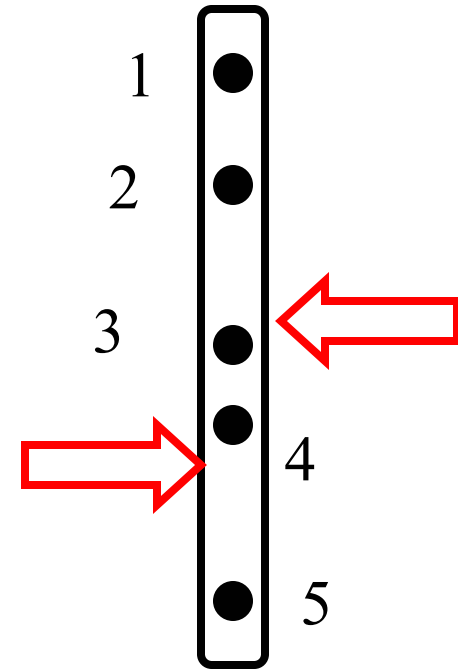
Where can you press with a third finger to keep the stick in static equilibrium?

(6) 1 and 2

(7) 1 and 4

(8) 1, 2 and 5

(9) 1, 2, 4, AND 5

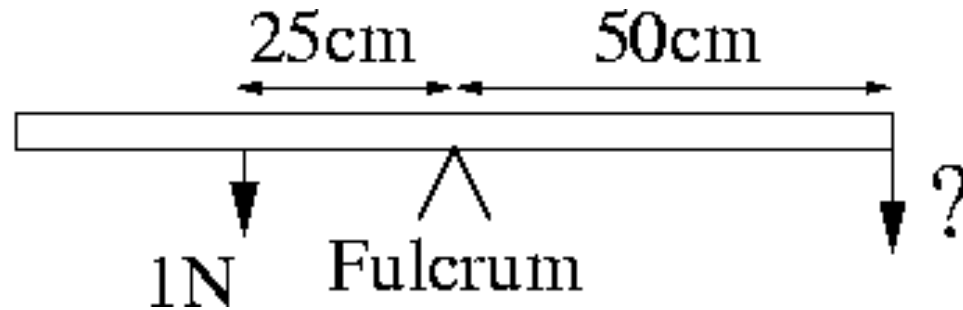


What force is required to balance the meter stick?

(1) 0.5N

(2) 1.0N

(3) 2.0N



Equilibrium: Sum of Forces = 0; Sum of Torques = 0

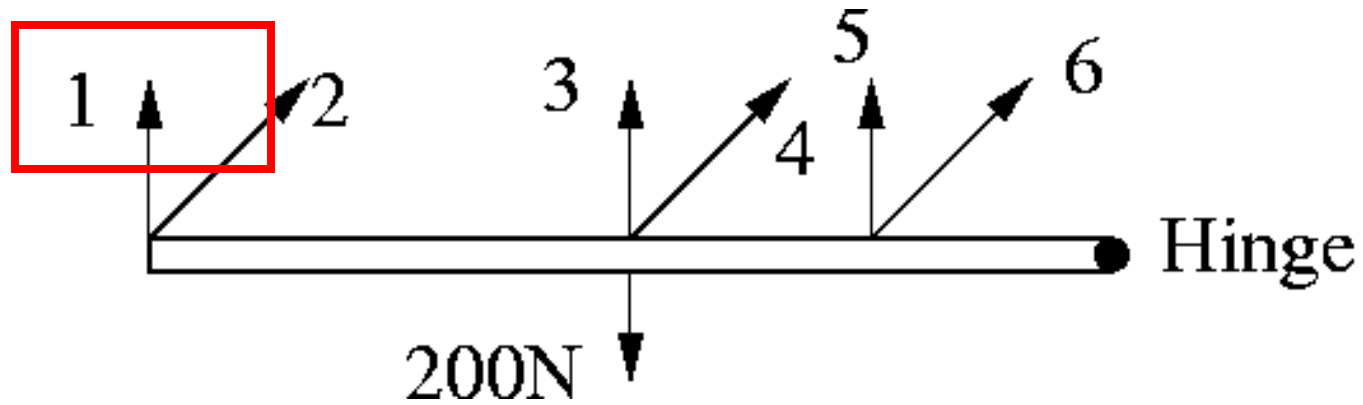
Torques: (pick axis of rotation, I choose middle point:)

$$\tau_1 = + 1 \text{ N} \times 0.25 \text{ m} = 0.25 \text{ Nm} , \tau_2 = - F_2 \times 0.50 \text{ (CW motion)}$$

$$\tau_1 + \tau_2 = 0 \rightarrow 0.25 - 0.5F_2 = 0 \rightarrow F_2 = 0.5 \text{ N}$$

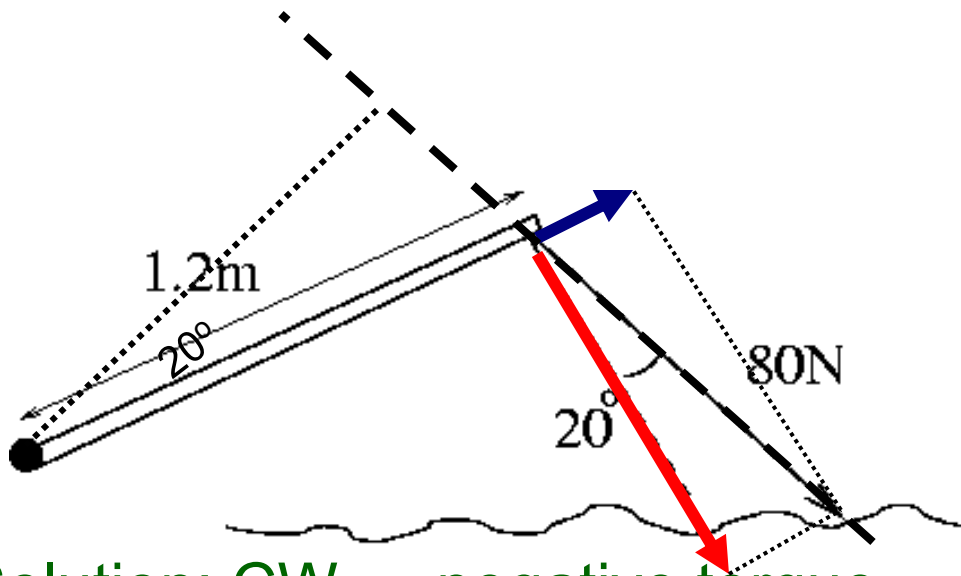
Follow-up question: what is balancing the forces?

At which point and at which direction would the least amount of force be required to hold the lever stationary?



Longest lever arm

What is the torque on the fishing pole? Use the end where it is held as the axis of rotation.



(1) $80\text{N} (\cos 20^\circ) 1.2\text{m}$

(2) $80\text{N} (\sin 20^\circ) 1.2\text{m}$

(3) $80\text{N} * 1.2\text{m}$

(4) $-80\text{N} (\cos 20^\circ) 1.2\text{m}$

(5) $-80\text{N} (\sin 20^\circ) 1.2\text{m}$

(6) $-80\text{N} * 1.2\text{m}$

Solution: CW → negative torque

1- Component of Force **perp** to line connecting hinge and point of application: $F_{\text{perp}} = F \cos 20$

$$\tau = -F_{\text{perp}} \times d = -F (\cos 20) (1.2)$$

OR 2 - find lever arm: $\ell = d(\cos 20)$

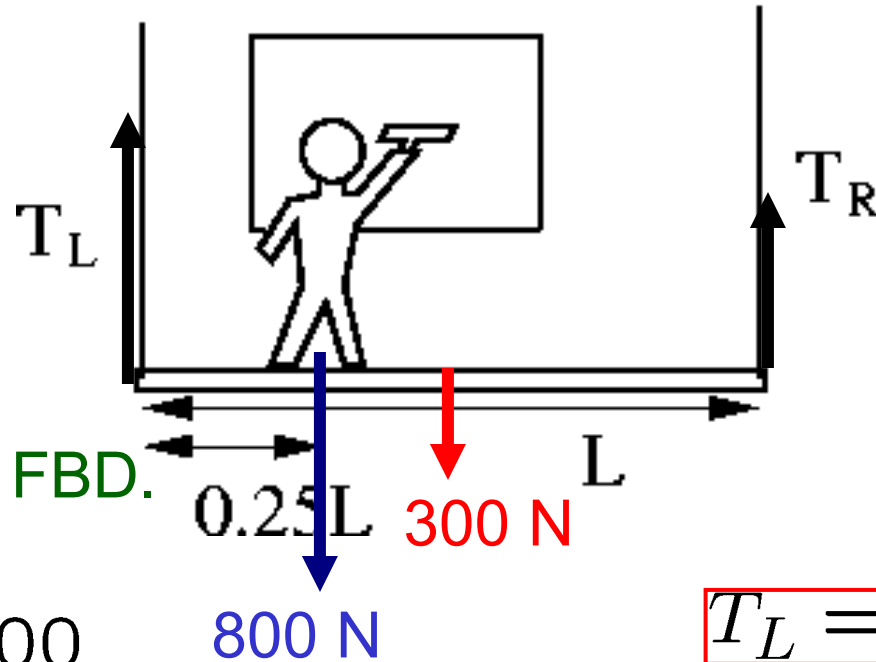
$$\tau = -F \times \ell = -F (\cos 20) (1.2)$$

A 800N painter stands on a 300N scaffold (a uniform rectangular board) with one rope supporting each end. The painter stands one-quarter of the length of the scaffold from the left side. Compare the tension in the two ropes.

(1) $T_L > T_R$

(2) $T_L < T_R$

(3) $T_L = T_R$



Solution: Static problem, FBD.

$$\sum F_y = 0$$

$$T_L + T_R = 800 + 300$$

$T_L = 750N$

$\sum \tau = 0$ (Choose axis at scaffold endpoint)

$$T_R \times L - T_L \times 0 - 800 \times 0.25L - 300 \times L/2 = 0$$

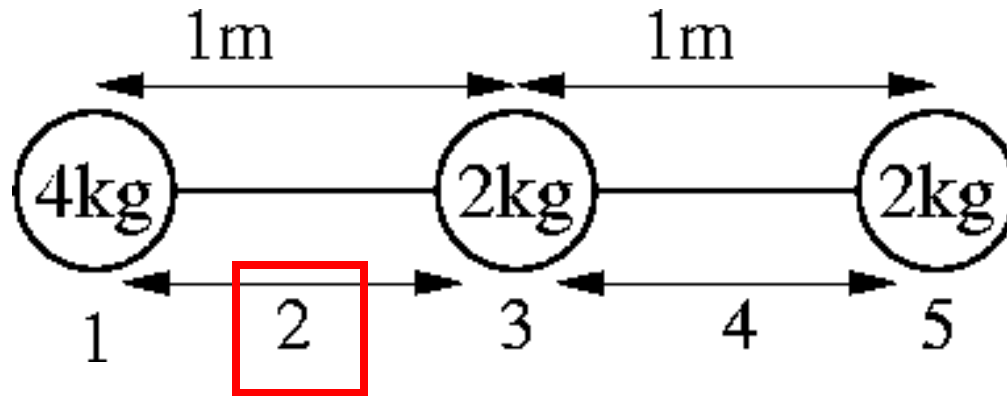
$T_R = 350N$

Center of Gravity (Center of Mass)

- Average location of the mass in a body/system.
- If extended object, can choose one place to apply Force due to Gravity (weight) and calc. Torque.
- If symmetrical, along point/line/plane of geometrical symmetry – "real center"
- If irregular, can balance or hang
- Support and Stability
- Numerically:

$$x_{cg} = \frac{M_1x_1 + M_2x_2 + M_3x_3 + \dots}{M_1 + M_2 + M_3 + \dots}$$

Roughly where is the center of gravity located? Point 1, 3, or 5 or region 2 or 4?



Solution:

$$x_{cg} = \frac{M_1x_1 + M_2x_2 + M_3x_3}{M_1 + M_2 + M_3}$$

$$x_{cg} = \frac{4 \times 0 + 2 \times 1 + 2 \times 2}{4 + 2 + 2} = \frac{6}{8} = 0.75 \text{ m}$$